PROBLEM-BASED INSTRUCTION FOUNDED ON MATH

DISCOURSE PRINCIPLES: A CURRICULUM

HANDBOOK FOR THE THIRD

GRADE CLASSROOM

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ABSTRACT

PROBLEM-BASED INSTRUCTION FOUNDED ON MATH DISCOURSE PRINCIPLES: A CURRICULUM HANDBOOK FOR THE THIRD GRADE CLASSROOM

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Problem-based instruction (PBI) is a proven pedagogical approach with promising applications in the elementary math classroom. Problem-based instruction engages students in inquiry and connects in-class learning to the world outside the classroom. When PBI is partnered with math discourse principles, students are able to engage in inquiry while expanding their thinking by participating in meaningful discussions with their peers. Although instruction that utilizes a problem-based approach has proven, positive effects on student learning, implementing such instruction is challenging for many teachers. This curriculum handbook clarifies and demystifies how PBI partnered with math discourse practices can be fully realized in third grade classrooms. The curriculum
handbook provides tools by which elementary teachers can learn to use PBI founded on math discourse principles to enhance math instruction and deliver engaging and real world math lessons. This curriculum handbook serves as a guide, a resource, and a supplemental curriculum to be used by innovative teachers interested in utilizing a problem-based approach to math instruction. The methods used to develop the lessons contained in the handbook are research based, and teachers are provided with the research behind the instructional choices described in the handbook.
CHAPTER I

INTRODUCTION

Background

Math instruction that promotes real world problem solving, mathematical reasoning, and collaboration is necessary in today’s math classrooms. The Common Core Standards are at the heart of current math reform. The Standards for Mathematical Practices (SMPs) call for students to apply mathematical thinking to real-world issues and problems. Also, the standards promote instruction that prepares students to think and reason mathematically. In addition, students will be asked to collaborate about mathematics and to critique the mathematical reasoning of their peers (www.corestandards.org). These standards will be fully adopted during the 2014-15 school year, and instructional practices that support the SMPs should become commonplace in all classrooms. This report will introduce the concept that problem-based math instruction founded on math discourse practices can address current math reform and improve math achievement.

Problem-based instruction (PBI) has been described by Hung (2006) as an instructional pedagogy that engages students in active, meaningful learning, resulting in deeper understanding and longer retention. In PBI these results are achieved by allowing students to acquire understanding of content by actively participating in the process of problem solving (Hung, 2006). Boss’s (2013) definition of PBI explains the nature of
problem solving and inquiry as it relates to the execution of PBI in the classroom. PBI is an instructional approach that utilizes complex problems that cannot be solved by using one correct strategy. Unlike traditional instruction, PBI puts the student in charge of the inquiry process by discovering answers to complex questions (Boss, 2013). Due to the fact that the SMPs will require students to make sense of problems and persevere in solving them, when PBI is applied to math instruction, it has the potential to meet the requirements of the Common Core Standards for Mathematical Practices.

Collaboration and discourse are fundamental components of PBI. Barron et al. (1998) described problem-based instruction as instruction that utilizes authentic problems that students can explore collaboratively. Although Barron et al.’s (1998) description of problem-based instruction mentions collaboration, it is important to emphasize the fact that collaboration is at the heart of problem-based learning. Papanikolaou and Boubouka (2010) studied the collaborative aspect of problem and project-based instruction. Problem based instruction and project-based instruction are similar due to the fact that in both instructional methods authentic, real world problems are the core of the learning experience. However, project-based learning uses a driving question that focuses all learning activities toward the construction of a concrete artifact that answers the original question in an authentic context (Papanikolaou & Boubouka, 2010). Papanikolaou and Boubouka’s (2010) study involved observing 82 students during a month-long study on problem- and project-based learning. They found that well-orchestrated collaboration during the problem solving and inquiry processes is essential. When reviewing the definitions of problem-based instruction provided by Barron et al. (1998), Boss (2013), and Papanikolaou and Boubouka (2010), it is apparent that PBI is an instructional
practice that utilizes authentic problem solving, inquiry, and collaboration. Because PBI provides students with opportunities to participate in real world problem solving scenarios and to collaborate about such problems, it is an instructional practice that meets the expectations of the common core standards for mathematical practices.

Problem-based instruction is a proven pedagogical approach that has positive effects on math achievement. Barron et al. (1998) carried out research on problem-based instruction at Vanderbilt University. They found that when students participated in problem-based learning experiences, they earned higher math scores when compared to the control group (Barron et al., 1998). Roti, Trahey, and Zerafa (2000) also found a connection between problem-based instruction and math achievement. When a problem-based approach to math instruction was partnered with collaboration, the percent of students who could solve math word problems at a proficient level increased from 6% to 39% (Roti et al., 2000). Therefore, practices that utilize a problem-based approach to instruction, including collaboration, can have positive effects in today’s math classrooms.

Purpose and Significance of the Project

PBI is an instructional approach that has the potential to improve math achievement, it is endorsed by reputable education associations such as the National Council of Teachers of Mathematics (NCTM) and the American Association for the Advancement of Science (AAAS), and it is a useful tool for meeting the expectations of current education reform. Therefore, why aren’t problem-based instructional practices apparent in most classrooms? This report will highlight two reasons why problem-based instructional practices are not being fully realized in today’s classrooms. Based on the
reasons presented, an argument will be made that support materials are needed for problem-based instructional to reach its potential in the elementary classroom. First, the fact that problem-based instruction requires teachers to adopt change will be presented. Secondly, the challenges of creating meaningful classroom collaboration as a necessary component of problem-based instruction will be brought to light.

Problem-Based Instruction and Instructional Change

Problem-based instruction (PBI) is a research based approach with much support. However, it requires teachers to adopt change and deal with various instructional uncertainties. Barron et al. (1998) state that the NCTM and the AAAS, support the use of PBI. Even so, Barron et al. (1998) cautioned educators by stating that there is a danger of failing to realize the potential of problem-based instruction. Barron et al.’s (1998) fear was based on the fact that PBI requires instructional changes. They worried that if educators did not embrace these changes, the benefits of PBI would not be attained. In hopes of promoting the use of PBI, Barron et al. (1998) determined and define specific design principles that should be inherent in good PBI. These design principles included frequent opportunities for formative and self-assessment and social organization that promotes collaboration and results in a sense of community (Barron et al., 1998). Melville, Kajander, Kerr, and Holm (2013) discovered that due to the uncertainties inherent in carrying out PBI, teachers struggle to properly implement such instruction. They found that PBI can be utilized properly when teachers are given opportunities to learn PBI through collaboration with other teachers. PBI can pose challenges for many
teacher, but when support is in place, instruction that is based on investigation and problem solving can be achieved.

**Problem-Based Instruction and Student Collaboration**

Collaboration and discussion are inherent qualities of PBI, and they are qualities that are at the heart of current math reform. Brendefur and Frykholm (2000) acknowledge that classroom discussions, in the form of discourse, play a large role in today’s math classroom. Discourse supports students’ mathematical development (Brendefur & Frykholm, 2000). As teachers embrace math reform, they will need to facilitate meaningful math discussions, or math discourse.

Although meaningful discussions can promote learning, discussions can be difficult to manage. Hufferd-Ackles, Fuson, and Sherin (2004) carried out research on math discourse, and they acknowledge that many teachers are not able to facilitate the type of math discussions that are aligned to the expectations of math reform. In order for high level math discussions to occur, a math discourse community must be created. The creation of a discourse community is a formidable task for most teachers and requires teachers to execute a very demanding pedagogical approach to math instruction (Hufferd-Ackles et al., 2004). Parrish (2011) explains that although the NCTM encourages math instruction that promotes math discussions, such instructional practices have not yet been embraced (Parrish, 2011). Promoting math discourse is a challenging instructional strategy, and learning to orchestrate meaningful math discourse is a challenging, yet rewarding practice. Support materials are needed for such rewards to be attained.
A curriculum handbook that provides practical information on the implementation of problem-based math instruction and math discourse could provide support to teachers so that PBI can be fully utilized in elementary classrooms. Philipp (2008) states that developing deep understanding of elementary math is more difficult than most people realize. Many teachers believe that students learn math by being shown how to solve problems (Philipp, 2008). When teachers learn to carry out problem-based math instruction founded on math discourse, they will be using an instructional approach that goes far beyond teaching problem solving rules and procedures. A curriculum handbook that provides practical examples of how to implement problem-based math instruction in the elementary classroom can help teachers move past procedural math instruction and provide instruction that addresses the Common Core Standards for Mathematical Practices.

Description of the Project

As a culminating project for the master’s program at California State University, Chico, I have created a math curriculum handbook. The math curriculum handbook is titled, *Problem-Based Math Instruction Founded on Math Discourse Principles: A Curriculum Handbook for the Third Grade Classroom*. The math curriculum handbook is an example unit that includes problem-based math lessons for third grade. The curriculum handbook explains how to carry out each problem-based math lesson while promoting meaningful math discussions that attain a high level of math discourse. This master’s project is limited in the sense that it only contains one math unit, and that unit exemplifies problem-based math lessons founded on math discourse.
principles as it pertains to multiplication. Although the curriculum handbook only provides one example unit on multiplication, the handbook is designed such that the strategies found in the handbook could be applied to other math concepts. Once teachers become familiar with the example unit, they could apply the problem-based instruction and math discourse practices described in the handbook to other math units.

The curriculum handbook contains eight third grade level lessons that show teachers how to implement problem-based math lessons founded on math discourse principles. The five basic principles of high-level tasks, questioning strategies, formative assessment, writing, and demonstrations of learning are interwoven to create a strong problem-based unit of instruction.

The Curriculum Handbook’s Five Principles

The handbook’s design is based on five principles for implementing PBI founded on math discourse principles: 1) a commitment to high-level tasks, 2) questioning techniques 3) formative assessment, 4) writing in math, and 5) demonstrations of learning in which students are allowed to showcase what they have learned. These five principles are inherent components of problem-based instruction and math discourse practices. The five principles are research based, and they provide a clear structure for teachers as they learn to properly implement PBI and math discourse.

The first principle, a commitment to high-level tasks, is used throughout the handbook. Hiebert and Wearne (1993) explain that math tasks can differ greatly in the demands they make on comprehension and strategy development. The math problems contained in the handbook allow students to grapple with math concepts and solution
methods. Also, the problems are designed to help students connect math problem solving to real world situations. The problems chosen for use in the handbook have been evaluated based on the information in the following literature review on high-level tasks.

The second principle on which the curriculum handbook is based involves using questioning strategies. Questioning techniques are used to encourage math problem solving and meaningful math discussions in both whole class and small group formats. The questioning techniques described in the literature review are explained and applied to each problem-based lesson contained in the handbook. Information from both Table 1 title, *Developing a Math Discourse Community* and Table 2 titled, *Overview of Questioning Techniques* are used in the handbook to promote high-level math discussions.

The third principle, formative assessment, is used in a variety of ways throughout the handbook. First, formative assessment is used throughout the curriculum handbook to inform instructional choices. In addition, formative assessment is used to promote student participation in the formative assessment process. Oberdorf and Taylor-Cox (2012) explain that when teachers learn to use formative assessment, they can begin to share results with students to enhance students’ ability to self-monitor progress. Papanikolaou and Boubouka (2010) explain that self-monitoring is an important aspect of both problem- and project-based instruction. They note that metacognitive skills, such as planning and self-monitoring, are necessary when engaging in open-ended activities that are inherent in problem- and project-based learning. The curriculum handbook uses formative assessment in a manner that supports the development of metacognitive skills. That is, formative assessment is used so that students learn to monitor, evaluate, and
improve their own thinking. For instance, the curriculum handbook contains two rubrics that are used as formative assessment tools. The teacher’s math discourse rubric can be used to inform instructional choices, but the student’s discourse rubric is provided as a means to help students self-evaluate their level of math discourse. Students are taught to use the student rubric to evaluate their own math discussions and those of their peers. Both rubrics are available in the resource section of the curriculum handbook. Both rubrics have been adapted from a rubric designed by professors at the University of California, Los Angeles (Park, Nava, & Applegate, 2011). The UCLA rubric was originally designed to evaluate math discourse at the high school level. Therefore, it has been simplified and adapted for use in the elementary classroom.

The fourth principle, writing in math, is based on the idea that writing can enhance math learning. The curriculum handbook contains lessons that showcase how writing can be incorporated into the math curriculum. Kostos and Shin (2010) explains that writing has a positive influence on students’ ability to communicate mathematical thinking, and it improves the use of math vocabulary. Therefore, the curriculum handbook contains lessons that utilize the think-talk-write process as described in the literature review by Huinker and Laughlin (1996). However, some writing lessons found in the handbook include an opportunity for students to reflect on their writing and the writing of their peers. This technique that partners writing with reflection is discussed by Lynch and Bolyard (2012) in the literature review. Therefore, a few writing lessons in the curriculum handbook depict a think-talk-write-reflect process.

The final principle, demonstration of learning, provides students with the opportunity to showcase what they have learned. Boss (2013) explains that in problem-
based learning, problems are complex and ask students to find their own solutions. Due to the nature of these problems, students should be given an opportunity to demonstrate what they have learned. Therefore, at the end of the unit contained in the curriculum handbook, students are given an opportunity to create a project that exemplifies what they have learned about solving real world math problems.

Implementation of the Five Principles

The first principle of using high-level math tasks, or complex math problems, occurs throughout the handbook in different forms. The handbook begins with an open-ended question that engages students in the inquiry process. The opening question is, how is multiplication used in the world around us? This question not only guides each lesson, but it helps students make connections between what is learned in the classroom and how it is applied in a real world setting. Also, each lesson is designed to “problematize” the skills to be learned (Hiebert et al., 1996). Instead of being taught multiplication concepts through direct instruction, students are presented with real world math problems that help them gain an understanding of multiplication concepts. For example, in lesson 1 students are introduced to multiplication concepts by solving and discussing a real world math problem about visiting a restaurant. Throughout the handbook, students are encouraged to grapple with math problems and determine their own solution method. After developing their own solutions, students engage in whole group or small group discussions. The math discussions allow for various understandings to be shared and encourage different levels of understanding to be defined, explored, and built upon. Cengiz, Kline, and Grant (2011) studied the connection between collaboration and mathematical thinking. They found a connection between math discussions and mathematical understanding.
Therefore, the curriculum handbook is designed so that, through discussion, students are given opportunities to grapple with, learn from, and become successful at solving high-level math tasks. By solving and discussing high-level math tasks, students are given opportunities to develop their own understanding of multiplication concepts.

The second principle addresses how questioning techniques are utilized throughout the curriculum handbook to promote problem solving and math discourse, and the third principle involves the use of formative assessment. Questioning techniques and formative assessment are interwoven throughout the curriculum handbook. Primarily, questioning techniques are taught and practiced during whole class and small group discussions. The curriculum handbook provides specific question and statement prompts for each whole class and small group discussion. The question and statement prompts encourage teachers and students to use a variety of questioning techniques that enhance problem solving and math discourse. For example, a few question prompts are designed to support teachers as they learn to facilitate math discourse. In addition, some question and statement prompts are designed to encourage students to explain and reflect upon how they are solving math problems, share mathematical reasoning and evidence, and make connections to previously learned material or alternative methods. Although questioning techniques are primarily taught and practiced as a means to improve problem solving and math discourse, a few lessons are provided so that students can learn how to evaluate and improve their questioning techniques and math discussions. As discussed earlier, the student’s discourse rubric that has been adapted from the UCLA rubric (Park et al., 2011) is used by students to evaluate math discussions. The goal is to learn to evaluate math discussions so the information gained can be used to improve the level of
discussion. The handbook provides lesson plans that allow students to first solve high-level math tasks then use question and statement prompts to discuss the high-level tasks with a partner while recording the math discussion. Recorded math discussions are critiqued by the students independently and by the class as a whole. The student’s math discourse rubric is used to guide both the independent and whole class critique. The goal of these recorded conversations is to support students as they develop the ability to engage in math discourse that supports their mathematical thinking and understanding.

The fourth principle involves the use of writing in the math classroom. In the curriculum handbook, writing is used to enhance vocabulary development and mathematical thinking. Two lessons on the think-pair-write-reflect are provided. Students are given a chance to think about one of the high-level math tasks presented in the handbook. They are first given time to think about what was difficult about the math problem, what helped them solve the problem, and what they learned. Then they share their thoughts with a partner. Finally, they write down a response. Later, the students have a chance to share their written statement with their classmates. This provides an opportunity for students to engage in a reflective discussion comparing the math challenges they faced and what was learned through the problem-solving process.

The final principle used in the handbook involves allowing students to showcase what they have learned. Therefore, students are able to create a project to answer the original questions, *how is multiplication used in the world around us?* Students are given a variety of options that provide them with the opportunity to demonstrate what they have learned. The options include creating a Google presentation, creating a poster, designing a diorama, or developing a play or a skit. These projects are
shared during an exhibition for an audience. The projects exist as a celebration of what students have learned and created. Although the final project is not assessed, assessment does occur throughout the curriculum handbook, and the pre- and post-assessment are used as evaluative tools. The pre- and post-assessments are designed so they can be used as formative or summative assessments. At the end of the unit contained in the curriculum handbook the assessments can be used to determine if students gained both skill and confidence when solving high-level math problems. Teachers can use the information gained from the pre- and post-assessment to evaluate students’ current ability and confidence level or students’ improvement over the course of the problem-based lessons.

Limitations of the Project

This curriculum handbook is one way of providing teachers with the knowledge of how to successfully implement problem-based math lessons founded on math discourse principles. When teachers learn to use this style of instruction, they are better prepared to meet the expectations of current math reform and the Standards for Mathematical Practices. However, training and professional development that supports teachers as they learn to implement the strategies as described in the handbook could strengthen and improve the use of such strategies. Brendefur, Thiede, Strother, Bunning, & Peck (2013) explain that teachers lack exposure to the necessary, rich math experiences that could help them develop an instructional base that adheres to the recent demands of education reform. Rigorous professional development is needed (Brendefur et al., 2013). Therefore, the implementation of the lessons and strategies contained in the
handbook is the first step toward the attainment of problem-based teaching practices. However, the handbook should be followed by training and professional development opportunities that supports teachers as they learn to implement the instructional practices contained in the handbook.

Definition of Terms

**Key Terms**: problem-based instruction, project-based learning, math discourse, high-level tasks, formative assessment, metacognition.

**Formative Assessment**

An assessment system in which feedback, student involvement, and a clear progression of learning are key components (Marzano, 2010).

**High Level Tasks**

Math tasks that allow students opportunities to do math. Math tasks that encourage high-level mathematical thinking and reasoning (Henningsen & Stein, 1997).

**Math Discourse**

Math discussions in which students make conjectures, talk, question, agree or disagree about a problem in order to discover important math concepts (Stein, 2007).

**Metacognition**

Knowledge of one’s own cognitive processes (Schoenfeld, 1992)

**Problem-Based Instruction (PBI)**

(1) Instruction that provides opportunities for students to be challenged to solve problems or do simulations that mimic real life. The problems are often defined in advance by the instructor, and they tend to be complex, even messy, and cannot be solved
by one “right” or easy-to-find answer (Boss, 2013); (2) problem-based instruction is an effective instructional pedagogy that inherently engages students in active, meaningful learning, resulting in deeper understanding and longer retention (Hung, 2006).

Project – Based Learning (PBL)

Learning experiences in which a problem or question drives learning activities toward the construction of a concrete artifact in an authentic context (Papanikolaou & Boubouka, 2010).
CHAPTER II

LITERATURE REVIEW

Introduction to the Research

Current education reform, including math reform, calls for instructional practices that promote real-world problem solving and mathematical reasoning skills. Melville et al. (2013) state that in current education reform there is a renewed enthusiasm for instructional approaches that connect understanding to a meaningful context. This style of instruction allows students to connect what they learn in an educational setting to real world experiences. Parrish (2011) mirrors this view when she explains that the new standards for math practice (SMPs) call for math instruction that is grounded in conceptual understanding and mathematical reasoning (Parrish, 2011). By learning conceptually and through reasoning, students will be better prepared for real world problem solving. Boss (2013) supports the concept that reasoning and analyzing prepare students for authentic learning experiences when she explains that problem based learning allows student to investigate and analyze authentic problems while creating solutions that are presented to authentic audiences. She goes on to explain that problem-based learning is a proven and practical teaching strategy that addresses 21st century education reform (Boss, 2013). Education reform will require teachers to create classrooms that promote real world problem solving and mathematical reasoning. Many researchers acknowledge the difficulty in making the instructional changes necessary
for education reform to be fully realized. Barak and Mesika (2007) explain that critical thinking and problem-solving are overlooked in traditional classrooms, and teachers are not adequately trained to promote critical and creative thinking. Melville et al. (2013) acknowledge that the new emphasis on problem solving and critical thinking has posed very real challenges for many teachers who are not accustomed to teaching in this manner. Clearly, education reform is imminent, and teachers should learn to promote critical thinking and reasoning while learning meaningful content.

Learning that occurs as a result of examining and solving a problem, or problem-based learning (PBL), could be the perfect conduit for providing instruction that meets the demands of education reform. Estes, Mintz, and Gunter (2011) explain that when teachers utilize PBL, the education experience becomes better matched to real-world and authentic learning. The problem-based approach takes the emphasis off finding the answer, and it focuses learning on the learner’s own inquiry. When implementing this technique, learning is no longer about filling in the blank with the correct answer. It is about critically thinking about and studying different aspects of an interesting problem (Estes et al., 2011). Project-based learning is one approach that allows students to learn through a problem-based approach. Papanikolaou and Boubouka’s (2010) describe project-based learning as learning that uses a problem, or an open-ended question to guide the learning experience. Students who learn through a project-based learning experience, learn by solving problems and engaging in inquiry. In project-based learning, as students solving problems, they construct and analyze information, and they create products that reflect their understanding (Papanikolaou & Boubouka, 2010). Melville et al., (2013) carried out a research study that supported the idea that project or problem-
based learning can prepare students for real world problem solving. They found that structuring learning within a problem situation can make schooling more useful and readily applied to the world (Melville et al., 2013). Project or problem-based instruction is a promising approach that meets the requirements of education reform.

This literature review explores problem-based instructional techniques as they apply to math instruction. The positive aspects of teaching math through a problem-based approach will be brought to light. The claim that problem-based instruction (PBI) meets the requirements of education reform in the field of mathematics will be supported. It will be determined that a problem-based approach to math instruction and math discourse are natural partners, and both PBI and math discourse support students as they develop a true and deep understanding of mathematics. This study reviews how using a problem-based approach to math instruction that is founded on math discourse principles allows students to develop and apply math understanding while developing reasoning and critical thinking skills.

The Benefits of a Problem-Based Approach to Math Instruction

There are many positive outcomes to implementing a problem-based approach to math instruction. Melville et al. (2013) stated that students learn mathematics most effectively when they are given opportunities to explore math concepts through the problem solving process. Therefore, it is important to take a deeper look into the positive effects of a problem-based approach to math instruction. First, the possibility that a problem-based approach to math instruction can help students become creative and critical thinkers will be explored. Secondly, the idea that a problem-based approach can
promote deeper understanding of math topics will be reviewed. Finally, the view that problem-based approach to math instruction prepares students for real world problem solving outside the classroom will be examined.

A problem-based approach to math instruction can help students develop creative and critical thinking skills. Brendefur et al. (2013) discussed the importance of adopting a problem-based approach to math instruction in their recent study on mathematical thinking. Their study focused on teacher training. The research showed that when teachers are trained to adopt a problem-based approach to math instruction, students learn to critically examine their own and other’s math strategies. When solving novel problems and examining multiple approaches, students can become mathematical thinkers, and these critical thinking skills can be applied inside and outside the classroom (Brendefur et al., 2013). Barron et al. (1998) found similar results in their study. They found that when problem-based learning is organized and scaffolded, students pay attention to important considerations that are embedded in the problem. Students develop critical thinking skills such as they are able to think through alternatives or view problems from various perspectives. In doing so, students can develop well-formed opinions. Also, students are able to reflect on their own thinking in relation to a larger community (Barron et al., 1998). This type of reflecting thinking is inherent in a well-planned, problem-based approach to instruction, and it can support the development of critical thinking skills.

A problem-based approach to learning can promote deeper understanding of math topics. Hiebert et al. (1996) describe a problem-based approach to math instruction when they discuss “problematizing math.” When math is “problematized,” students learn
and acquire mathematical understanding through solving problems. In their research, Hiebert et al., (1996) provide an example of a problem-based approach to instruction when they illustrate how to “problematize” math in a second grade classroom. In the example provided, students learn subtraction by solving a meaningful math word problem. This approach is in opposition to the idea that math computation and procedures should be taught and acquired first then applied to math word problems. When allowing students to grapple with and search for a solution to a meaningful math problem, students develop a true and deep understanding of math concepts (Hiebert et al., 1996). Roti et al.’s (2000) research supports this concept. They carried out an action research project in which open-ended and engaging math problems were used to improve math problem solving. They found that when a problem-based approach to math instruction is utilized, the percentage of students who could solve math word problems increased (Roti et al., 2000). Clearly, when teachers utilize a problem-based approach to math instruction, students can develop a deeper understanding of math topics.

A problem-based approach to math instruction can prepare students for real world learning. Hutchinson (2010) uses real world problems to enhance math instruction. By choosing math problems, or questions, that are embedded in authentic situations, math problems can have more relevance for the students, and students are allowed to see math through the lens of real world experiences (Hutchinson, 2010). Barron et al., (1998) supports this use of real world problem solving by explaining that problem or project-based learning makes schooling more useful and readily applied to the world. Authentic problem solving has been a driving concept behind education reform for many years.
(Barron et al., 1998). Clearly, problem-based learning can prepare students for problem solving inside and outside the classroom.

How to Implement a Problem-Based Approach to Math Instruction

When teachers implement a problem-based approach to math instruction, there are a few important teaching strategies to consider. First, the nature of the math problem is key. Not all problems achieve the goals of problem-based instruction. Secondly, a problem-based approach to math instruction should be partnered with math discourse practices.

The Nature of the Problem is Key

Engaging math problems are at the heart of problem-based learning. Both Hiebert et al. (1996) and Smith and Stein (2011) believe that math lessons should begin with a quality problem that allow students to grapple with the problem. Singer (2007) utilizes this problem-based approach to math instruction with his own students. He begins his math lessons with a problem that has no clear right or wrong answer. It is not the kind of problem that students can solve alone at their desks. Singer (2007) uses math problems that force students to use their minds to grapple with solution pathways. This is the kind of problem that lies directly inside of psychologist Lev Vygotsky’s zone of proximal development (Singer, 2007). Smith and Stein (2011) support this problem-based approach to math instruction when they explain the importance of the task selection process. Task selection refers to the act of finding quality problems and anticipating how such problems can enhance learning and promote meaningful math discussions. When selecting a task, teachers should choose math problems that have the potential to enhance
students’ thinking (Smith & Stein, 2011). It is clear that selecting math problems that engage students in high-level discussions and high level thinking is an important part of problem-based math instruction.

**Partnering Problem-Based Instruction with Math Discourse**

Once a quality math problem has been selected, math discourse can be interwoven into the problem solving process. Partnering problem-based instruction with math discourse can have a few positive effects on students’ mathematical understanding. First, engaging math problems that promote math discussions allow students to evaluate their own mathematical thinking. Secondly, partnering math problem solving and math discourse can help students develop a deeper understanding of math concepts.

Papanikolaou and Boubouka (2012) found when teaching through a problem or project-based approach, collaboration can improve students’ progress because meaningful discussions help them evaluate and organize their own thinking as they grapple with a problem. However, students’ progress depends on the depth and intensity of discussions that occur between students as they work through a problem-solution cycle (Papanikolaou & Boubouka, 2012). When Papanikolaou and Boubouka’s (2012) research is applied to math, it is clear that math problems must be engaging and rich math discussions should be encouraged. Chapin et al. (2009) support this view when they state that a problem in math must be puzzling and intriguing. Math problems should not have apparent solutions. Such problems provide opportunities for students to engage in math discussions. When students engage in math discussions, their thinking is revealed. Teachers can use this information to make instructional decisions that enhance students’
understanding of math concepts (Chapin et al., 2009). When engaging math problems are complemented by meaningful math discussions, or math discourse, problem solving is scaffolded in a manner that promotes mathematical understanding.

Research supports the notion that a problem-based approach to instruction complemented by math discourse has positive outcomes on student achievement. Murray, Olivier, and Human’s (1998) research showed that when problem-centered, or problem-based, learning is complemented by high quality student interactions and discussions, the combination can have a positive influence on the students’ mathematical achievement. In Murray et al.’s (1998) research, students who learned through a problem-based approach that included quality math discussions, did well on math assessments when compared to students who experienced traditional teaching approaches. More recently Roti et al.’s (2000) conducted an action research project that combined a problem-based approach to math problem solving with math discourse. They found that math discussions that occur during cooperative group problem solving and whole group discussion produce higher student achievement in math problem solving. The math discussions cultivated higher level thinking skills that allow students to become strong math problem solvers (Roti et al., 2000). Schmoker (2006) agrees with the fact that math discourse can improve higher level thinking skills when he explains that quality discussions provide an opportunity for students to become critical thinkers. Research shows that a problem-based approach to math instruction should be closely partnered with meaningful math discussions, or math discourse, in order to yield better problem solving and critical thinking skills.

Examples: Partnering Problem-Based Instruction with Math Discourse. To clarify how a problem-based approach to learning and math discourse work together to
promote problem solving skills, it is worth reviewing a few examples. A deeper look into Hiebert and his colleague’s classroom example can shed light on how meaningful problems can be complemented by math discourse. Hiebert et al.’s (1996) research highlighted a second grade classroom in which the students were engaged in problem solving in a manner that allowed them to generate their own procedures for subtracting double digit numbers. A quality problem was presented and students were given the opportunity to develop their own methods for solving the problem. This problem was, *find the difference in the height of two children who are 62 inches tall and 37 inches tall.* Once the students wrestled with the problem, the teacher engage the students in a discussion about their approaches. Students shared results of their inquiries and explained and justified their results (Hiebert et al., 1996). When evaluating this lesson, it is clear that the teacher not only selected a high-level problem that promoted critical thinking, but the teacher also encouraged meaning math discussions. Throughout the problem-based lesson, the teacher encourages meaningful discussions by integrating math discourse into the problem-based lesson. The teacher carefully chose students to share their methodologies, and she made connections between the methods. She questioned students in a manner that prompted them to clarify and extend their thinking. The students explained their own thinking, and they discussed similarities and differences between their approach and those of their classmates (Hiebert et al., 1996). Hiebert et al.’s (1996) lesson provides an example of how quality problems inherent in the problem-based approach to mathematics instruction can be connected to math discourse principles to promote students’ problem solving skills.
The concept of combining quality problems with math discourse was more recently studied by Mueller and Maher (2009). They carried out a research study during an after school math program. A problem-based approach complemented by math discourse was used to teach math problem solving. It was found that students became accustomed to the practice of convincing others of the validity of their ideas such that “proof-making” became a fundamental part of problem solving tasks. The nature of the problem and the manner in which the students were engaged in thinking and processing with others while problem solving played a large role in the development of problem solving skills (Mueller & Maher, 2009).

The *Navigating through Problem Solving and Reasoning Grade 3* supplemental curriculum (Yeatts, Battista, Mayberry, Thompson, and Zawojewski, 2004) is an investigations-based curriculum. It provides an example of how problem solving and math discourse can be interwoven to create engaging lessons. The authors of this book (Yeatts et al., 2004) provide four investigation-based lessons. Each lesson intricately links problem solving, reasoning, and discussion to promote students’ mathematical understanding. When reviewing the lessons, it became clear that the authors linked problem solving to instruction practices that promote discussion. This supplemental curriculum is one example of how problem solving and math discourse can be incorporated into math instruction.

Sorting and discussing word problems is another approach that has been used to promote students’ problem solving skills and math discussion abilities. Fuchs et al. (2004) carried out a research project on problem solving and the development of math schemas. Fuchs et al. (2004) found that students can learn to see math patterns and create
schemas by sorting and discussing math problems. Structuring lessons to allow students to seek and notice patterns can help students make connections between math problems. Sorting problems can help students begin to make connections between concepts as they are presented in math problems. This research found that sorting problems can help students develop math schemas that provide support for future problem solving (Fuchs et al., 2004).

The paired sorting activities applied by Fuchs et al. (2004) also helped students evaluate math problems on a deeper level. Roti et al. (2000) explain that students often have difficulty understanding the relationship between words and math symbols in complex math problems. Students often rely on insignificant aspects of the math problem that lead to incorrect solutions. In the study completed by Fuchs et al. (2004) students were able to look past superficial cues and develop a deeper understanding of what each math problem was asking. Although this research merely touched on student discussions as a component to the sorting process, it is possible that the discourse that occurred had an impact on the development of math schemas. Roti et al. (2000) found that when students participate in discourse with others in order to identify the relevant information in math problems, they become better independent problem solvers. Therefore, it is possible that the practice of sorting and discussing word problems could have positive effects on students’ problem solving skills.

An Introduction to Math Discourse

It is now clear that a problem-based approach to math instruction and math discourse are natural partners. Problem-based teaching and math discourse can help
students develop essential problem solving skills, enhance reasoning and critical thinking skills, and acquire deep mathematical understanding. Therefore, it important to take a thorough look at math discourse.

What is Math Discourse?

The NCTM describes the ability of students to construct viable arguments and critique the arguments of others as the useful term, mathematical discourse. The organization further describes mathematical discourse as ways of representing, talking, agreeing, and disagreeing about a math task or topic (de Garcia, 2011). Brendefur and Frykholm (2000) expand on the concept of mathematical discourse by explaining that classroom discourse refers to math discussions that support students’ conceptual development. Both verbal and written communication allow students to engage in rich discussions that cultivate comprehensive mathematical understanding (Brendefur & Frykholm, 2000).

The Common Core Standards (www.corestandards.org) contain the standards for mathematical practice (SMPs) that are referred to as the “backbone” of the Common Core Standards. The SMPs define math discourse and describe its function in the classroom. The third SMP is titled Construct Viable Arguments and Critique the Reasoning of Others. This SMP explains that mathematically proficient students should be able to make conjectures and build a logical progression of statements to explore the truth of their conjectures (www.corestandards.org). Also, students should recognize multiple methodologies and sort them into examples and non-examples. Students should justify their conclusions and communicate their conclusions to others. In addition, mathematically proficient students should be able to respond to the mathematical
arguments of their peers (www.corestandards.org). When evaluating the definition of mathematical discourse, it becomes clear that math discourse involves crafting and critiquing mathematical discussions, claims, and methods. Also, math discourse is fundamental to the implementation of math reform.

**Why Implement Math Discourse in the Classroom?**

Properly implemented math discourse can have multiple benefits. First, math discourse provides useful information to teachers so that it can be used to inform and differentiate instruction (Small, 2012). Secondly, math discourse can be used to foster a social environment, or community, that encourages positive interaction and learning (Chapin et al., 2009). Thirdly, math discourse can provide an opportunity for students to develop reasoning skills and a deeper understanding of math concepts (Brendefur & Frykholm, 2000). Lastly, students themselves can become aware of their level of understanding, that is, discourse allows students to assess their own thinking and the thinking of others (Chapin et al., 2009). In short, math discussions give students the opportunity to develop, deepen, and evaluate, their mathematical understanding and reasoning skills in a safe environment.

Differentiated instruction is a cornerstone of math discourse. Marian Small (2012) notes that education systems expect teachers to be aware of individual student needs and to address these needs through well-planned, differentiated instruction. She explains that questioning techniques that are an inherent part of a discourse community allow diverse students to access math lessons at different levels. Chapin et al. (2009) support this claim by stating that students can learn at different paces and in different...
ways when teachers apply math discourse principles to the math curriculum. In addition, Smith and Stein (2011) discuss how teachers can differentiate instruction when implementing math discourse practices. They explain that in a discourse community, teachers monitor students as they discuss high-level tasks. Monitoring student work provides teachers with information about students’ mathematical thinking (Smith & Stein, 2011). In summary, teachers can use information obtained from math discourse to modify lessons and meet the needs of students at all levels of understanding.

Chapin et al. (2009) present the notion that mathematical discourse can have positive social outcomes. Well-orchestrated discourse has the ability to bring students together to form a learning community. Discourse provides students with the opportunity to learn active listening skills, to be respectful of classmates, and to be patient with others as they develop mathematical understanding (Chapin et al., 2009). In a discourse community, individuals work together to assist one another’s learning experience (Hufferd-Ackles et al., 2004). Clearly, math discourse has the propensity to build a strong community of students who learn from and with one another.

When students engage in math discourse, their conversations can deepen their mathematical understanding. Brendefur and Frykholm (2000) carried out a research study during which various types of math discourse were evaluated. They found that through reflective discussions, students can begin to incorporate ideas presented by others into their own, and they can build upon math conversations in ways that lead to deeper understandings of math concepts. Smith and Stein (2011) emulate these same sentiments when they explain that math discussions that allow students to compare their own reasoning with that of others, provide opportunities for students to enhance their
understanding of mathematics. There is a noticeable connection between math discussions and students’ ability to go beyond initial, shallow math understandings to deeper more complete understanding (Smith & Stein, 2011).

Math conversations also give students practice in reflecting on their own thinking processes. Chapin et al. (2009) explain that math conversations allow students to develop as expert thinkers. Cengiz et al.’s (2011) research supports the notion that math discourse can enhance students’ math understanding and critical thinking skills. They reviewed teachers who used math discourse to elicit and extend student thinking. When teachers extended students’ thinking through math discussions, students were able to move beyond their initial mathematical observations and develop deeper understanding of math concepts (Cengiz et al., 2011). Also, the use of mathematical discourse creates a class climate that promotes mathematical reasoning. Mueller and Maher’s (2009) research showed that in settings where learners were encouraged to use each other as resources, students learn to extend initial math ideas to create deeper forms of reasoning. Chapin et al. (2009) support this notion when they state, “discussions can play a critical part in helping students improve their ability to reason logically” (p. 7). In summary, math discourse has positive effects on both mathematical understanding, mathematical reasoning skills, and reflective thinking. Therefore, it is worth examining how to implement math discourse in the classroom.

Math Discourse in the Classroom

Facilitating math discourse in the classroom, is a challenging, but worthwhile undertaking. Brendefur et al. (2013) and Brendefur and Frykholm (2000) studied teacher
training specifically focusing on teachers’ ability to teach math in a manner that engages students in communicating and reasoning collaboratively about math. Brendefur et al. (2013) found that teachers enter the teaching field unprepared to teach math in the manner that utilizes math discourse to improve students’ mathematical understanding and problem solving abilities (Brendefur et al., 2013). However, many math educators and math researchers have developed proven teaching practices that support teachers as they learn to implement math discourse in the classroom. At UCLA, educators have created a rubric that is used to evaluate student teachers as they learn to implement math lessons. Math discourse is at the heart of this rubric, and the rubric clearly states both teaching strategies and student behaviors that should be apparent in classrooms when math discourse is properly utilized (Park et al., 2011). Exposure to these math discourse principles complimented by examples of and practice with their implementation will improve teachers’ ability to fully implement meaningful math discourse. Through the review of literature both challenges that teachers faces along with strategies to address these challenges will be exposed.

Implementing Math Discourse a Challenge for Many Teachers

Many teachers do not know where to begin when creating a classroom environment in which discourse practices are implemented at the level as described by advocates for math reform (Hufferd-Ackles et al., 2004). Hufferd-Ackles et al. (2004) explain that in order for students to engage in high quality math discussions, teachers must change the nature of their teaching, yet they acknowledge that implementing quality mathematical discourse is a demanding pedagogical approach. The question arises, why
is math discourse so difficult to implement? Smith and Stein (2011) attempt to answer this question by comparing traditional classrooms to classrooms that utilize math discourse. They describe traditional math classrooms as places where a math problem is given and where students carry out a method that has been previously taught. Math discourse drastically changes this style of teaching. When students engage in a math discourse there is not one way or one method for solving a math problem. In a discourse community, teachers monitor students as they discuss high-level tasks, and each contribution to the discussion is valued. Communication and collaboration are essential elements of math discourse, and communication is used to enhance math understanding so that all students can develop as mathematical thinkers (Smith & Stein, 2011). Clearly, learning to utilize math discourse is a challenging, yet worthy, undertaking.

Implementing Math Discourse Consulting Research Based Practices

To provide a path for teachers as they begin to orchestrate classroom discussions, Chapin et al. (2009) describe five “talk moves” teachers can use to encourage math discourse. Revoicing is one talk move that can be used to clarify math discussions. When revoicing, teachers restate a student’s comments and ask for clarification. Repeating is another talk move that involves asking a student to restate someone else’s reasoning. This talk moves allows for processing time, and it provides students access to one another’s thinking. The talk move, reasoning, asks students to apply their own reasoning to someone else’s math statement. The teacher asks a student to agree or disagree with a claim made by another student and to provide reasons for their stance. The talk move called adding on involves asking students to provide additional
input to previous statements. Wait time is another talk move. Although wait time does not involve speech, it can enhance math discourse (Chapin et al., 2009). De Garcia (2011) briefly describes Chapin and her colleagues’ five talk moves in her paper titled, How to Get Students Talking. She explains that teachers can use the five talk moves to engage students in discussions, orchestrate the discussions, develop the art of questioning, and facilitate a supportive environment.

Smith and Stein (2011) also offer five practices that teachers can use when implementing mathematical discourse. The five practices are anticipating, monitoring, selecting, sequencing, and connecting. Anticipating involves choosing a math task, or problem, and envisioning how students might mathematically approach the math problem. Monitoring involves closely monitoring what students do as they work. While monitoring, teachers ask a variety of questions. The questions make students thinking visible, help students clarify their own thinking, and press students to consider important aspects of the math problem. Selecting refers to teachers choosing student work to share or discuss as a whole group. When selecting work, teachers should consider choosing work that depicts multiple strategies, contributes to the learning goal, and enhances the math discussion. Sequencing is part of the selection process. Teachers must select student work and determine the sequence in which it will be presented. Teachers must sequence work so that it achieves the optimum level of discussion and mathematical understanding. Connecting is the final step, and it involves teachers helping students make connections between various methods and to the key mathematical idea of the lesson. By implementing these five discourse practices, the teacher attempts to help the students see mathematical patterns and create a deeper understanding of math concepts (Smith &
Stein, 2011). Each of the five practices builds on the other and requires thought and planning on the part of the teacher.

Hufferd-Ackles et al. (2004) carried out a yearlong study during which four components of mathematical discourse were discovered. The four components are questioning, explaining mathematical thinking, altering the source of mathematical ideas, and accepting responsibility for learning. These discourse components can be applied in the classroom to develop different levels of mathematical conversations. The continuum by which these practices are developed can be found in Table 1.

Table 1

*Developing a Math Discourse Community*

<table>
<thead>
<tr>
<th>Questioning</th>
<th>Explaining Mathematical Thinking</th>
<th>Source of Math Ideas</th>
<th>Responsibility for Learning</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level 0-3 A shift from teacher as questioner to</td>
<td>Level 0-3 Students increasingly</td>
<td>Level 0-3 There is a</td>
<td>Level 0-3 Students increasing</td>
</tr>
<tr>
<td>students and teachers as questioners</td>
<td>explain, articulate, and justify</td>
<td>shift from the teacher</td>
<td>responsibility for learning.</td>
</tr>
<tr>
<td></td>
<td>their mathematical thinking</td>
<td>as the source of</td>
<td>They evaluate their own</td>
</tr>
<tr>
<td></td>
<td></td>
<td>math ideas to</td>
<td>thinking and the thinking</td>
</tr>
<tr>
<td></td>
<td></td>
<td>students’ ideas</td>
<td>of others. Math sense is the</td>
</tr>
<tr>
<td></td>
<td></td>
<td>having an influence</td>
<td>criterion for evaluation</td>
</tr>
<tr>
<td></td>
<td></td>
<td>on the direction of</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>the lesson</td>
<td></td>
</tr>
</tbody>
</table>

As can be seen, discourse principles, practices, and components can be termed and framed differently in various research reports. However, there are some essential practices that warrant further investigation. First, it is important to look into classroom climate including the transfer of control. Secondly, the importance of questioning and how it can be used to propel the math discussion and promote mathematical thinking is worth further review. Thirdly, the practice of selection and how this applies to high-level tasks as well as error analysis is an inherently important principle of math discourse. Fourthly, exploring both verbal and non-verbal formats by which students can explain their mathematical thinking will shed a light on how to enhance student’s thinking. Finally, the concept of using math discourse to inform instruction can further improve teaching practices that promote math problem solving and mathematical thinking. By investigating these five discourse principles, the path to implementing engaging math discussions will become less ambiguous.

**Math Discourse and Class Climate.** A classroom that incorporates discourse principles can be an engaging and motivating environment for students. Kazemi and Stipek (1998) explain that students can become motivated when educators are teaching conceptually with the goal of promoting deep understanding. Through research, they discovered a few instructional practices that promote student motivation. These practices include asking students to seek alternative solutions, providing opportunities for students to engage in mathematical conversations, and teaching for understanding. These three practices are characteristics of a math discourse community, and they can be found in Smith and Stein’s (2011) definitions of the selecting, sequencing, and connecting practices of math discourse. Bostic and Jacobbe’s (2010) research supports the notion
that a classroom culture that fosters math discourse promotes student engagement and motivation. They implemented a problem-based discourse curriculum and found that a classroom culture that promotes problem-solving discourse also provides students with the opportunity to become highly involved in the learning experience (Bostic & Jacobbe, 2010). It is clear that well-planned math discussions can enhance student engagement and motivation.

When teachers create a discourse community, there is a switch that occurs in both the teaching structure and authority structure of the classroom. Simon and Blume (1996) note that when teachers begin to encourage students to share their mathematical evidence and justifications, students become more accountable for classroom learning and the authority switches away from the teacher. The teacher is no longer the sole authority for the right answers. This switch in classroom structure allows students to view math as created by a community of learners (Simon & Blume, 1996). Therefore, classroom discourse can change the dynamic of a classroom, and it can provide opportunities for students to take an active role in learning.

Although it is clear that mathematical discourse has the power to create a positive classroom climate and enhance learning, teachers might ponder the steps involved in creating such a classroom culture. Hufferd-Ackles et al. (2004) developed a framework that can be used as a guide as teachers begin to implement math discourse and create a classroom community of empowered students. Hufferd-Ackles et al. (2004) describe such a classroom as a discourse community. Their framework describes three developmental levels of mathematical discourse and defines instructional practices that can help students move to a level 3 discourse community. Teachers can achieve a level 3
discourse community by focusing on four instructional practices that include questioning, explaining mathematical thinking, the source of ideas, and responsibility for learning. An overview of the Hufferd-Ackles et al.’s (2004) framework can be found in Table 1. As the community of students grows from level 0 to level 3, teachers should be prepared to take less of a commanding role as students begin to take charge of creating their own mathematical thinking and understanding.

**Questioning Skills and Math Discourse.** To successfully create a discourse community, teachers must change typical questioning patterns. Although Smith and Stein (2011) do not use the term questioning as a key principle for developing math discourse, they do describe questioning as a key component of the anticipating, monitoring, and connecting practices of math discourse. To illustrate the necessary change in questioning patterns, Smith and Stein (2011) explore questioning techniques. IRE is a questioning pattern that refers to initiating, responding, and evaluating. This style of questioning involves the teacher initiating a question, the student responding, and the teacher evaluating a short response by the student. When implementing math discourse, questioning takes on a different form. During math discourse questions should be used to probe student thinking, explore mathematical relationships, and expose mathematical ideas (Smith & Stein, 2011). Questioning techniques that are inherent in math discourse differ greatly from traditional questioning techniques.

Small et al. (2004), describe questioning as a key component of a discourse community. They explain that for teachers to create a math discourse community, they must ask a variety of well-planned questions. Three questioning styles are described. The first questioning style involves helping students develop metacognition skills by
prompting students to reflect on their mathematical thinking. Another questioning style focuses on helping students make connections between math concepts and strategies. The third style of questioning provides opportunities for students to develop their reasoning skills by encouraging them to make predictions, acknowledge generalizations, and recognize inconsistencies. Learning how and when to implement the different levels of questioning is an important teaching practice in a discourse community.

Small (2012) explains that good questioning can provide an opportunity for different students to access a complex math problem at diverse, yet appropriate, zones of proximal development. A question can provide an opportunity for students to solve a problem and access mathematical understanding at different levels (Small, 2012). Therefore, good questioning can become a catalyst for creating a classroom community in which all students are part of the math conversation.

It is clear that questioning is a complex, but integral principle of mathematical discourse. An overview of various questioning techniques can be found in Table 2.

Selecting and Maintaining High Level Math Tasks. It has already been established that a problem-based approach to math instruction and math discourse are natural partners, and they should be combined to create engaging math lessons that promote deep math understanding. However, it is important to take a thorough look into the nature of complex math problems that are inherent in a problem-based approach to math instruction. Some researchers refer to such math problems as high-level math tasks. Stein, Smith, Henningsen, and Silver (2009) explain that the math task or problem chosen by the teacher can affect the students’ level of engagement. Although high-level math tasks can enhance student engagement and mathematical understanding, they can cause
<table>
<thead>
<tr>
<th>Question Style</th>
<th>Possible Questions</th>
<th>Goals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reflection</td>
<td>What did you do first? Why? Can you tell me what you did and why? What were you thinking when you…? Why did you change your approach? How did you know to …? What was the most helpful clue? Why? Can you say more about your thinking? What hint would you give to a friend who was stuck?</td>
<td>Promotes student reflection Improves metacognition Helps students understand what they are doing and why.</td>
</tr>
<tr>
<td>Connection</td>
<td>Does this problem remind you of another? Is there another way to solve this problem? Can you create a problem that would be solved the same way? Can you represent this in a different way?</td>
<td>Deep mathematical understandings Develop a schema for future problem solving</td>
</tr>
<tr>
<td>Reasoning</td>
<td>Do you think this pattern will continue? Can you state a general rule you have discovered? Would this still be true if you change…? Do you agree or disagree with…?</td>
<td>Make generalizations Recognize logical math proofs</td>
</tr>
<tr>
<td>Exposure</td>
<td>Can you model what you were thinking? How do you know you can do that? How can we represent that with symbols?</td>
<td>Expose student thinking Expose important math topics</td>
</tr>
<tr>
<td>Revoicing and Repeating</td>
<td>So what you are saying is…? The teacher restates part of what was said by a student then asks, is this what you meant? Can you restate what…said?</td>
<td>To clarify or to deal with confusion.</td>
</tr>
</tbody>
</table>

angst for some students. Teachers must learn how to manage student anxiety by creating a safe environment in which mistakes can be exposed and discussed while the teacher maintains the demands of the high-level task or complex math problem (Stein et al., 2009).

Stein et al. (2009) explain that high-level math tasks must build on students’ previous understanding and provide opportunities for students to extend their existing math understanding. When students are asked to extend their existing knowledge, some anxiety can occur. Philipp (2008) asserts that when teachers choose a high-level task or challenging math problem, there can be a temptation to provide too much support when students begin to struggle with the task. Teachers should not make the mistake of providing too much support to create a feeling of safety for their students. Stein et al. (2009) acknowledge that high level tasks can create a level of uncertainty for many students, but teachers must learn how to provide an appropriate level of support that maintains the “integrity of the math task” (p. 49). Scaffolding must support student thinking in a manner that “enables the student to complete the task, but that does not reduce the overall complexity or cognitive demands of the task.” (p. 527). Learning to maintain the high-level task throughout its life in a classroom requires good questioning skills and scaffolding skills on the part of the teacher.

Smith and Stein (2011) describe the anticipating principle of math discourse as one that can enhance the implementation of a high-level math task. Once teachers have chosen a high-level math task that meets a math goal, teachers must predict and analyze possible student strategies. Teachers should prepare to address issues, misconceptions, and various levels of understanding that might arise as students attempt high level math
tasks (Smith & Stein, 2011). When teachers plan high-level math tasks by anticipating student thinking, the math task has the possibility of maintaining its rich mathematical nature throughout the learning process.

Another aspect of using high-level math tasks is the issue of error analysis. Some educators might be uncomfortable with the idea of exposing student errors. However, discussing errors and misconceptions are important components of a discourse community, and errors are a natural part of the learning process when students are faced with challenging math problems. Henningsen and Stein (1997) address the issue of error analysis when they explain teachers should focus on the thinking process not on the accuracy of the answer. Both correct and incorrect answers can help students engage in mathematical thinking. Smith and Stein (2011) touch on the necessity to discuss misconceptions when they explain the discourse practices of selecting and sequencing. Again, selecting involves teachers systematically selecting students to share their work. During the selection process errors or misconceptions will be selected and discussed. Determining the sequence of methods and the exposure of errors is crucial. The selecting and sequencing processes “determine what ideas students will have the opportunity to grapple with and ultimately to learn” (Smith & Stein, 2011, p.43). Liping Ma (1999) explains how error analysis is viewed in China. Chinese teachers put errors on the board and invite students to engage in class discussions that examine the errors. Through these discussions students learn both correct procedures and underlying misconceptions of the math task. It is clear that discussing multiple approaches, including approaches based on flawed thinking, is an important principle of math discourse. Exposure to correct and incorrect responses should be carefully planned.
Teachers should not fret, errors can be brought to light in a discourse classroom because the classroom dynamic is one of safety, value, and learning. Kazemi and Stipek (1998) explain that learning environments should focus on understanding, and errors should be viewed as a natural part of the learning process. Viewing errors as a normal part of learning, helps to create a safe environment that focuses on thinking rather than correct answers. Munkhjargal (2007) explains that in a math discourse community each student should be valued as a unique math learner. When each student is valued for their mathematics, a safe environment can be create in which errors can be analyzed for the sake of learning. Hoffman, Breyfogle, and Dressler (2009) carried out a research project on the power of incorrect answers. They found that using incorrect answers in the classroom gave students the opportunity to explain and justify their reasoning. When students are valued for their contributions, both correct and incorrect methods can be acknowledged without making value statements about the learner.

A problem-based approach to math instruction requires the use of high-level math tasks. Math discourse principles including questioning that scaffolds the learning experience and error analysis can be applied to such math tasks so that students can become empowered to grapple with and complete the task.

Connecting Verbal and Non-Verbal Math Discourse. It is clear that oral math discourse can have benefits. Therefore, the question arises, can written discussions support mathematical thinking and problem solving? Burns (2004) notes that the NCTM acknowledges writing as an important component of math instruction. Through her own research, Marilyn Burns found support for this statement when she proved that the writing process was able to help students organize, clarify, and reflect on their own
thinking about mathematics (Burns, 2004). More recently, Kostos and Shin (2010) carried out a research study in which the use of math journals had a positive influence on the students’ communication of mathematical thinking and the use of math vocabulary. It appears that mathematical writing has the potential to enhance students’ mathematical thinking skills. Therefore, when implementing math discourse practices, it is important to consider how to connect writing to the discourse practices.

As teachers connect writing lessons to math curriculum, they should consider the relationship between writing and math discourse. Wilcox and Monroe’s (2011) suggest using math learning logs to incorporate a think-write-share strategy. Before students write, the teacher poses a reflective question such as, What did you learn about (state a math topic)? The students are first given think time. In Wilcox and Monroe’s (2011) model, the students write after they have been given think time, and they discuss their response with a partner once the writing component is complete. However, Huang and Normandia’s (2007) research suggests that writing activities should be supported by classroom discussions. Huinker and Laughlin (1996) describe a strategy that utilizes math discussions before the writing process. This strategy is referred to as the think-talk-write process. They explain that “the think-talk-write strategy builds in time for thought and reflection and for the organization of ideas and the testing of those ideas before students are expected to write” (p. 82). Time for thinking and discussing will engage the thought processes. Once students’ minds are engaged, they will have the tools to better explain their own understanding of the topic in writing (Huinker & Laughlin, 1996). When teachers connect mathematical thinking, math discourse, and writing, there are positive outcomes.
Lynch and Bolyard (2012) suggest adding another component to the think-talk-write model. They suggest encouraging students to share their completed writing assignment in small groups or with a pen pal. Through research, they found that getting meaningful feedback from a partner or pen pal can help students expand or revise their explanations and deepen their thinking. During their research study, math explanations were shared with a pen pal, and the pen pal responded by using questioning techniques. When questioning techniques were used, students began to give more detailed descriptions of their mathematical thinking and reasoning. The students’ explanations began to go beyond procedures and into the reasoning behind the procedures (Lynch & Bolyard, 2012). When writing, math discourse, and mathematical tasks are partnered, there is potential for deep mathematical thinking.

Burns (2004) makes a few suggestions for incorporating writing into the math curriculum. She suggests that teachers use math writing as a formative assessment. Teachers can evaluate students writing by asking a few simple questions. What was the level of understanding amongst the students? Was the math task accessible to all students? Borasi and Rose (1989) have a similar view of writing as a tool to inform instruction. They advocate for teachers to read students’ math journals. Through evaluation of math writing, teachers become aware of the individual needs of students, and they can plan instruction in response to students’ level of understanding (Borasi & Rose, 1989). When teachers use writing to inform instruction, it can become a powerful teaching tool.

Math Discourse as a Formative Assessment. Instruction that pairs math discourse with formative assessment has the capability of bringing teaching and learning
together in a manner that creates a shared classroom learning experience. Munkhjargal, (2007) explores the notion that is there a reciprocal relationship between math discourse and the curriculum. In a math discourse community students are involved in both creating and implementing the curriculum, and students learn more than merely what they are taught (Munkhjargal, 2007). This reciprocal relationship is similar to the relationship that can be found between formative assessment, teaching, and learning. This view is emulated by Margaret Heritage (2007) in her article on formative assessment. She states that formative assessment has the power to “transform both teaching and learning” (p. 140). A reciprocal relationship can be created between instruction and assessment, and the relationship can enhance the learning experience (Heritage, 2007). It is possible that combining classroom discourse with formative assessment has the potential to change the relationship between teaching, learning, and the creation of quality math discussions.

Formative assessment can be used as a motivational tool for students. Moss and Brookhart (2009) explain when formative assessment is used to motivate, it has the possibility to close the achievement gap. They state that the use of formative assessment should partner the teacher and the students so that evidence of learning can be gathered in a manner that attains the mutual goal of improving student achievement. Formative assessment practices that guide the decisions of teachers and students as they progress toward a learning goal, can be motivating. Oberdorf and Cox (2012) explain that teachers should share expectations, progress, and results with students to enhance their ability to self-monitor progress toward educational goals. It is possible that applying formative assessment methodologies to a math discourse community could have powerful effects on students’ math achievement.
Cai, Lane, and Jakabcsin (1996) connect the use of formative assessment and evaluative feedback to the math classroom. They discuss how teachers can use scoring rubrics to help students evaluate their own mathematical thinking (Table 3). Through the use of classroom discourse and scoring rubrics, students come to understand different levels of mathematical proficiency. With this awareness, students are able to evaluate their own performance, and they are empowered to improve their own math proficiency.

Connecting the instructional practice of formative assessment to mathematical discourse can have a few positive effects. It can partner the teacher and the student in a manner that enhances student learning, and it can be motivating for students. By creating and using a formative assessment scale, teachers can provide meaningful information to students as they become adept at carrying out math discussions.

Conclusion

A problem-based approach to math instruction complemented by math discourse principles has many positive outcomes. A problem-based approach develops students’ critical thinking skills, it develops deeper understanding of math concepts, and it prepares students for real world problem solving. However, when engaging problems are presented in a manner that promotes discussion these positive outcomes can be magnified. When teachers implement mathematical discourse principles, thinking is exposed and teachers can tailor instruction to varied levels of understanding, the climate of a classroom can be transformed into a motivating and engaging educational environment, critical thinking and reasoning skills can be brought to light and improved, mathematical understanding can be enhanced, and students become empowered to
Table 3

*Example of a Math Discourse Scoring Rubric*

<table>
<thead>
<tr>
<th>Communication Levels</th>
<th>Math Discourse Practices</th>
</tr>
</thead>
</table>
| Communication Level 4 | - Uses appropriate math terminology  
- Identifies the important elements of the problem and demonstrates an understanding of their relationships  
- Gives clear evidence of a solution process  
- Provides a complete response with a clear, unambiguous explanation or description  
- Communicates effectively to the audience  
- Presents strong support arguments that are logically sound and complete  
- Discussion may include examples and non-examples |
| Communication Level 3 | - Uses nearly correct math terminology  
- Identifies most of the elements of the problem and demonstrates a general understanding of the relationships  
- Gives clear evidence of a solution process  
- Provides a fairly complete response with reasonably clear explanation or description  
- Generally communicates effectively to the identified audience  
- Presents supporting arguments that are logically sound, but may contain some minor gaps |
| Communication Level 2 | - Uses some math terminology  
- Identifies some elements of the problem and shows a limited understanding of their relationships  
- Gives some evidence of a solution process  
- Makes significant progress toward completion of the problem, but the explanation or description may be somewhat ambiguous or unclear  
- Communication may be somewhat vague or difficult to interpret  
- Arguments may be incomplete or may be based on logically unsound premise |
| Communication Level 1 | - May misuse or fail to use math terminology  
- Fails to identify elements of the problem and does not show an understanding of their relationships  
- Gives incomplete evidence of a solution process  
- Communication has some satisfactory elements but it is not complete or it may omit significant parts of the problem  
- Explanation or description may be missing or difficult to follow |

evaluate and improve upon their own math understanding. These positive outcomes of a problem-based approach to math instruction that is founded are discourse principles are aligned to the expectations of current education reform. In short, a problem-based approach to math instruction complemented by math discourse supports student achievement while aligning classroom practices to new expectations.

It is apparent that the creation of a math discourse community is quite an undertaking. Some refer to the process of implementing math discourse as daunting and formidable. Conversely, when describing the effects of a discourse community on student learning, researchers use positive terminology to describe the outcomes. Discourse communities are described as engaging and motivating entities that promote deep mathematical thinking. Although the creation of a discourse community might be a daunting task, it is apparent that the undertaking will be worth the effort. As teachers embark on this path, it will be helpful to focus on the essential principles of math discourse outlined in this paper. First, teachers can learn to evaluate and improve the classroom climate. This will engage students in math discussions while empowering them to take control of their mathematical understanding. Secondly, teachers can develop and fine-tune their questioning skills. When teachers cultivate good questioning techniques, they are able help students develop metacognitive skills, make connections between math concepts, and develop reasoning skills. Quality questions can also provide differentiated access to complex word problems taking into account the needs of a diverse student population. Third, teachers should learn to select engaging math problems that encourage students to grapple with complex math concepts. When selecting such high-level tasks, teachers must learn to see both correct and incorrect answers as opportunities to enhance
learning. By doing this, teachers allow students to approach problems in multiple ways, accept, and even relish, errors as an opportunity to learn, and learn to justify their mathematical thinking. Fourthly, teachers should find ways to encourage students to write during math lessons. When writing is interwoven with math discourse, mathematical thinking can be improved upon and fully articulated. Finally, teachers should learn how to use math discourse as a formative assessment. By doing so, instructional practices can be tailored to student’s needs. By working on classroom climate, questioning techniques, the problem selection process, the writing process, and informed instruction, teachers can begin the process of creating meaningful math discussions around engaging math problems.

This research review has exhibited a few studies that show the positive results of connecting a problem-based approach to math instruction with math discourse principles. These studies show that when teachers learn to choose quality problems and engage students in meaningful math discourse, students’ problem solving skills, mathematical understanding, and reasoning skills can be improved. Since a problem-based approach to math instruction that is founded on math discourse principles can be challenging for many teachers to implement, a few examples of how a problem-based approach to math instruction can be partnered with math discussions were shared. It is clear that math instruction that is rooted in a problem-based approach and incorporates discourse principles can be attained.
CHAPTER III

METHODOLOGY

The core of this thesis project involves the creation of a third grade curriculum handbook titled, *Problem-Based Math Instruction Founded on Math Discourse Principles*. The curriculum handbook contains eight lessons designed to teach third grade level multiplication through a problem-based instructional (PBI) approach. The lessons combine mathematics problem solving and math discourse to deepen students’ understanding of multiplication. In this chapter, the methods used to create the problem-based lessons will be clarified, and the assessment system designed to inform instruction throughout the eight lessons will be explained. In addition, the methods used to evaluate the curriculum handbook for the purposes of this project will be made clear.

Methods for Creating the Curriculum Handbook

Through the review of literature, five principles stood out as essential components of PBI and math discourse practices. The lessons contained in the curriculum handbook are based on these five research based principles. The five principles are: high-level math tasks, questioning techniques, formative assessment, writing in mathematics, and demonstration of learning. First, the lessons in the curriculum handbook contain high-level math tasks that were chosen based on Henningsen and Stein’s (1997) definition of high-level tasks and Hiebert et al.’s (1996) description of meaningful math
problems. In addition, instructional strategies and materials are provided to promote the maintenance of such high-level math tasks (Stein et al. 2009). Secondly, the lessons are designed to support questioning techniques that promote problem solving and enhance math discourse. Small et al. (2004), describe questioning as a key component of math discourse. Thirdly, the lessons are designed to include formative assessment practices. Oberdorf and Cox (2012) advocate for a formative assessment system that allows teachers to share expectations, progress, and results with students to enhance their ability to self-monitor progress toward educational goals. Fourthly, the lessons are designed so that writing is used to enhance math instruction. Marilyn Burns (2004) explains that the writing process can help students organize, clarify, and reflect on their own thinking about mathematics. Finally, the lessons are designed based on the project-based learning practice of allowing students to demonstrate what they have learned. Boss (2013) explains that due to the fact that students participate in an extensive inquiry process as part of the problem-based learning experience, students should be given an opportunity to demonstrate what they have learned.

**Principle 1: Methods for Using and Maintaining High-Level Math Tasks.**

The curriculum handbook contains complex math problems that can be described as high-level tasks as defined by Henningsen and Stein (1997). These high-level math tasks, or complex math problems, allow students to learn math concepts by engaging in the problem solving process, and they are at the heart of the PBI approach found in the handbook. Henningsen and Stein (1997) define high-level tasks as math tasks that encourage high-level mathematical thinking and reasoning. Hieber et al. (1996)
describe high-level math tasks when they discuss “problematizing” math instruction. They state that students should be given math problems that allow them to grapple with and search for a solution. When solving this type of complex math problem, students develop a true and deep understanding of math concepts (Hiebert et al., 1996). Each math problem contained in the curriculum handbook has been evaluated based on Henningsen and Stein’s (1997) definition of high-level math tasks as well as Hiebert et al.’s (1996) explanation of math problem solving. Many of the math problems have been adapted from research based resources such as About Teaching Mathematics (Burns, 2007), Teaching Student-Centered Mathematics (Van de Walle & Lovin, 2006), the Inside Mathematics website that is supported by the Noyce Institute (2005 a, b), and the Illustrative Mathematics website that is an initiative of the Institute for Mathematics and Education. Furthermore, the math problems contained in the curriculum handbook have been chosen based on their ability to help students connect math problem solving to real world situations.

The curriculum handbook combines high-level math problems with instructional strategies, support materials, and research-based advice that are meant to maintain the high-level math problems so that the complexity of the math problem does not decline throughout the duration of the lesson. Stein et al. (2009) explain that teachers must learn how to maintain the cognitive demands of a high-level math task. Through research, Stein et al. (2009) determined instructional factors that can maintain or cause decline in high-level math tasks. The instructional factors that affect high-level math tasks can be found in Table 4.
Table 4

**Instructional Factors that can Maintain or Cause Decline in High Level Math Task**

<table>
<thead>
<tr>
<th>Maintaining High Level Math Tasks</th>
<th>Factors that Cause Decline in</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Scaffolding of student thinking and reasoning</td>
<td>1. Problematic aspects of the task become routinized.</td>
</tr>
<tr>
<td>2. Students are provided with means to monitor their own progress</td>
<td>2. The teacher focuses on the answer rather than emphasizing understanding of the concept.</td>
</tr>
<tr>
<td>3. The teacher or capable students model high-level performance</td>
<td>3. Students aren’t given enough time to grapple with the math problem.</td>
</tr>
<tr>
<td>4. The teacher should push for explanations, connections, and justification through questioning, comments, and feedback.</td>
<td>4. The math task is not in the zone of proximal development for the students.</td>
</tr>
<tr>
<td>5. The task should build on prior knowledge and be within the zone of proximal development.</td>
<td>5. Students aren’t held accountable for high-level products.</td>
</tr>
<tr>
<td>6. Sufficient time is given for students to explore and grapple with the math task.</td>
<td>6. Classroom management affects student engagement in the task.</td>
</tr>
</tbody>
</table>


The curriculum handbook contains instructional materials including a variety of question frames and statement frames. Each question and statement frame has a specific purpose, and a few of these question and statement frames were specifically designed to help teachers scaffold student’s thinking in a manner that maintains the high-level math task throughout the problem solving process as described by Stein et al. (2009). The question frames titled, *Questions Frames that Promote Understanding* are adapted from Small et al. (2004), and they are designed to maintain high-level math problems during independent problem solving. When students are working independently
on a complex math problem, the teacher uses these questioning strategies to support student’s thinking while maintaining the cognitive demands of the high-level math problem. The three separate question frames titled, *Revoicing, Restating, and Exposure and Connections* are adapted from the work of Chapin et al. (2013), Chapin et al. (2009), Smith and Stein (2011), and Small et al. (2004), and they are provided so that teachers can promote math discourse and scaffold the problem solving process during whole group math discussions. These three question frames help students access the mathematical thinking of others allowing students at all levels to become successful when solving complex math problems. These question frames are discussed further in the section titled, *Question Techniques in a Safe Environment that Promotes Engagement* because through scaffolding students are supported in a manner that creates a safe math discourse environment.

The curriculum handbook contains question and statement frames that are designed to support math discourse during peer math discussions. However, these question and statement frames can be used to scaffold the problem solving process as well. The question frames titled, *Question Frames that Promote Explanations, Reasoning, and Evidence* and the statement frames titled, *Statement Frames that Promote Comparisons and Connections* are adapted from Small et al. (2004), Smith and Stein (2011), and Chapin et al. (2013), and they are designed so that students can participate in math discourse with their peers in a manner that allows the maintenance of the high-level math problems. These question and statement frames are discussed more thoroughly in the next section on questioning techniques.
Instructional strategies that maintain high-level math tasks, or complex math problems, are embedded in the curriculum handbook. First, the eight lessons are sequenced in a manner that allows students to build on previously learned information. Secondly, each lesson begins with a section titled, *Set the Stage*. This section is designed with a few purposes in mind. The first purpose is to provide students with opportunities to review previously learned material while connecting prior knowledge to new concepts. For example, in lesson 2, the students review and reflect upon what was learned in lesson 1 before diving into new material. The second purpose of the section titled, *Set the Stage* is to grab student interest in a manner that allows students to reason mathematically in a meaningful context. By allowing students to reason mathematically about real and meaningful situations, teachers can scaffold students’ mathematical thinking. For example, in lesson 1, the teacher acts out a real world scenario to engage students in the problem solving experience allowing students to reason mathematically about a real and meaningful math problem. The real world setting allows students to make sense of mathematics. Thirdly, the students are taught to evaluate, reflect upon, and improve their math discourse as a means of maintaining high-level math problems. This is achieved by using a math discourse rubric. This rubric will be discussed further in the section titled, *Refining Questioning Techniques with a Math Discourse Rubric*.

The curriculum handbook includes dialogue boxes titled, *Research Based Advice*. A few of these dialogue boxes prompt teachers to maintain high-level math tasks and provide research to support the instructional practices being described in the handbook. The instructional strategies, support materials, and research-based advice contained in the curriculum handbook can assist teachers as they learn to use instructional
strategies that promote the maintenance of high-level math tasks, or complex math problems.

**Principle 2: Methods for Using Questioning Techniques**

The curriculum handbook is designed to provide both teachers and students with discussion promoting strategies, primarily questioning techniques, which promote math problem solving and meaningful math discourse. In the curriculum handbook each lesson is divided into different sections that include whole-group discussions, peer discussions, and independent work. In each section various questioning techniques are used to enhance problem solving and math discourse. The curriculum handbook is designed so that questioning techniques can be used to support math discussions, students can be questioned and question their peers in a positive and safe environment that promotes engagement, and questioning techniques can be refined by using a math discourse rubric that allows for self-evaluation and progress.

**Questioning Techniques Are Used to Support Math Discussions.** In both the whole group and peer discussion sections of each lesson, examples of question frames and statement frames are provided so that teachers can learn to use questions to promote and improve math discourse. Each question and statement frame is also available in the resources section of the curriculum handbook. Because math discourse includes a variety of discussion practices, each question and statement frame begins with a heading. The heading depicts the math discourse practice that is supported by the given question or statement frame. There are question frames that promote math explanations, mathematical reasoning, and mathematical evidence. There are statement frames that
promote mathematical connections and comparisons. Question frames that promote math predictions, conjectures, and justification are also included. The curriculum handbook describes how to use these question and statement frames to promote math discourse in the whole group discussion and peer discussion sections of each lesson.

Questioning Techniques in a Safe Environment That Promotes Engagement

Other questioning strategies such as revoicing and restating, and question strategies that promote understanding, are used during whole group discussions and independent work times. Revoicing and restating can be used to clarify mathematical reasoning and increase the likelihood that students at all levels are following the math conversation (Chapin et al., 2009). Question frames that promote revoicing, restating, and understanding are embedded in lesson 1, and they can be found in the resources section of the curriculum handbook. These math discourse practices not only maintain high-level math tasks and promote strong math discussions, but they foster a safe and positive classroom climate. Revoicing and restating question frames are provided so that students can engage in, and learn from, math conversations without the pressure of answering difficult math questions. The question frames that promote understanding are provided so that students can become successful when solving challenging math problems. When all students are allowed to successfully participate in math discourse at their level, an inclusive and safe discourse community can be created (Simon & Blume, 1996). These questions frames are designed to help students find entry points into challenging problems and break math problems down into manageable chunks of information. By supporting students’ mathematical understanding, a safe environment in which students are honored for their mathematical thinking, not just for correct answers, is created.
The curriculum handbook is structured in a manner that provides students with the opportunity to use questioning techniques in a safe and positive classroom environment. As mentioned earlier, the curriculum handbook contains question frames and statement frames that promote different discourse practices, and these question and statement frames are used to scaffold the learning experience. Scaffolding allows students to receive the support they need to become successful while maintaining a low stress learning environment. Also, the curriculum handbook is structured in a way that creates a progression that allows students to learn, practice, and build upon discourse practices. For example, the process by which students learn to make justifications, conjectures, and predictions is broken down into smaller mini-lessons throughout the curriculum handbook. The process begins in lesson 1 with a mini-lesson during which the teacher models how to make mathematical justifications, conjectures, and predictions using the question frames. Then, in lesson 4, students are given opportunities to attempt these challenging discourse practices without assessment or evaluation. Next, in lesson 7, the discourse practices of providing justification and making conjectures and predictions are modeled, reviewed, and discussed again during a mini-lesson before the students are expected to put them into practice. By providing question frames that scaffold math discussions and organizing the process by which challenging discourse practices are taught, a safe, supportive environment in which students can become successful is created.

Refining Questioning Techniques with a Math Discourse Rubric. After learning how to participate in math discussions that include the discourse practices of explaining, reasoning, providing evidence, and making connections and comparisons,
students learn how to use a discourse rubric to evaluate and improve upon their math discussions. Recorded math discussions are evaluated using a math discourse rubric that was adapted for the elementary classroom (Park et al., 2011). At first, recorded discussions are evaluated by the whole class. The student math discourse rubric is referred to during these whole group discussions in an attempt to give students practice analyzing and improving math discourse. During the whole group discussion, students are asked to give constructive advice as to how the math discussion could be improved. While discussing and sharing constructive advice, students are prompted to refer back to the question and statement frames. This allows students to build upon the questioning strategies that were taught previously. Once students have practiced analyzing math discussions in a whole class setting, they are given the opportunity to use the student rubric to analyze their peer discussions. When working with a partner to record and evaluate math discussions, students are asked to score their discussions using the math discourse student rubric. The students circle two things on the rubric. First, they circle or highlight the aspects of the discussion that they feel they have achieved. Then they circle or highlight the level of discussion that they have attained. By analyzing math discussions as a whole group and with a peer, students can learn to improve their own math discussions in a manner that allows them to engage in meaningful math discourse that supports mathematical problem solving.

**Principle 3: Methods for Using Formative Assessment.**

There are three main assessments and many informal assessment opportunities that are described in the curriculum handbook. The three main assessments include a pre-
assessments, and two rubrics. Throughout the curriculum handbook the pre-assessment and both rubrics can be used to inform instruction. They are formative assessments, and the curriculum handbook explains how to use the pre-assessment and both rubrics to guide instructional choices. Although these three main assessment pieces provide feedback to teachers, and this feedback can be used to inform instructional decisions, there are other informal assessment opportunities embedded in each lesson. All formative assessment possibilities are discussed in the formative assessment section of each lesson.

The Pre-assessment: A Formative Assessment. The curriculum handbook contains a pre-assessment that can be used as a formative assessment tool. The pre-assessment provides information in three ways. First, it provides information about each student’s problem solving abilities. Secondly, it provides information as to whether or not students are confident about their math problem solving abilities. Finally, it provides information about how well the students are participating in math discourse in a manner that promotes mathematical problem solving.

The first section of the pre-assessment contains three multiplication word problems that reflect real world situations and two questions about students’ confidence. The real world problems have been adapted from problems that are available on the Inside Mathematics website (Noyce Institute, 2005a, b). These problems and the Inside Mathematics website are based on research by the Noyce Institute (www.insidemathematics.org). After the students solve the three word problems, they answer two questions about their problem solving confidence. These two questions ask about student’s confidence and specifically refer to the three multiplication word
problems contained in the assessments. The information from the pre-assessment can be used to inform instruction throughout the unit. For example, teachers can use the information to determine which students may need extra support to solve complex math problems and which students might benefit from extra encouragement due to low confidence levels. Also, information from the pre-assessment can be used to create peer discussion groups at the beginning of the unit. A pre-assessment scoring sheet is provided, and advice on how to use the scoring sheet to create peer discussion partners is provided.

The second section of the pre-assessment provides students with the opportunity to engage in math discourse. After completing the first section of the pre-assessment, students are allowed to engage in a math discussion with a partner. During the discussion, students are allowed to make changes to their original answers. However, they are asked to make any changes with a colored pencil. If a student chooses to make changes to their original answers, they must explain why they decided to make these changes in writing. Also, students are asked if the math discussion helped them solve the three problems. This section of the pre-assessment has two purposes. First, data gained from allowing students to engage in a math discussion and change any answers based on the discussion is used to determine if math discourse improved student’s understanding of the real world multiplication problems. Secondly, since students have the opportunity to evaluate whether or not the math discussion helped them solve the three problems in section one of the pre-assessment, this information can be used to determine student’s comfort level with using math discourse to improve math problem solving. For instance, if data from the pre-assessment shows that math discourse did not improve math problem
solving, the teacher knows that the students will need a lot of support to develop math discourse skills. In this case, the teacher can use question frames and statement frames that promote revoicing and restating before moving on to question frames that promote more complex thinking. If the data from the pre-assessment shows that students already know how to use math discourse to improve problem solving or that they feel comfortable using math discourse to improve problem solving, the teacher can focus on improving the level of discussions such that they promote higher level reasoning, connections, conjectures, and justification.

**The Post-Assessment: A Formative or Summative Assessment.** The data from the post-assessment can be used in a few ways. The pre-assessment and post-assessment were designed in a similar format. They were designed in this manner so that the data from the pre-assessment is easily compared to the data of the post-assessment. At the end of the unit, the post-assessment is administered, and the data can be used for two purposes. First, it can be used to determine how each child improved over the course of the unit. There are three aspects of improvement to evaluate. Teachers can review pre- and post-assessment scores to determine growth in student’s problem solving ability, student’s ability to use math discourse to solve complex problems, and student’s confidence with math problem solving. Secondly, the post assessment can be used to evaluate student’s current level of problem solving ability, level of problem solving confidence, and each student’s ability to engage in a math discussion that promotes math problem solving skills. This information can be used formatively or as a summative assessment. The teacher can use the data gained from the post-assessment to inform instruction as the class moves on to a new unit of study. The information can be used to
determine new math discourse partners for the next math unit. Also, this data gained from the post-assessment can be used to determine the level of instruction when teaching math problem solving and math discourse practices during the next unit. However, this information can also be used as a summative assessment, and it can provide a score for the end of the unit.

The Math Discourse Rubrics, Formative Assessments. Both math discourse rubrics can be used to improve math discourse, and to inform the instructional choices of the teacher. They achieve these goals in different manners. There are two rubrics because one rubric is designed for student use, and it is referred to as the student rubric, and one rubric is designed for the teacher, and it is referred to as the teacher rubric. The student rubric was discussed in the above section on questioning techniques. Both the student rubric and the teacher rubric were adapted from Park et al.’s (2011) rubric used at the University of California, Los Angeles (UCLA). The original UCLA rubric was designed for the secondary level, so it was adapted for use in the elementary classroom. Both rubrics are provided in the curriculum handbook for the specific purpose of improving math discourse.

As explained earlier, the student rubric allows students to self-evaluate their math discussions for the purpose of improving math discourse. However, the student rubric can also be used by the teacher as a formative assessment tool. For instance, when students use the rubric to evaluate their discussion, they give themselves a score, and they circle the asterisks next to the aspects of math discourse that they feel went well during the conversation. These scores can be used by the teacher as a window into student’s own feelings about their ability to engage in math discourse. For example, if the teacher
notices that students are circling the asterisk next to the phrase, *I did not use these connections to compare strategies, ideas, or answers with those of others*, the teacher can address this in a whole class lesson. During the next whole class discussion, the teacher can model how to use sentence frames that prompt connections and comparison. Also, the teacher can use scores from the student rubric to differentiate instruction or form new math discourse partners.

The teacher rubric is used by the teacher to evaluate the level of individual student’s math discourse. This information can be used to make instructional decisions, and it can be used to provide clear and immediate feedback to students. For example, if the teacher notices that many students are receiving a score of 2 on the teacher rubric, the teacher can focus on questioning strategies that encourage students to make connections, analyze errors, and provide reasoning and evidence to support their mathematical thinking. By doing so, the teacher creates opportunities for students to learn and practice level 3 and level 4 math discourse practices. Also, the teacher rubric can be used to provide feedback to students as to what each student is doing well and what they could do to improve. The teacher can achieve this by highlighting strategies on the rubric that a student has achieved, and circling strategies that the student can improve upon.

**Principle 4: Methods for Incorporating Writing into Mathematics**

In the curriculum handbook writing is used to complement math lessons in a variety of ways. First, students are allowed to reflect on their math lessons by writing about what they’ve learned. By asking students to reflect on their math problem solving experience, students are able to think more deeply about mathematics. For example, in
lesson 1 students write about their problem solving experience. They are asked to think about what they learned and respond in writing. Lesson 2 begins by allowing student to reread their written response to lesson 1 and reflect upon what was learned with their peers. This sets the stage for a new learning experience, and it allows students to deepen their understanding of what was learned in lesson 1. Secondly, throughout the curriculum handbook students are allowed to reflect upon their understanding of math vocabulary through writing. The vocabulary activities are based on Marzano’s research on academic vocabulary development. Marzano (2004) explains that when students copy the teacher’s definition of a term instead of generating their own explanation, the results are not as strong. Ideally, student explanations should come from the students. Having students represent their understanding of a new term by drawing a picture or using a symbolic representation leads to academic gains (Marzano, 2004). When students are asked to write about their math vocabulary, they are allowed to create a collaborative definition based on classroom learning experiences and feedback from peers. They are also able to illustrate math vocabulary to deepen their understanding of each word. The curriculum handbook suggests that teachers allow their students to use a math journal, and examples of math journal entries are provided. Finally, when learning math vocabulary, students are given the opportunity to engage in kinesthetic learning. Shrum and Glisan (2000/2010) explain that teachers can build vocabulary by presenting vocabulary in a manner that using actions to enhance the meaning of vocabulary words. Therefore, the vocabulary lessons in the handbook allow students to create actions that show the meaning of each vocabulary word. The definitions and the actions are practiced at various
points throughout the unit to reinforce the meaning of the vocabulary words and to encourage the application of the vocabulary word to new learning experiences.

**Principle 5: Methods for Allowing Students to Demonstrate their Understanding**

The high-level math tasks used throughout the curriculum handbook are designed to help students connect in-class learning to the world outside the classroom. The high-level math tasks are complex math problems set in real world situations. The type of problem solving used throughout the curriculum handbook can be described as problem-based learning. Problem-based learning can be explained as instruction that provides opportunities for students to be challenged to solve problems that mimic real life (Boss, 2013). This lessons in the curriculum handbook begin with a driving question, *how is multiplication used in the world around us?* Throughout the unit, as students solve complex, real world math problems, they address this question and make a list of ways multiplication is used in the world outside the classroom. At the end of the unit, students will be given the opportunity to create a project that shows their answer to this original question. They are given a variety of formats in which they can showcase their answer to this question. The students can create posters, Google presentations, dioramas, or skits that show their understanding of multiplication concepts and how they are used in real world situations. On the last day of the unit, students have an opportunity to share their projects with others.
Methods for Evaluating the Curriculum Handbook

For the purpose of evaluating the curriculum handbook titled, *Problem-Based Math Instruction Founded on Discourse Principles*, the data gained from the pre-assessment and post-assessment and the data gained from both the student rubric and the teacher rubric could be used to determine the success of the curriculum handbook. Although the curriculum handbook was not evaluated as a component of this master’s project, evaluation resources are available and ready to be used. In fact, the pre- and post-assessment were designed so that the data gained from each assessment can easily be used to evaluate student improvement. As a follow-up to this project, data gained from the pre- and post-assessments and the two rubrics could be used during an action research project during. Through action research the curriculum handbook and the instructional strategies that it contains could be thoroughly evaluated.
CHAPTER IV

SUMMARY, CONCLUSIONS AND
RECOMMENDATIONS

Summary

Much sought after math reform has finally reached our public schools. For many years, the National Council for Mathematics (NCTM) has been advocating for math reform (Barron et al., 1998). Now the Common Core Standards for mathematics (CCSM) will be fully implemented and assessed in California’s public schools during the 2014-15 school year. The CCSM will ask students to learn and apply demanding math concepts and procedures (www.corestandards.org). At last, the long-desired changes in math instruction are upon us, and they have the potential to create positive changes in classroom math instruction.

The curriculum handbook titled, Problem-Based Math Instruction Founded on Math Discourse Principles was specifically designed to meet the CCSM and promote change in classroom math instruction. The curriculum handbook was designed to address the CCSM by utilizing a problem-based approach to math instruction combined with math discourse principles. The problem-based instructional approaches used in the curriculum handbook allow students to grapple with meaningful math problems. By grappling with complex math problems, students can develop a true and deep understanding of math concepts (Hiebert et al. 1996). The math discourse practices
The curriculum handbook can support teachers as they learn to implement the CCSM and carry out new instructional strategies aligned to reform mathematics. To clarify the connection between the lessons contained in the handbook and the CCSM for teachers, each lesson begins with a list of the Common Core Standards addressed in that lesson. To clarify the reform-based instructional practices contained in the handbook, each lesson begins with a research-based rationale. In addition, research-based advice is included throughout each lesson to clarify the reasoning behind instructional practices. The research-based advice is meant to shed light on instructional practices that might represent change for some teachers. When teachers have the opportunity to read the research-based rationale and the research-based advice, they gain a deeper understanding of the reform-based instructional practices contained in the handbook. By making the CCSM and the reform-based instructional practices clear to teachers, the curriculum handbook supports teachers as they learn and apply new pedagogical practices that meet the expectations of today’s math classrooms.

The curriculum handbook exists as a first step for teachers as they learn to implement problem-based math lessons and math discourse practices. The curriculum handbook provides teachers with the opportunity to learn, rehearse, and apply research-
based instructional practices in a manner that supports independence. After teaching the problem-based lessons contained in the curriculum handbook, teachers could apply problem-based math instruction to future math units. By addressing the CCSM and explaining how problem-based math instruction and math discourse principles can be used to promote instructional changes, this curriculum handbook provides materials and strategies that allow teachers to begin the process of adopting current practices in math instruction.

Conclusions

Brendefur et al. (2013) explain that teachers lack exposure to the necessary and rich math experiences necessary to develop an instructional base that adheres to the demands of education reform. The curriculum handbook is one resource that addresses this issue. The lessons contained in the handbook provide exposure to, and practice with, problem-based math instruction and math discourse principles so that teachers can learn to addresses the Common Core Standards and the new expectations in math instruction through the use of these instructional practices. The lessons in the curriculum handbook have been created based on five research-based principles that are inherent qualities of problem-based instruction (PBI) and math discourse practices. The handbook utilizes these five principles in a manner that allows teachers to successfully implement problem-based math lessons founded on math discourse principles while meeting the expectations of the CCSM. The five principles are high-level tasks, questioning techniques, formative assessment, writing in mathematics, and demonstration of learning.
The Common Core Standards call for students to make sense of problems and persevere in solving them (www.corestandards.org). High-level tasks contained in the handbook allow teachers to engage students in meaningful problem solving, and teachers are provided with research-based strategies so that the high-level tasks can be maintained throughout the duration of the learning experience.

The CCSM ask students to construct viable arguments and critique the arguments of others (www.corestandards.org). The curriculum handbook contains questioning techniques, question frames, and statement frames that support teachers as they learn how to facilitate constructive math discussions. The materials and questioning strategies contained in the curriculum handbook demonstrate how to engage students in meaningful math discussions, and they support students as they learn how to communicate mathematically with their peers.

The CCSM ask students to apply the mathematics they know to new, real world math problems. Students should apply mathematics to problems that arise in everyday life, society, and the workplace (www.corestandards.org). A demonstration of learning is an integral component of PBI, and the curriculum handbook utilizes this PBI component in a manner that allows students to showcase what they have learned about multiplication and apply their understanding to real world situations.

The CCSM ask students to communicate precisely to others. Students should use clear definitions in discussions with others and in their own reasoning (www.corestandards.org). The writing activities contained in the curriculum handbook provide opportunities for students to use writing to develop a deeper understanding of
multiplication concepts and vocabulary, and students then use their writing to support precise communication with their peers.

Formative assessment is an important component of PBI. Formative assessment is used throughout the handbook in a few ways, but it is used as a component of PBI when it is used to support student’s metacognitive skills. Metacognitive skills, such as planning and self-monitoring, are necessary skills when engaging in problem- and project-based learning (Papanikolaou & Boubouka, 2010). The curriculum handbook provides opportunities for students to learn how to self-evaluate and improve upon their math discourse and their math problem solving skills. For example, students are able to record and evaluate their own math discussions using a student discourse rubric as a guide to direct and enhance the evaluation process. In addition, students are often allowed to make changes to their math problem solving methods after engaging in math discussions. Students make changes using colored pencil so that they are able to see how their own math problem solving was improved as a result of the math discourse process. This type of structured self-evaluation is necessary when supporting student’s metacognitive skills throughout the problem solving process inherent in PBI.

Recommendations

The curriculum handbook has been designed to meet the expectations of the CCSM using a problem-based approach to math instruction founded on math discourse principles. As part of this master’s project, the curriculum handbook has not been evaluated in the classroom setting. In the future, the lessons and strategies contained in the curriculum handbook should be evaluated using data gained from the pre-assessment,
post-assessment, the student rubric, and the teacher rubric. By evaluating the data gained from these four sources, the success of the curriculum handbook could be determined, and changes could be made to the curriculum handbook to further improve the lessons and strategies. Wiggins and McTighe (2011) discuss curriculum and unit design. They describe curriculum and unit design as a system of continuous improvement. Student performance can be used to inform needed adjustments in the curriculum (Wiggins & McTighe, 2011). The curriculum handbook should be evaluated based on student performance so that this system of continuous improvement can occur.

In the future, the curriculum handbook should be evaluated by comparing the pre-assessment scores to the post-assessment scores. The pre-assessment and post-assessment were designed in a similar format. Both the pre-assessment and the post-assessment contain three separate sections. The first section of the pre- and post-assessment should be compared to determine growth in student’s problem solving ability. The second section of the pre- and post-assessment can be used to determine if student’s confidence with math problem solving improved over the course of the lessons found in the curriculum handbook. The third section of the pre- and post-assessment should be compared to determine if student’s ability to use math discourse to solve complex problems improved over the course of the unit contained in the curriculum handbook.

The curriculum handbook should be evaluated using the teacher’s discourse rubric. The teacher’s discourse rubric can be used to evaluate the curriculum handbook in three ways. At two points throughout the curriculum handbook, students are asked to record their math conversations. These recorded conversations can be evaluated using the teacher’s discourse rubric. First, the scores from the teacher’s discourse rubric can be
used to find the percentage of math discussions that contain various math discourse practices. For instance, if the teacher evaluated the first set of recorded math discussions and found that only 20% of the students were making connections and comparisons, this information could be compared to data collected later in the unit to see if the percentage of students making mathematical connections and comparisons increased over time. This type of evaluation should occur for each discourse practice found on the teacher’s math discourse rubric including error analysis, comparisons and connections, reasoning, explanations, evidence, and conjectures and predictions. By evaluating each discourse practice in this manner, it can be determined whether each math discourse practice improved or did not improve throughout the course of the lessons. Secondly, the scores can be used to determine if the overall level of math discourse improved throughout the course of the lessons. For example, if the teacher evaluated the first set of recorded discussions and found that, three out of 22 students received a score of one, eight out of 22 students received a score of two, and 11 out of 22 students received a score of three using the teacher’s math discourse rubric, this information could be compared to scores received at the later recording period. Finally, the teacher’s discourse rubric could be used to determine how many students improved over the course of the lessons. For instance, each student’s two scores could be evaluated to determine how many student’s math discourse scores improved over the course of the lessons.

The student’s discourse rubric could be used to determine the success of the curriculum handbook. At two points throughout the curriculum handbook, students are asked to evaluate their recorded math discussions using the student’s discourse rubric. The student rubric allows students to self-evaluate their math discussions providing a
window into each student’s confidence level with each discourse practice contained on the rubric. By comparing student’s scores at two points throughout the unit, the handbook can be evaluated to determine if the lessons and strategies improve student’s own confidence with math discourse practices.

Future evaluation of the curriculum handbook titled, *Problem-Based Math Instruction Founded on Math Discourse Principles*, will lead to further improvement of the strategies contained within the handbook. By assessing and improving the curriculum handbook, teachers will gain valuable information about the best PBI practices and how these practices can be implemented to support the CCSM and reform math instruction.
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REFERENCES


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APPENDIX A
Curriculum Handbook: Multiplication in the World Around Us

Problem-Based Math Instruction Founded on Math Discourse Principles

Level: Third Grade
By Signe Miller
California State University, Chico
Fall 2014
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SECTION 1

Overview of the Curriculum Handbook

This curriculum handbook contains problem-based math lessons that address multiplication at the third grade level. The problem-based lessons are structured around real world mathematics problems and math discourse principles. These principles are integrate into the problem solving process through the use of a driving question. The driving question is used to help students make connections between in-class problem solving and mathematical problem solving in the world outside the classroom. For instance, the driving question how is multiplication used in the world around us, is presented during the first lesson, and it sets the stage for the real world problem solving that students will experience throughout the lessons. At the end of the unit, each child is given the opportunity to synthesize what they have learned as they create a project to answer the original question, how is multiplication used in the world around us?

The problem-based lessons contained in this curriculum handbook are designed to provide students with opportunities to learn multiplication through real world problem solving while participating in meaningful math discussion with their peers. Throughout the lessons, students will have the following learning opportunities.

Students will have the opportunity to:

- Connect in class learning to real life experiences by solving complex, real world math problems.

- Engage in math discussions with their classmates. The discussions provide opportunities for students to better understand math concepts and to better understand that real world problem solving can be collaborative.
• Participate in math discourse by learning to make connections between math ideas and strategies, compare various approaches to math problems, provide reasoning and evidence to support mathematical thinking, make conjectures or predictions about mathematical ideas, and justify conjectures when engaging in math discourse with others.

• Record and evaluate mathematical discussions

• Learn math vocabulary as it pertains to the concept of multiplication.

• Use writing to reflect on what they have learned about multiplication and problem solving.

• Sort and discuss complex math problems.

• Write math word problems that call for the application of one’s understanding of multiplication as well as real world applications of multiplication concepts.

• Demonstrate an understanding of how multiplication is used in the real world by creating an exhibit showing how multiplication is used in the real world.

Overview of the Lesson Structure

The problem-based lessons contained in the curriculum handbook are organized into various sections. The sections include; setting the stage, posing the problem, independent problem solving, peer discussions, whole group discussions, apply what you’ve learned, writing in math, extension activities, and formative assessment. These sections are organized in various formats depending on the objectives of each lesson. Setting the stage is a section of the lesson dedicated to capturing student interest while tapping into students’ previous understanding and focusing that understanding on a new learning experience. The independent problem solving section provides an opportunity for students to independently explore math problems. Independent exploration allows students to make sense of a problem situation, and it opens up the possibility for multiple solution methods. The peer discussion section enables students to engage in math discourse while solving math problems. By discussing math problems in a small group setting, students are able to access the mathematical ideas of their peers. The whole group
discussion section provides an opportunity for students to learn and practice math discourse principles while using the whole group discussion to deepen their understanding of the math concept. Some lessons provide opportunities for students to apply what they have learned. Students try out their new mathematical understanding in new, novel situations. The writing in math sections allows students to organize and reflect upon math vocabulary, the math discourse process, and mathematical thinking. The extension section provides activities that can be used for early finishers or as ways to practice math concepts in new situations. The formative assessment section describes how to use information gained from lesson discussions and activities to inform instructional choices and practices.

**Flexibility in Using this Curriculum Handbook**

Teachers are professionals and all classrooms are unique. Therefore, teachers must be free to make professional decisions and to determine the best way to utilize the problems and strategies contained in this handbook. The goal of the curriculum handbook is to provide support, yet flexibility, for teachers who are interested in using a problem-based approach to math instruction founded on math discourse principles. Teachers can consider the following possibilities when determining how to use the curriculum handbook to best meet their instructional needs.

**As a Supplement Unit**

Most teachers will have an adopted math program. The curriculum handbook can be used in its entirety to supplement an adopted math program. Not all adopted math programs, will contain the problem-based instructional practices and math discourse principles that can be found in this curriculum handbook. Therefore, this curriculum handbook can be used as a supplemental unit to an adopted math program.

**As a Resource**

The curriculum handbook can be used as a resource. Teachers can use this handbook as a
resource to enhance an already existing math program. For example, a teacher may choose to use various strategies contained in the curriculum handbook and apply the strategies to lessons found in an adopted math program. A teacher may choose to apply the math discourse strategies to math problems found in an adopted math program. A teacher may choose to use the driving question and exhibition day to provide an opportunity for students to inquire about multiplication in the real world and demonstrate what they have learned. A teacher may choose to use the sorting or writing strategies but apply them to problems contained in an adopted math program. Therefore, the curriculum handbook can be used as a resource to enhance an existing math program.

As a Guide

The curriculum handbook can be used as a guide. This curriculum provides an example of how to use problem-based instructional strategies founded on math discourse principles, and it can be used as a guide to create other math units. The curriculum handbook shows how to use a driving question to engage students in the inquiry process, demonstrations of learning to exhibit what students have learned, complex math problems that allow students to grapple with and develop understanding of math concepts, math discourse to promote collaboration and problem solving, and formative assessment practices that bring students into the assessment process. This information can be used as an example to create other math unit.

Overview of Research Based Rationale for the Curriculum Handbook

The curriculum handbook was created based on research on the topics of mathematical discourse, problem and project-based learning, writing in math, and formative assessment.

Overview of Mathematical Discourse

The Common Core Standards for Mathematics (CCSM) promote the use of mathematical discourse by calling for students to construct viable math arguments and critique the
mathematical thinking of others. Also, the CCSM advocate for meaningful math discussions that address problems set in real-world situations (www.corestandards.org/math/practices). Many teachers struggle to incorporate meaningful math discussions into the elementary classroom due to the fact that creating math discourse is a challenge at the elementary level. Hafferd-Ackles, Fuson, and Sherin (2004) studied math discourse communities, and they found that many teachers were not properly prepared to facilitate the type of math discussions that are aligned to the expectations of math reform. This curriculum handbook attempts to simplify and demystify the implementation of math discourse in the elementary classroom. By combining problem-based instruction strategies with math discourse practices, this handbook provides third grade teachers with the necessary tools to engage students in meaningful math discussions.

This curriculum handbook provides opportunities for teachers to learn how to incorporate math discourse principles into their math instruction. Stein (2007) describes math discourse as math discussions in which students talk, question, agree or disagree, and make conjectures about a problem in order to discover important math concepts. The lessons provided in this supplemental handbook showcase the use of math discourse principles during whole group discussion, peer discussions, and one-on-one instruction. During whole group instruction, questioning strategies are provided as a means to engage students in quality math discussions. Questioning strategies such as revoicing and restating are provided to help students clarify and reflect upon their own mathematical thinking as well as connect to the mathematical thinking of others. During peer discussions, students use question frames, statement frames, and a student discourse rubric to engage in and improve their math discourse. During one-on-one instruction, questioning strategies that promote understanding are provided as a means to scaffold students’ mathematical thinking and bolster problem solving confidence. Teachers are able to use theses
questioning strategies to promote understanding of multiplication concepts while students work through high-level math tasks. Chapin, O’Connor, and Anderson (2009) state that for years the NCTM has encouraged teachers to use mathematical discourse in elementary classrooms to improve students’ ability to reason mathematically. They explain that math discussions can directly promote student learning by providing access to ideas, strategies, procedures, and more (Chapin et al., 2009). This curriculum handbook supports teachers as they learn to implement and enhance meaningful math discussions and learn to use math discourse to help students reason mathematically and improve their understanding of multiplication concepts.

**Overview of Problem-Based Instructional Practices and use of High Level Tasks**

This curriculum handbook utilizes problem and project-based instructional practices that allow students to learn, practice, and apply multiplication in a meaningful context. Problem-based learning can be explained as instruction that provides opportunities for students to be challenged to solve problems that mimic real life (Boss, 2013). The problem-based unit that is contained in the curriculum handbook begins with an open-ended question that engages students in the inquiry process. The opening question is, *how is multiplication used in the World Around Us?* This question connects the in-class learning experience to the world outside the classroom.

Although the driving question sets the stage, other problem-based instructional practices are used throughout the curriculum handbook to promote continued engagement. For example, complex math problems, or high-level math tasks, are used to facilitate discussion and promote understanding of multiplication concepts. The focus of the unit is not to teach specific multiplication procedures, but rather to develop mathematical understanding through the problem solving process. Murray, Oliver, and Human (1998) suggest using quality problems and posing such problems in a setting in which students are allowed to engage in inquiry. Heibert,
Carpenter, Fennema, Fuson, Wearne, Murray, Olivier, and Human (1997) state that the chosen math task is the key to the math learning experience. The chosen task must allow and encourage students to engage in reflection and mathematical communication. The math task must allow students to treat the problem as something they need to think about rather than a prescription they need to follow (Heibert et al., 1997). Many of the math tasks and problems found in this handbook are adapted from reputable sources such as About Teaching Mathematics by Marilyn Burns, Teaching Student Centered Mathematics by Van de Walle and Lovin, The Illustrative Mathematics website, and the Inside Mathematics website. The math problems have been chosen for their ability engage students in problem solving while discussing real world math topics.

Demonstrations of learning are a key component of problem and project-based learning. Boss (2013) explains that students should be given an opportunity to demonstrate what they have learned. At the end of the math unit contained in the curriculum handbook students are given the opportunity to demonstrate what they have learned by creating a project that addresses the original driving question, how is multiplication used in the world around us?

**Overview of Writing and Math**

This unit connects writing to math problem solving and math discourse practices. Kostos (2010) explains that writing has a positive influence on students’ ability to communicate mathematical thinking, and it improves the use of math vocabulary. Therefore, this unit contains lessons that utilize writing in a manner that encourages students to think, talk, write, and reflect on their learning experiences. By incorporating writing into math lessons, students are given the opportunity to apply their understanding of math vocabulary in written form, and they are given the chance to reflect on their mathematical thinking.
Overview of Formative Assessment

This curriculum handbook contains formative assessment tools including a pre-assessment, a student rubric, and a teacher rubric. Formative assessment tools are used to provide meaningful feedback to teachers and to help students take an active role in their learning experience. Oberdorf and Taylor-Cox (2012) explain that when teachers learn to use formative assessment, they can begin to share results with students in a manner that enhances students’ ability to self-monitor progress. Throughout the unit contained in the handbook, students are given the opportunity to assess and reflect upon their own math discourse. Their self-assessment is supported by a rubric. The rubric is a tool by which students are taught to evaluate their own math discussions. Through this self-assessment, students are given the opportunity to become active participants in the assessment process. Also, the teacher will use both a pre-assessment and a rubric to assess students’ math discussions. The information gained from these formative assessment tools can be used to inform instructional decisions.
Multiplication in the World Around Us
Math Standards Addressed in this Curriculum Handbook

Standards for Mathematical Practice (SMPs) Addressed in this Unit

SMP #1 – Make sense of problems and persevere in solving them
SMP #2 – Reason Abstractly and Quantitatively
SMP #3 – Construct Viable Arguments and Critique the Reasoning of Others
SMP #4 – Model with Mathematics
SMP #6 – Attend to Precision
SMP #7 – Look for and Make Use of Structure

Standards for Mathematical Practice (SMPs) Explicitly Taught in this Unit

SMP #1 - Make sense of problems and persevere in solving them: In this unit students will solve complex, real world problems. They will share methods used to solve such complex problems, and learn to approach complex problems using a variety of mathematically correct approaches. Students will learn to analyze and make sense of complex problems.

SMP #3 - Construct viable arguments and critique the reasoning of others: In this unit students will listen to the mathematical arguments of their peers. When engaging in mathematical arguments, students will learn to ask questions to clarify, build on, or improve the arguments of their peers.

SMP #6 - Attend to precision: In this unit students try to use clear definitions in discussions with others. They will state the meaning of the symbols they choose, including using the equal sign consistently and appropriately

Operations and Algebraic Thinking, Grade 3 (OAT)

OAT Standard #1: In this unit students will interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each.

OAT Standard #3: In this unit students will use multiplication within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities.

Geometric Measurement, Grade 3 (MG)

MG Standard #7: In this unit students will relate area to the operations of multiplication and addition.
SECTION 2
Pre-Assessment and Post Assessment

Pre-Assessment a Formative Tool for Teachers

The pre-assessment, post assessment, and the pre-assessment scoring sheet can be found in the resource section of the curriculum handbook. The pre-assessment is designed to be an instructional tool. The information gained from the pre-assessment can be used in a few ways. Here are three possible suggestions for the pre-assessment.

1) Pre-Assessment to Determine Current Problem Solving Ability

The pre-assessment can be used as a window into each student’s current problem solving ability. This information can be used to differentiate levels of support during the problem-solving lessons.

2) Pre-Assessment to Determine Current Math Discourse Level

The pre-assessment can be used to determine each student’s ability to engage in and use mathematical discourse. It provides information as to how well students are able to use math discourse to improve mathematical problem solving. This information can be used to guide questioning, voicing, and restating choices used by the teacher. It can also be used to create math partners.

3) Pre-Assessment to Determine Current Problem Solving Confidence

The pre-assessment provides information to teachers as to how confident students are with their problem solving skills. This information can be used to provide encouragement to those students who might need it.

A pre-assessment scoring sheet is provided. The scoring sheet can be used to organize information so that it can be used by the teacher to make instructional decisions.
Post Assessment Overview

The post assessment can be used as a formative or summative assessment. Here are three possible suggestions for using the post assessment.

1) Post Assessment as a Tool to Show Growth

The post assessment is designed similarly to the pre-assessment. It was designed in this manner so that the data from the post assessment is easily compared to the data of the pre-assessment. The data from the post assessment can be compared to the data from the pre-assessment to determine areas of growth. There are three areas of growth that can be evaluated. Teachers can review pre- and post-assessment scores to determine growth in problem solving ability, improvement in student’s ability to use math discussions to solve complex problems, and improvement in math confidence.

2) Post Assessment a Tool to Inform Instruction

Although the post assessment is designed to be given at the end of the lessons contained in the curriculum handbook, it can still be used to inform future instruction. The post assessment can be used to evaluate students’ current level of problem solving ability, level of problem solving confidence, and each student’s ability to engage in a math discussion that promotes math problem solving skills. This information can be used to provide intervention to students who will need continued support in mathematical problem solving and math discourse as the class moves on to new math concepts.

3) Post Assessment a Summative Tool

The post assessment can be used to give students a final score on mathematical problem solving. A post-assessment scoring sheet is provided so that teachers can organize data for scoring purposes.
Multiplication in the World Around Us

Lesson 1 – We’re Going to a Restaurant

Lesson 1 Goals

Problem-Based Learning Goals
- Students will solve and discuss real world math problems.
- Students will create methods for solving real world math problems by drawing pictures or using manipulatives.
- Students will demonstrate their understanding of multiplication by solving a real world assessment problem.

Math Discourse Goals
- The students will ask questions and share connecting comments by using the question and sentence frames provided.

Writing in Math Goals
- The students will demonstrate their understanding of repeated addition and multiplication when writing definitions and drawing pictures of each vocabulary word.
- Students will reflect on their learning experience in writing.

Math Standards Addressed in Lesson 1

Standard for Mathematical Practice (SMP)
SMP #1 - Make sense of problems and persevere in solving them

Operations and Algebraic Thinking, Grade 3 (OAT)
OAT Standard #1 – In this lesson students will interpret products of whole numbers,
OAT Standard #3- In this lesson students will use multiplication within to solve word problems in situations involving equal groups, arrays, and/or measurement quantities.
Lesson 1 – We’re Going to a Restaurant

Research Based Rationale

Throughout lesson 1, questioning strategies will be used to promote math discourse. Small, Sheffield, Cavanagh, Dacey, Findell, and Greens (2004), explain that for teachers to create meaningful math discourse, they must ask a variety of well-planned questions. Also, revoicing and restating strategies will be used to help students access other’s mathematical thinking, promote student reflection, and provide clarification of math concepts. Chupin, O’Connor, and Anderson (2009) explain that revoicing and restating, or repeating, are “talk moves” that can be used to encourage math discourse. This lesson provides question and statement frames that can be used to promote math discourse.

In order for math discussions to reach their fullest potential, a safe environment must be established. In this lesson the teacher will create an environment in which it is safe to discuss ideas and make mistakes. Kazemi and Stipek (1998) explain that errors should be viewed as a natural part of the learning process. Viewing errors as a natural part of learning helps to create a safe environment that focuses on thinking rather than correct answers. Munkhjargal (2007) explains that when each student is valued for their mathematical thinking, a safe environment can be create in which errors can be analyzed for the sake of learning. Lesson 1 includes advice regarding how to create a safe environment that focuses on understanding rather than answers. For example, during whole group discussions multiple methods are honored to show that all students’ thinking is valued. In addition, the teacher models how mistakes can be used to enhance learning. Also, the first lesson provides structure so that students can learn how to communicate mathematically with their peers creating safety through understanding.

This lesson allows students to refine their understanding of math vocabulary through writing. The vocabulary activities are based on Marzano’s research on academic vocabulary development. Marzano (2004) explains that when students copy the teacher’s definition of a term instead of generating their own explanation, the results are not as strong. Ideally, student explanations should come from the students. Having students represent their understanding of a new term by drawing a picture or using a symbolic representation leads to academic gains (Marzano, 2004). This lesson also uses kinesthetic learning to enhance vocabulary development. Shrum and Gilian (2010) explain that teachers can build vocabulary by presenting vocabulary in a manner that using actions to enhance the meaning of vocabulary words.

This lesson provides an opportunity for students to reflect on their learning experience through writing. The written reflection component to this lesson is based on some of Marilyn Burn’s research. Burns (2004) found that writing was able to help students think more deeply about mathematics. The writing process can help students organize, clarify, and reflect on their own thinking about mathematics.

At various points throughout the lesson the teacher will focus on selecting student work. Work examples that promote understanding and facilitate meaningful whole class math discussions should be selected. Smith and Stein (2011) explain that teacher selection determines the ideas students will have the opportunity to grapple with and ultimately master.
Lesson 1 – Going to a Restaurant

Focus: Using Problem-Based Instruction Founded on Mathematical Discourse Principles to Learn About Multiplication in the World Around Us

Materials
* Large chart paper with the driving question, How is multiplication used in the world around us, written at the top (teacher creates)
* Chart paper to display the problem titled, Going to a Restaurant (teacher creates)
* Blank paper for student work
* Manipulatives (blocks, toothpicks…)
* 1 copy of Question Frames to promote explanations, reasoning, and evidence to display (see resources)
* Student copies of Question Frames to promote explanations, reasoning, and evidence (see resources)
* 1 copy of Statement and Question Frames to promote connections and comparisons to display (see resources)
* Student copies of Statement and Question Frames to promote connections and comparisons (see resources)
* Poster of Independent Thinking, Peer Discussion, Whole Group Discussion
* 1 copy of the Explain Your Method poster to display (see resources)
* Document camera or projector, large display screen (preferable)
* One copy of question frames that promote predictions, conjectures, and justification to display (see resources)
* Vocabulary worksheet to display (see resources)
* Vocabulary journals or student copies of vocabulary worksheets (see resources).
* Directions for written reflection (see resources)
* Lesson 1 Evaluation/Assessment to display (see resources)
* Student copies of Lesson 1 Evaluation/Assessment (see resources)

Set the Stage: Real World Math Problems

Explain: Tell the students that multiplication is in the world all around us. Explain that today the students will begin to discover how multiplication is used in the real world.
- Post the large chart paper with the driving question, how is multiplication used in the world around us, written at the top. Call on a student to read the driving question.

Explain: Explain that throughout this unit students should discover different ways in which multiplication is used in the word outside the classroom. Introduce the idea that at the end of this unit, the students will create a project to answer this question, How is multiplication used in the world around us?
- Save the chart paper with the driving question for later lessons.
- The teacher will introduce the first real world math problem by acting out a scenario to engage students’ thinking. The teacher will pretend to answer the telephone and participate in a conversation.
- When the teacher gets off the phone, the teacher will tell the students that the entire class has been invited to eat at a restaurant.
Post the first math problem titled, Going to A Restaurant (teacher created).
The students will read the math problem chorally.

**The problem titled, Going to a Restaurant**

“Our class has been invited to eat at a restaurant, but the restaurant has a rule about chopsticks. The restaurant requires everyone to eat with chopsticks, and everyone must bring their own chopsticks.”

Pause and model how to use chopsticks. Then continue reading the problem.

“Our teacher will go to the store to pick-up the necessary chopsticks, but you must determine how many chopsticks our class will need. Also, you must show how you determined how many chopsticks are needed. You can use pictures, numbers, manipulatives, or equations, but once you have a method, you must write it down. Be ready to share your method with a partner.”

*Source: Adapted from: Burns, M. (2007) *About Teaching Mathematics*, The Chopstick Problem*

**Independent Problem Solving: Real World Problem Solving**

- Display the paper that states, Independent Thinking to Peer Discussion to Whole Group Discussion (see resources).
- **Explain**: Explain that students will work independently first, and then have time to share their ideas with a partner. Solutions and methods will be shared as a whole class. During the whole class discussion the focus will be to learn different methods from each other. The focus is NOT about finding the one right answer or method. Explain that it is perfectly fine if they don’t know the answer right away. Tell students not to worry about mistakes.

**Research-Based Advice**

**Advice**: Take time to explain that when sharing one’s math ideas with others it is important to realize that mistakes are a natural part of the learning process, and that the focus is on learning and not on correct answers. This will begin the process of creating a safe environment that supports math discourse.

**Research**: When teachers begin to encourage students to share their mathematical thinking and evidence, students become more accountable for classroom learning and the authority switches away from the teacher to the students. This switch in classroom structure allows students to view math as created by a community of learners (Simon & Blume, 1996).

- Pass out blank paper to all students. Make blocks and toothpicks available for students to use.
- **Monitor**: Give students about 10-15 minutes to complete their method for solving the math problem. Monitor students and engage in questioning to promote mathematical understanding (see resources and example). Encourage early finishers to find a second method for solving the problem.
Research Based Advice

Advice: Try to maintain the high level of the math task. When students are working independently, allow them to grapple with the problem. Don’t be tempted to provide too much support.

Research: Teachers must learn how to provide an appropriate level of support that maintains the “integrity of the math task” (Stein, Smith, Henningsen, & Silver, 2009, p. 49). Scaffolding must support student thinking in a manner that “enables the student to complete the task, but that does not reduce the overall complexity or cognitive demands of the task.” (Stein, et al., p. 527).

Example of questions that promote mathematical Understanding

<table>
<thead>
<tr>
<th>Possible Questions to Promote Mathematical Understanding</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>What do you know about this problem?</em></td>
</tr>
<tr>
<td><em>What do we need to find out?</em></td>
</tr>
<tr>
<td><em>What information is necessary for you to solve this problem?</em></td>
</tr>
<tr>
<td><em>How can you use what you know to solve this problem?</em></td>
</tr>
<tr>
<td><em>Finally – What could be your first step? Try taking a first step and go from there.</em></td>
</tr>
</tbody>
</table>

Source: Adapted from Small, et al., 2004

*What do you know about this problem?*
(Possible response: There are 20 students in our class, and they each need to bring 2 chopsticks to the restaurant.)

*What do we need to find out?*
(Possible Response: We don’t know how many chopsticks we need to bring to the restaurant.)

*What information is necessary for you to solve this problem?*
(Possible Response: Everyone needs two chopsticks)

*How can you use what you know to solve this problem?*
(Possible response: I could draw picture of two chopsticks or I could use the toothpicks and put them in groups of 2)

*What could be a first step or initial try you could attempt to solve this problem? Try taking a first step and go from there.*
(Possible Response: students begins an attempt to solve the problem using pictures, blocks, or toothpicks.)

- Give a 5 minute “wrap up” notice and remind students that they should be prepared to share their work with a partner.

Peer Discussion: Set the Stage for Problem Solving and Math Discourse

Research Based Advice

Advice: Remember to create safe environment in which thinking and mistakes can be shared.

Research: Munkhjargal (2007) explains that in a math discourse community each student should be valued as a unique math learner. When each student is valued for their mathematics,
a safe environment can be create in which errors can be analyzed for the sake of learning.

- **Create a Safe Environment:** The teacher will set the stage for a positive, safe classroom discourse.
  *Reinforce the idea that all mathematical think is good thinking.
  *Remind students that math problem solving is not about finding the correct answer right away. It is about thinking through a complex problem. It might require multiple attempts.
  *Remind students that mistakes are a natural part of the problems solving process.
  *Remind students that there are many ways to solve the problem.
  *Ask students to choose a partner who they feel comfortable working with.
  *Tell students that if they don’t have a solution yet, they can work in a group of three with the goal of listening to and learning from their partners.
- **Provide time for students to find a partner. Monitor to make sure everyone has a partner.**

**Explain:** Explain the math discussion procedures.

- *Explain that each partner will have about 5 minutes to have a math discussion.
- *Explain that the math discussion will include first an explanation and then questions.
- *Display the poster titled, How to Have a Math Discussion, Part 1 (see resources and example).

Review the poster together. Ensure that students understand that explaining a method is more than just telling the procedures.

---

**Example of the How to Have a Math Discussion Poster, Part 1**

<table>
<thead>
<tr>
<th>How to Have a Math Discussion</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Part 1: Explain Your Method</strong></td>
</tr>
<tr>
<td>1. Explain <strong>how</strong> you solved the problem. Include <strong>math reasons</strong> that tell <strong>why</strong> you solved it this way. Possible sentence starters…</td>
</tr>
<tr>
<td>“First I ______ because ______.”</td>
</tr>
<tr>
<td>“Then I ______ because ______.”</td>
</tr>
<tr>
<td>“Also, I ______ because ______.”</td>
</tr>
<tr>
<td>2. Provide <strong>evidence</strong> that shows why your method makes sense. Possible sentence starter…</td>
</tr>
<tr>
<td>“… makes sense because ______.”</td>
</tr>
<tr>
<td>“This strategy works because ______.”</td>
</tr>
<tr>
<td>3. Provide <strong>connections</strong> and <strong>comparisons</strong>. Does this math problem or method remind you of another? How are they similar or different? Possible sentence starters…</td>
</tr>
<tr>
<td>“I used a similar method when I ______.”</td>
</tr>
<tr>
<td>“This problem reminds me of ______.”</td>
</tr>
<tr>
<td>“This problem reminds me of ______, but it is a little different because ______.”</td>
</tr>
<tr>
<td>“Honestly, I don’t think I’ve ever solved a math problem like this before.”</td>
</tr>
</tbody>
</table>

**Explain:** Explain that once one student has explained his/her method, the other partner will ask questions.

- **Display the question frames that promote math explanations, reasons and evidence (see resources and example).**
- **Read the question frames together, and tell the students that they will choose 2-3 questions from the question frames to ask their partner.**
Example of Question Frames that Promote Explanations, Reasoning, and Evidence

<table>
<thead>
<tr>
<th>Questions Frames – That promote math Explanations, Reasoning and Evidence</th>
</tr>
</thead>
<tbody>
<tr>
<td>* Why did you …?</td>
</tr>
<tr>
<td>* Why did you think that strategy would work?</td>
</tr>
<tr>
<td>* What is your evidence?</td>
</tr>
<tr>
<td>* How did you know to …?</td>
</tr>
<tr>
<td>* Can you prove to me that … makes sense.</td>
</tr>
<tr>
<td>* Can you explain why you …?</td>
</tr>
<tr>
<td>* What convinced you that was the right answer?</td>
</tr>
</tbody>
</table>

*Source: Adapted from Small et al., 2004, Smith and Stein, 2011, Chapin et al., 2013*

**Explain:** Explain that each student should choose questions that help them understand their partner’s method, or they should choose questions about things that were not covered by their partner in the original explanation.

*Explain that students will switch roles. Tell the students that the teacher will give a 1 minute wrap-up signal before they are expected to switch roles.*

- **Model:** The teacher will model how to participate in a math discussion with a partner. Ask for a student volunteer to come to the front. Tell the volunteer to listen to the explanation and be ready to ask follow up questions. The teacher will use examples and non-examples when modeling a math discussion.

<table>
<thead>
<tr>
<th>Research Based Advice</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Advice:</strong> When the teacher is modeling, show a few examples and non-examples of a good math discussion.</td>
</tr>
</tbody>
</table>

**Non-Example of Explaining a Math Method:** The teacher models explaining the answer in very basic terms, purposefully omitting an explanation of the method used to find the answer.

**Possible Wording:** “We need 40 chopsticks because there are 20 students, and they all need 2. Then the teacher should ask the class, “The question frames make it clear that I should explain my method. Did I explain my method?”

**Example of Explaining a Math Method:** Have a whole class discussion about how to improve the math discussion so that it includes an explanation of the method. Use information from the conversation to provide a true example. Here is one possible example of explaining the method. “Since there are 20 students, and they all need 2 chopsticks. I drew a picture of 20 stick figures, and I drew 2 chopsticks for each stick figure. I counted the chopsticks, and I got 40.”

**Non-Example of Explaining the Reasoning or Providing Evidence:** After discussing the non-example, the teacher should model how to explain the method, but only explain how you solved the problem (procedural understanding). Don’t explain the reasoning or provide evidence.

**Possible Wording:** “Last time, I forgot to explain my method, but then I did a better job of explaining my method. First, I drew a picture of 20 stick figures. Then I put two chopsticks with each stick figure. Finally, I added all the chopsticks. I got 40.” Then say, “Ok. I now I explained everything I did. I think I’m done.” Pause and let students think about your
explanation for a few seconds. Then say, “The question frame says I should provide reasoning and evidence. Did I ever explain why I added or provide evidence that adding made sense?” Have a whole class discussion about how to improve the math discussion so that it includes reasoning and evidence. Refer to the question frames that promote explanations, reasoning, and evidence as needed.

Example of Explaining the Reasoning or Providing Evidence: “First, I drew a picture of 20 stick figures. Then I put two chopsticks with each stick figure. Finally, I added all the chopsticks. I got 40. I know 40 makes sense because it is like counting by 2, so if I count by 2s 20 times I get 40. It looks like this: 2 + 2 + 2 + 2... = 40.”

Research: Estes, Mintz, and Gunter (2011) state that students need practice and guidance to learn the process of thinking. In order for math discourse to enhance mathematical thinking, students will need guidance and practice. Chapin, O’Connor, and Anderson (2013) explain that talking about math concepts allows teacher to address misconceptions and confusions.

- Now the volunteer will use the question frames to ask 2-3 follow-up questions. Remind the volunteer to ask questions that help them understand the method or that help clarify information that was not covered in the explanation.

Peer Discussion: Problem Solving and Math Discourse

- Pass out student question frames that promote explanations, reasoning and evidence (see resources). Pass out How to Have a Math discussion, Part 1 (see resources). Ask the students to begin. (Students will need to keep these resources for future lessons)
- Monitor: Listen to the conversations. Prompt students to use the How to have a Math Discussion poster and question frames that promote explanations, reasoning, and evidence as needed. Look for possible solutions methods to select for the whole class discussion.
- When most students are finished, give a 1 minute wrap-up signal to let the students know they will be switching roles soon.
- Ask the partners to switch roles, and give a few minutes for the new partners to share and ask questions.
- Give students a 1 minute wrap-up signal.

Peer Discussion: Problem Solving and Math Discourse / Making Connections

Explain: Tell the students that they will be given a few minutes to make connections between math methods. Explain that making connections and comparing math problems are two more important practices when having a math discussion.
- Model: Display the statement and question frames that promote connections and comparing (see resources and example). Ask for a volunteer to come to the front to be the teacher’s partner. Model how to use the statement and question frames to make connections between math methods. First, the teacher will quickly recap his/her method. Then the student can use the statement and question frames to make connections or compare methods. Then the
student and the teacher switch roles.

**Explain:** Tell the students that they will continue to work with their chosen partner. Explain that they will now compare their methods and look for connections. Tell students that the teacher will be monitoring the discussions and will select a few partners to come to the front to share their methods during the whole group discussion.

- Pass out the connecting and comparing statement frames (see resources and example). Ask the students to begin.

**Example of statement and question frames that promote connections and comparing**

<table>
<thead>
<tr>
<th>Statement and Question Frames: Promote Connections and Comparing</th>
</tr>
</thead>
<tbody>
<tr>
<td>I used a different method.</td>
</tr>
<tr>
<td>*I... and you..., but I understand how they both make sense.</td>
</tr>
<tr>
<td>*I... and you..., I am having a hard time understanding your method.</td>
</tr>
<tr>
<td>*I... and you... I like how you... that thinking is new to me.</td>
</tr>
<tr>
<td>We used the same method.</td>
</tr>
<tr>
<td>*You... and I...</td>
</tr>
<tr>
<td>My method is similar to your method.</td>
</tr>
<tr>
<td>*You and I both... but I didn’t...</td>
</tr>
<tr>
<td>*You and I both... but I...</td>
</tr>
<tr>
<td>*I... but I didn’t...</td>
</tr>
</tbody>
</table>

**Questions that promote Connections and Comparison**

*Does this math problem remind you of another problem?*

*Have you solved a math problem like this before?*

*Have you used this math method before?*

*Can you create another problem that can be solved the same way?*

**Source:** Adapted from Small et al., 2004 and Smith and Stein, 2011

- **Monitor:** The teacher should monitor the conversations. When monitoring listen for interesting methods and common errors. As the teacher monitors students, look for possible solutions methods to select for the whole class discussion.

- Give a 1 minute wrap-up signal. Then ask students to switch roles.

**Whole Group Discussion: Problem Solving and Math Discourse**

**Explain:** Tell students that you will be calling a few students to the front to share their methods.

<table>
<thead>
<tr>
<th>Research Based Advice</th>
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</thead>
<tbody>
<tr>
<td><strong>Advice:</strong> It is important to reinforce a safe environment throughout the lesson. Before calling students to the front remind the students that mistakes are fine.</td>
</tr>
</tbody>
</table>

| Research: Chapin, O’Connor, and Anderson (2013) state that by revealing and clarifying misconceptions or partial understandings, students are able to better develop conceptual understanding. |

- Call one student to the front of the class. This should be a student that the teacher has selected during the monitoring process. Avoid beginning with a complex solution method. This first example should be one that is accessible to most students.

- Ask the volunteer to explain his/her method using the How to Have a Math Discussion
Poster, Part 1 (see resources) for support.

- As the student explains his/her method, the teacher can use revoicing techniques, restating techniques, and exposure and connecting techniques as needed to promote a meaningful whole group math discussion (see resources and examples).

Examples of revoicing, restating, and exposure and connecting techniques

| Revoicing Techniques: Help students access other’s thinking and provide clarification. |
|-------------------------------|---------------------------------|
| *... pointed out that... Do you agree or disagree? |
| *So you are saying that... |
| *Can ... say what ...said so I can be sure I understand. |
| *Who can put that into their own words? |
| *Let me see if I understand. Are you thinking...? |
| *Who can repeat what ...said? |
| **Source:** Adapted from Chapin et al., 2009 and Chapin et al., 2013 |

| Restating Techniques: These promote student reflection and improve metacognition. |
|-------------------------------|---------------------------------|
| *What did you do first? Why? |
| *What were you thinking when you wrote this? |
| *How did you know to ...? |
| *Can you tell me what you did? |
| **Source:** Adapted from Chapin et al., 2009 and Smith and Stein, 2011 |

| Question Frames: Promote Exposure and Connections amongst students |
|-------------------------------|---------------------------------|
| * Can you model what you were thinking? |
| * How do you know you can do that? |
| *How can we represent that with symbols or with a formula? |
| *Who can add on? |
| *Does this method make sense to you? Why? |
| *Does anyone want to contribute more evidence to explain why this method works? |
| *Does anyone have a different way to explain this? |
| **Source:** Adapted from Small et al., 2004, Smith and Stein, 2011, and Chapin et al., 2013 |

**Advice Based on Research**

**Advice:** Revoicing, restating, and questions that promote exposure and connecting can be used to help students at all levels access the math discussion. These techniques can be used to draw students into the conversation, and they can be used to help students at all levels of math ability understand the math concepts that are being discussed during the whole groups discussion.

**Research:** Chapin, O’Conner, and Anderson (2013) explain that questioning techniques that help students connect to the thinking and reasoning of others can also help students become responsible for attending to each other’s contributions, they can help everyone in a class deepen their own understanding, and they help students learn from one another.

- When the first student is finished presenting his/her method, the student should be prompted to call on other students to ask questions.
- The students should use the question frames that promote explanations, reasoning, and evidence or question and statement frames that promote connections and comparisons (see resources) when asking questions. Students should have these question frames at their desks.
**Explain:** After one volunteer has shared a method and called on other students to ask questions, the teacher should explain that there are many ways to solve problems. Tell students that you will be exploring a variety of different methods that were used to solve today’s math problem.

- As students explain their methods, leave each method in front of the class so that it can be referred to later.
- **Selecting Volunteers:** Ask 3-4 students to share their methods in this same manner using the same questioning and statement strategies and frames as needed. Try to select work that shows multiplication, repeated addition, and at least one error that students can use to enhance their understanding of multiplication.

**Whole Group Discussion: Problem Solving and Math Discourse / Reinforce Vocabulary**

- Use questioning strategies to introduce the vocabulary words, repeated addition and multiplication.

<table>
<thead>
<tr>
<th>Research Based Advice</th>
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</thead>
<tbody>
<tr>
<td><strong>Advice:</strong> During the whole group discussion, the teacher will help the students understand the connection between multiplication and repeated addition. The teacher will use questioning techniques rather than direct instruction to develop these math concepts.</td>
</tr>
<tr>
<td><strong>Research:</strong> Van de Walle and Lovin (2006) explain that to teach math effectively teachers should engage students in a manner that allows them to develop new ideas and understanding. Students should be allowed to make sense of mathematics.</td>
</tr>
</tbody>
</table>

- Refer to the displayed methods. Point and ask, “Did anyone use this method? Ask for thumbs up. Ask, “Did anyone use this method, but do anything a little differently?” If so, have student explain while the teacher records the student’s approach on the front board.
- Do this for each method that was discussed and displayed, and use questioning strategies to help students discover similarities and differences between the methods.
- Introduce the vocabulary repeated addition and multiplication.

**Possible Wording:** “Which method do you think is referred to as repeated addition?” Pause and wait for a response. “Why?”

- Write the vocabulary word, repeated addition on the board.
- Help students make a connection between repeated addition and multiplication.

**Possible Wording:** “How are repeated addition and multiplication similar?” Pause and wait for a response. Get input from a few students.

- Write the vocabulary word, multiplication on the board.
- Through discussion, help students understand why one might use multiplication instead of repeated addition.

**Possible wording:** “Which method would help us solve the problem quickly?” Pause and wait for a response. “Which method works, but takes more time?”

- At this point, the teacher is hoping that the students realize that both repeated addition and multiplication can be used to solve the problem, but repeated addition takes more time.
• Leave up one student example of repeated addition and one student example of multiplication.

**Whole Group Discussion: Problem Solving and Math Discourse / Math Conjectures**

• Display the question frames that promote predictions, conjectures, and justification (see resources and example)
• **Explain:** Tell the students that they will have a whole class discussion about these two methods. The goal is to make conjectures or predictions.
• The teacher will model how to use questioning strategies that promote math predictions, conjectures, and justification, making math predictions and conjectures can be challenging for third grade students. This lesson introduces these challenging math discourse practices. These math discourse practices will be further developed in later lessons.

Example of question frames that promote predictions, conjectures, and justification

<table>
<thead>
<tr>
<th>Question Frame: Promote Predictions, Conjectures, and Justification</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>Can you state a general rule you have discovered?</em></td>
</tr>
<tr>
<td><em>Would this still be true if you change…?</em></td>
</tr>
<tr>
<td><em>How do we know this for sure?</em></td>
</tr>
<tr>
<td><em>Can you give another example of your thinking?</em></td>
</tr>
</tbody>
</table>

*Source: Adapted from Small et al., 2004 and Smith and Stein, 2011*

**Possible Explanation:** “To make conjectures or predictions, we will use the clues or patterns from the math problem to discover math ideas and to justify our mathematical thinking.”

• **Can you state a general rule you have discovered about repeated addition and multiplication?** (possible response: multiplication and addition are similar)
• **How can we know this for sure?** (possible response: when you add 20 groups of 2 you get 40 chopsticks, but some people did 20 x 2 and you still get 40 chopsticks)
• **Would this still be true if students brought different amounts of chopsticks?** For example, what if some students brought 1, 2, 3, or 4 chopsticks? Could we still add? Could we still multiply? (possible response: you could still add, I don’t think you could multiply)
• **Can you give another example of a situation in which you could add, but you couldn’t multiply?** (possible answer: If 6 friends brought 2 cookies to school you could multiply, but if they all brought different amounts of cookies they could only add)
• **Can you state a NEW general rule you have discovered about multiplication, repeated addition, and addition?** (possible response: you can use repeated addition and multiplication when you are adding equal amounts, but you can’t multiply when you have different amounts)

**Note:** If students don’t see the similarities and differences between repeated addition and multiplication, give an example and non-example.

**Non Example:** “If I bought 2 chopsticks to the restaurant and you bought 8 chopsticks, how many would we have?” Draw a picture to show a group of 2 chopsticks and a group of 8
chopsticks. Ask, “In this situation do we have equal groups? (discuss) How would we figure out how many chopsticks we had?” (discuss) “Could we add? Could we multiply?”

**Example:** “If I brought 4 chopsticks, Jenny brought 4 chopsticks, and Julio brought 4 chopsticks, how many would we have?” Draw a picture to show the groups of chopsticks. Ask, “In this situation do we have equal groups?” (discuss) “Could we add? Could we multiply?” (discuss) “When you multiply do you have to have equal groups?” (discuss)

Once you have given an example and non-example, ask the question again, can you state a general rule you have discovered about multiplication, repeated addition, and addition?

- Once the students have a general understanding about the similarities and differences between repeated addition and multiplication, the teacher should reinforce the concept of **equal groups**.

**Math Vocabulary Development: Kinesthetic Learning**

<table>
<thead>
<tr>
<th>Research-Based Advice</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Advice:</strong> When teaching vocabulary, it can be helpful to allow students to act out the vocabulary words. This provides an opportunity for students to learn kinesthetically.</td>
</tr>
<tr>
<td><strong>Research:</strong> Shrum and Glisan (2010) explain that teachers can build vocabulary by presenting vocabulary in a manner that uses actions to enhance the meaning of vocabulary words.</td>
</tr>
</tbody>
</table>

- Add the vocabulary word, **equal groups** to the board.
- Chorally read the three vocabulary words on the board: repeated addition, multiplication, equal groups.
- Then put actions to, or make gestures for, each vocabulary word. This provides an opportunity for students to practice vocabulary kinesthetically.
- First, model how to make an addition sign with your fingers (+). Then show how to repeat this sign. This will be the action for repeated addition. Make the repeated addition action and say, “repeated addition.”
- Show how to make a gesture for the word, groups, and a gesture for the word, equal sign. (The teacher can create these gestures or ask for recommendations from the class). Once the gestures for repeated addition, equal, and groups have been established, the teacher should model the gestures while saying, “In repeated addition there are equal groups, and you add the same amount more than twice.” Then practice as a whole class making the gestures while saying, “In repeated addition there are equal groups, and you add the same amount more than twice” as a whole class.
- Explain that when the teacher makes the addition sign and says “Teach repeated addition,” the students must say the corresponding words with the corresponding gestures. Practice doing this once so the students know what to do when the teachers says, “Teach repeated addition.”
- Ask the question, “How is multiplication related to repeated addition?” Discuss whole class. Then ask, “Can you use multiplication when the groups are different amounts or do
the groups have to be equal when you multiply?"

- Create gestures for the words multiplication. Model how to make the multiplication sign and have the students practice making the gesture for the sign. Review the actions, or gestures, for the words equal and groups. Once the gestures have been established, the teacher should model how to make the multiplication and equal groups actions while saying, “When you multiply, there must be equal groups.” Then practice as a whole class doing the gestures and saying, “When you multiply, there must be equal groups.” Remind the student that when the teacher makes the multiplication sign and says, “Teach multiplication,” they will make the gestures while saying the corresponding words.

- Practice what you have learned so far. The teacher makes the plus sign and says, “Teach repeated addition.” The students make the gestures while saying “In repeated addition there are equal groups, and you add the same amount more than twice.” Then the teacher makes the multiplication sign and says, “Teach multiplication.” The students make the gestures while saying “When you multiply there must be equal groups.”

- Display one student example that depicts how to use multiplication to solve the chopstick problem. Explain that when you multiply, the first two numbers, the factors, have a meaning. Using the student example, show the first number in the equation (20x2). Ask, “What does the 20 refer to in this example?” The students might say that it refers to the number of students. Then ask “What does the 2 refer to in this example?” The students might say the number of chopsticks each student needs.

- Now point to the first factor and say, “In this situation the 20 refers to the number of students. In multiplication the first factor refers to the number of groups.”

- Write number of groups on the board next to the other vocabulary words.

- Now point to the second factor and say, “In this situation the 2 refers to how many chopsticks each student will need. In multiplication the second factor refers to the amount in each group, or how many in each group.”

- Write how many in each group.

- Repeat the original kinesthetic gestures for multiplication. Say, “Teach multiplication.” The students make the corresponding gestures while saying, “When you multiply there must be equal groups.”

- Now add more gestures. Make a one with your finger and say, “The first factor represents the number of equal groups.” When you say equal groups, make the predetermined equal groups gesture with your hands.

- Practice what you’ve learned. The teacher makes the multiplication sign and says, “Teach multiplication.” The students say, “When you multiply there must be equal groups.” The teacher puts up one finger and says, “Teach first factor.” The students make the gestures and say, “The first factor represents the number of equal groups.”

- Add more gestures. Make a two with your fingers and say, “The second factor represents how many in each equal group.” When you say, “how many in each,” shrug your shoulders? When you say, “equal groups” make the predetermined gestures with your hands. Practice doing the gestures and saying the corresponding words together whole class. Say, “Teach second factor.” The students make the corresponding gestures while saying, “The second factor represents how many in each equal group.”
• Practice what you’ve learned. Practice each gesture and the corresponding words.
  * The teacher makes the addition sign and says, “Teach repeated addition.” The students make the sign and practice saying the corresponding words with the corresponding gestures.
  * The teacher makes a multiplication sign and says, “Teach multiplication.” The students make the multiplication sign and practice saying the corresponding words with the corresponding gestures.
  * The teacher puts up one finger to represent the first factor and says, “Teach first factor.” The students make the sign and say the corresponding words with the corresponding gestures.
  * The teacher puts up two fingers to represent the second factor and says, “Teach second factor” the students make the sign and say the corresponding words and gestures.

**Writing in Math: Math Vocabulary Development**

• The teacher will leave the vocabulary words on the board.
• Display the math vocabulary worksheet (see resources and example).

**Note:** If the class uses interactive vocabulary notebooks, this vocabulary page can be placed in the notebook.

**Explain:** Tell students that they will write a definition for each vocabulary word. The definition can be written in their own words. Once they have a definition for repeated addition, multiplication, and equal groups, they will draw a picture to show the meaning of each vocabulary word.

• **Model:** Point to the vocabulary word, repeated addition, and call on a student to explain the vocabulary word in their own words. Once the first student is finished with his/her definition, ask if anyone wants to add anything to the definition. Model how to write a definition in the second column. Then ask the students how to draw a picture that shows the meaning of repeated addition. Take their suggestions, and draw a picture showing the meaning of the vocabulary word in the third column.

• Remove the displayed copy of the math vocabulary words, and pass out student copies of the math vocabulary worksheet (see resources)

• Give students time to write their own definitions and draw pictures that show the meaning of each vocabulary word.

**Example Vocabulary Worksheet to display**

<table>
<thead>
<tr>
<th>Repeated Addition</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Multiplication</td>
<td></td>
</tr>
<tr>
<td>Equal Groups</td>
<td></td>
</tr>
</tbody>
</table>

• Have student share their entries with their table group, and give the students an opportunity
to make revisions to their entries if they would like.

- Display the teacher’s copy of the math vocabulary worksheet and call on volunteers to explain the meaning of each word.
- The teacher will write in the student’s definitions on the displayed vocabulary worksheet. The teacher can combine various definitions to make a joint definition.
- Call one student at a time to come up and draw a picture of each vocabulary word

### Example of a Vocabulary Entries to Display

<table>
<thead>
<tr>
<th>Repeated Addition</th>
<th>Repeated addition means there are equal groups and you add the groups together</th>
<th>**** **** **** 4+4+4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multiplication</td>
<td>You use multiplication when you have equal groups. It means the number of groups and how many in each group.</td>
<td>**** **** **** 3x4</td>
</tr>
<tr>
<td>Equal Groups</td>
<td>Groups that have the same value or same amount.</td>
<td>*** *** ***</td>
</tr>
</tbody>
</table>

- Give students time to change or make revisions to their entries if they would like to.

### Writing in Math: Reflection on Learning

- The teacher will ask the students to write a reflection entry in their interactive journals next to the three vocabulary words for the day (if the classroom is not using interactive journals, use separate paper)
- Post the direction for the written reflection on the board (see resources)
  1. Think first
  2. Talk with a partner
  3. Write
    * What was difficult or easy about the math problem?
    * What helped you understand or become successful with the math problem?
    * Explain what you learned
    * Use at least one vocabulary word
- Read the directions.
- **Model Thinking First**: First the teacher will think out loud.
- **Possible Think Aloud Reflection**: “How can I explain my method and compare it to another? I solved the problem by thinking that each person must bring 2 chopsticks. We have 20 students in class, so I did 2+2+2+2...twenty times. George had a similar approach, but instead of adding 2+2+2, he just did 10 groups of 2 in his head, and he knew that was 20. Then he added another 20 for the next 10 students.”
- **Possible Think Aloud Reflection**: “How can I explain what I learned? I learned that you can solve a problem in many ways.
- **Model Talking with a Partner**: Model how to share your ideas with a partner before writing. Ask for a volunteer to come to the front to model how to discuss what your method and what you learned. This peer discussion will help those students who aren’t sure what to write. It will provide scaffolding as they learn how to reflect upon their mathematical thinking and learning in written form.
- **Model Writing a Reflection**: I solved the problem by thinking that each person must bring
2 chopsticks. We have 20 students in class, so I did 2+2+2+2...twenty times. George had a similar approach, but instead of adding 2+2+2, he just did 10 groups of 2 in his head, and he knew that was 20. Then he added another 20 for the next 10 students. I learned that you can solve a problem in many ways.

- **Model Reflection:** Review the written example to reinforce the expectation. “Did I explain my method?” (wait for a response and discuss) “Did I compare my method to another method?” (wait for a response and discuss) “Did I explain what I learned?” (wait for response and discuss) “Did I use a vocabulary word?” (wait for a response and discuss).
- The teacher will give students a few minutes to write a math reflection.
- Students keep their vocabulary words and reflections in their desk.

**Formative Assessment: Real World Problem Solving**

- Display the assessment titled, The Fortune Cookie Problem. Read it chorally whole class.

**Question:** My whole class went to a restaurant together. All 20 students ate 2 fortune cookies. How many cookies did our class eat?

**Explain:** Remind students that there are many, many different types of multiplication problems because multiplication is in the world around us. However, the fortune cookie problem is very similar to the chopsticks.

- Pass out the assessment / evaluation worksheet (see resources). Give students time to solve the problem. When they are finished, the students hand in their papers.
- Check the papers to assess students’ level of understanding.
- If students used multiplication to solve the fortune cookie problem, check to see if they used the first factor and second factor correctly. The concepts of equal groups and factors will be reinforced in the following lessons. At this point, it will be helpful to see how many students already understand the concept of factors and equal groups.

Teacher Notes Lesson 1:
Multiplication in the World Around Us
Lesson 2 – Math in the World of Restaurateuring

Lesson 2 Goals

Problem-Based Learning Goals
- After viewing a video that depicts one example of how math is used in a real world restaurant setting, students will solve a real world problem using a chosen method.
- Students will solve real world math problems by completing a worksheet that requires both repeated addition and multiplication to be applied to real world situations.

Math Discourse Goals
- Students will engage in mathematical discourse to discuss different solution methods.
- Students will discuss the relationship between repeated addition and multiplication.
- Students will practice identifying and correctly utilizing the vocabulary terms first factor and second factor.
- Students will use the question and statement frames that promote math discourse when engaging in both whole class and peer math discussions.
- After completing a worksheet on repeated addition and multiplication, students will engage in a math discussion with a peer, and students will use these math discussion to make changes to their math work as needed.

Math Standards Addressed in Lesson 2

Standard for Mathematical Practice (SMP)
SMP #1 - Make sense of problems and persevere in solving them.
SMP #3 - Construct viable arguments and critique the reasoning of others.

Operations and Algebraic Thinking, Grade 3 (OAT)
OAT Standard #1 – In this lesson students will interpret products of whole numbers
OAT Standard #3- In this lesson students will use multiplication within to solve word problems in situations involving equal groups, arrays, and/or measurement quantities.
Lesson 2 – Math in the World of Restaurateuring

Research Based Rationale

In lesson 2, students build upon their understanding of multiplication by engaging in problem solving complemented by math discussions. Melville et al. (2013) stated that students learn mathematics most effectively when they are given opportunities to investigate ideas and concepts through problem solving. Lesson 2 provides opportunities for students to develop a deeper understanding of multiplication concepts by engaging in math problem solving.

In lesson 2, students engage in both whole group and peer discussions. Through discussion, students are exposed to multiple problem solving strategies and a safe classroom climate in which students are empowered to learn math is fostered. Small (2012) explains that when students are allowed to explore math problems in a manner in which a variety of responses and approaches are shared, students at different stages of mathematical development can become part of the classroom math community. By allowing students to use math discourse to explore multiple approaches to math problems, lesson 2 helps further develop a safe and successful classroom math community.

During this lesson responsibility begins to shift from the teacher to the students, and students are given a larger role in managing their peer discussions. Hufferd-Ackles, Fuson, and Sherin (2004) explain that in a math discourse community students are empowered to take on the responsibility for learning. Teachers can achieve a well-functioning math discourse community by focusing on questioning strategies, allowing students to explain their mathematical thinking, allowing students to be the source of mathematical ideas, and allowing students to take responsibility (Hufferd-Ackles et al., 2004). In Lesson 2 instruction is structured in a manner that encourages students to take more control of their math discussions. When solving problem #1, the students think first then share whole class. The whole class discussion provides a review of how to participate in a meaningful math discussion. When approaching problem #2, the students think first, but engage in math discourse with a peer before sharing their thinking whole class. The goal is for the students to become empowered to participate in meaningful math discussions with their peers while deepening their understanding of multiplication.

Student are empowered to make choices. At various points throughout lesson 2 students can choose to make changes to their original method or to challenge themselves to develop new methods for solving math problems. These small choices empower students to take responsibility for their learning.

Since a safe and empowered classroom math community is developing, the teacher can now choose math discourse partners based on formative assessments. In lesson 1 students selected their own partners. In Lesson 2 students are asked to work with a partner who is chosen by the teacher. This can be done because a safe environment in which mathematical thinking is honored and errors are accepted is beginning to be established.
Lesson 2: Math in the World of Restaurateuring

Focus: Using Problem-Based Instruction founded on Mathematical Discourse to Learn About Multiplication in the World Around Us

Materials:
* Math discourse partners worksheet (see resources, prepare partners in advance)
* Vocabulary journals or worksheets (should be in student desks from lesson 1)
* 1 copy of the How to Have a Math Discussion poster, Part 1 for display (see resources)
* Student copies of the How to Have a Math Discussion, Part 1 (see resources)
* Partner work posters (see resources)
* Question frames that promote math explanations, reasoning and evidence (should be in student desks, make copies as needed, see resources)
* Statement frames that promote connections and comparing (should be in student desks, or see resources)
* Cookie manipulatives (see resources)
* Muffin manipulatives (see resources)
* Chart paper for respect poster (create)
* Projector, document camera, or large screen for displaying problems
* Internet connection and ability to show a web-based video on a large screen
* Large display of two math practice problems (create)
* Math worksheet (see resources)
* A teacher example of an answer to problem 2 that includes an error (create)
* Baker’s hat (create)
* Math discourse rubric for teachers (see resources)
* Chart paper with the driving question, How is multiplication used in the world around us, written at the top (from lesson 1).

Set the Stage: Real World Problem Solving

- The teacher will display yesterday’s vocabulary. The teacher will ask students to get their vocabulary worksheets, or journal, and written math reflections out of their desks.

Explain: Tell the students that they will be given a few minutes to review the reflections and vocabulary from lesson 1. Tell students to think about what helped them become successful with yesterday’s math problem and three things they learned in lesson 1.
- The teacher will give the students a few minutes to reflect on their learning experience during lesson 1.
- After a few minutes, ask volunteers to share their reflections.
- During the discussion, refer back to the vocabulary words displayed on the large screen: repeated addition, multiplication, equal groups to reinforce the meaning of the words.
- Ask students to put their vocabulary words and written reflections back in their desks (They will be adding to these later in this unit).
- Tell the students that they will view a video of how math can be used in a real world situation.
- Show video clip: http://www.thirteen.org/get-the-math/the-challenges/math-in
discuss the video. During the discussion ensure that students discuss and understand that there are many ways that a restaurateur might use multiplication.

- Display the blank poster with the driving question, *how is multiplication used in the world around us?* Ask students if they think anything should be added to the poster after watching the video. Add a few ideas, but leave space for more ideas to be added throughout the following lessons.

**Explain:** Tell students that they will pretend to be restaurant owners today. They will be solving math problems that a restaurant owner or employee might experience. Reinforce the concept that today’s math problems are just a few examples of how math can be used in the world of restaurateuring. Remind students that at the end of this unit, they will create a project to answer the question, *how is multiplication used in the world around us.* Encourage the students to be thinking of different ways in which multiplication could be used in restaurants.

**Pose the Problem: Real World Problem Solving**

- The teacher will put on the baker’s hat and act as a baker.

**Practice Problem #1:** “My customers love decorative cookies. In fact, I am famous for my cookie decorations. Right now, I am decorating my famous star-style cookies. Mrs. McCrown made a special order. She ordered 6 cookies, and she wants 4 star-style decorations on each cookie. All my decorations are pre-made and in the freezer. How many star style decorations will I need to get from the freezer?” The teacher will act as a confused baker, “Ahh, I was never good at math. Please help me! How can I figure out how many star decoration I will need today? If I don’t get it right, Mrs. McCrown will be very upset.”

**Independent Problem Solving: Real World Problem Solving**

**Explain:** Write *Independent Thinking to Whole Class Discussion* on the front board. Tell students that they will work independently first, but in a few minutes they will be asked time to share their ideas and methods with the class.

- Pass out blank paper to all students.
- Make the cookie and decoration manipulatives (see resources) available to students. Allow students to choose their own manipulatives.
- **Monitor:** The teacher will give students about 10-15 minutes to create and complete their method for solving the math problem. Monitor students and engage in questioning to promote understanding (see resources and example).
  *While monitoring, look for student work to select for the whole group discussion. Look for work that contains multiple methods and or work that contains common errors that could be exposed and discussed for learning purposes.
  *While monitoring try to select at least two examples that show a repeated addition method.
  *While monitoring try to select at least two examples that show a multiplication method.
  *Select at least one multiplication method that shows the correct usage of factors. This will give the teacher the opportunity to reinforce the vocabulary words and correct use of first
factor and second factor.

Research Based Advice

Advice: When students are working independently, maintain the high-level task. Use a questioning strategy to encourage students to work through the problem.

Research: See advice, Lesson 1

Advice: The information gained from the formative assessment used in lesson #1, can be used to help the teacher determine which students will benefit from questions that promote understanding. For example, if students struggled to solve the fortune cookie question, they may benefit from questions that promote understanding.

Research: Moss and Brookhart (2009) explain that formative assessment can be used to close the achievement gap. Formative assessment can be used to continuously and systematically gather evidence of learning with the express goal of improving student achievement.

Example of questions to promote mathematical understanding

<table>
<thead>
<tr>
<th>Possible Questions to Promote Mathematical Understanding</th>
</tr>
</thead>
<tbody>
<tr>
<td>*What do you know about this problem?</td>
</tr>
<tr>
<td>*What do we need to find out?</td>
</tr>
<tr>
<td>*What information is necessary for you to solve this problem?</td>
</tr>
<tr>
<td>*How can you use what you know to solve this problem?</td>
</tr>
<tr>
<td>* Finally – What could be a first step or initial try you could attempt to solve this problem? Try taking a first step and go from there.</td>
</tr>
</tbody>
</table>

Source: Adapted from Small, et al., 2004

• When most students are finished, give the students a 3 minute “wrap up” reminder. Remind the students that you will be asking students to explain their solution method and mathematical thinking in a few minutes.

Whole Group Discussion: Problem Solving and Math Discourse

Explain: Remind students that some errors might occur, and that errors can help everyone in the class learn. Remind students to be respectful of other’s math methods and mathematical thinking.

• Get out the blank chart paper. Create a poster of respectful math discussions by asking students questions about respectful math discussions.

Possible Wording: “When a classmate comes to the front to share their math method and mathematical thinking we want to be respectful. What does that look like?” (wait for responses and discuss) Record a summary of the student’s ideas on the chart paper. “What does it sound like if you are being respectful about a mistake?” (wait for responses and discuss). “What should we do if one of our fellow classmate makes a mistake?” Record a summary of the student’s ideas on the chart paper.

Example of a respect math discussions poster

<table>
<thead>
<tr>
<th>Respect Math Discussions Poster</th>
</tr>
</thead>
</table>
**Explain:** Display and review the How to Have a Math Discussion Part 1 poster and the student question and statement frames that were introduced in lesson #1 (see resources and example). Explain that first students will be asked to explain their method and mathematical thinking. Then other students will use the question and statement frames to ask follow up questions and make connections and comparisons.

Example of How to Have a Math Discussion, Part 1

<table>
<thead>
<tr>
<th>How to Have a Math Discussion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Part 1: Explain Your Method</td>
</tr>
<tr>
<td>1. Explain how you solved the problem. Include math reasons that tell why you solved it this way.</td>
</tr>
<tr>
<td>Possible sentence starters…</td>
</tr>
<tr>
<td>“First I _____ because _____.”</td>
</tr>
<tr>
<td>“Then I _____ because _____.”</td>
</tr>
<tr>
<td>“Also, I _____ because _____.”</td>
</tr>
<tr>
<td>2. Provide evidence that shows why your method makes sense. Possible sentence starter…</td>
</tr>
<tr>
<td>“ _____ makes sense because _____.”</td>
</tr>
<tr>
<td>“This strategy works because _____.”</td>
</tr>
<tr>
<td>3. Provide connections and comparisons. Does this math problem or method remind you of another? How are they similar or different? Possible sentence starters…</td>
</tr>
<tr>
<td>“I have used this method before. I used this method when I _____.”</td>
</tr>
<tr>
<td>“I used a similar method when I _____, but this time I _____.”</td>
</tr>
<tr>
<td>“This problem reminds me of _____, but it is a little different because _____.”</td>
</tr>
<tr>
<td>“Honestly, I don’t think I’ve ever solved a math problem like this before.”</td>
</tr>
</tbody>
</table>

- Students should have the How to Have a Discussion poster, Part 1 and the question and statement frames in their desk from lesson 1. Ask students to get these materials ready.

Examples of question and statement frames

<table>
<thead>
<tr>
<th>Questions Frames - Promote Math Explanations, Reasoning and Evidence</th>
</tr>
</thead>
<tbody>
<tr>
<td>*Why did you _____?</td>
</tr>
<tr>
<td>*Why did you think that strategy would work?</td>
</tr>
<tr>
<td>*How did you know to _____?</td>
</tr>
<tr>
<td>*Can you prove to me that … makes sense.</td>
</tr>
<tr>
<td>*Can you explain why you _____?</td>
</tr>
<tr>
<td>*What convinced you that was the right answer?</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Statement and Question Frames: Promote Connections and Comparing</th>
</tr>
</thead>
<tbody>
<tr>
<td>I used a different method:</td>
</tr>
<tr>
<td>*I … and you…, but I understand how they both make sense.</td>
</tr>
<tr>
<td>*I … and you… I am having a hard time understanding your method.</td>
</tr>
<tr>
<td>*I … and you… I like how you… that thinking is new to me.</td>
</tr>
</tbody>
</table>

*Source: Adapted: Chapin et al., 2013*
We used the same method.
*You...and I...*

My method is similar to your method.
*You and I both...but I didn’t...*
*You and I both...but I...*
*I...but I didn’t....*

**Questions that promote Connections and Comparison**
*Does this math problem remind you of another problem?*
*Have you solved a math problem like this before?*
*Have you used this method before?*
***Can you create another problem that can be solved the same way?***

*Source: Adapted from Small et al., 2004 and Smith and Stein, 2011*

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**Research Based Advice**

**Advice:** Before beginning the whole group discussion, the teacher should recreate the **safe environment** that was created during lesson 1 by reminding the students that mistakes are natural, and they help us learn. Although errors were discussed while creating the poster about respectful math discussions, the teacher will have noticed common errors while monitoring and selecting student work, and the teacher will be bringing these errors to light during the whole group math discussion.

**Research:** See Research, Lesson 1

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- **Selecting:** Encourage students who you noticed during the **monitor/selecting** stage to come to the front.
- During the discussion support the discussion as needed by prompting students to use the **How to Have a Math Discussion** poster part 1.
- During the discussion make sure to point out the correct use of the first and second factors and reinforce the meaning of these two vocabulary words.
- During the discussion make sure to point out a multiplication equation. The students are still learning how to use multiplication to solve real world problems. It is important to point out a multiplication equation so that students begin to understand how numbers and equations can be used to solve real world problems and how equations can be used to represent information.
- If students come to the front with pictorial representations of their method that lack numbers or equations, encourage them to add the numbers and equations as they go. **How could you represent that with an equation?**
- If a student comes to the front to discuss his/her method and an error is exposed, make sure students notice the value in the error. Focus on how the error can help the class learn and improve.
- Call on 4 or 5 students to share their method. After each student comes to the front, allow that student to call on others to ask questions are make comments using the question or statement frames.
- Display and discuss the various methods used. While discussing the methods reinforce the concepts of multiplication, repeated addition, equal groups, first factor, and second factor.
Research Based Advice

Advice: During this discussion the students are learning how to participate in a meaningful math discussion, and they are learning multiplication concepts. The students are learning important multiplication concepts such as the relationship between multiplication and addition, how to represent solutions using multiplication equations, and how to use factors correctly. These multiplication concepts are being taught through problem-based instructional approaches which includes exposure to real-world problems, discussion, and questioning strategies. It is important for the teacher to ensure that students are exposed to these concepts during the discussion.

Research: Chapin, O’Connor, and Anderson (2013) explain that students need to experience and interact with math ideas to understand and remember concepts. Therefore, take time to ask questions that allow students to interact with and understand these multiplication concepts.

Pose the Problem: Real World Problem Solving

Explain: Display the poser, Independent Thinking to Peer Discussion to Whole Class Discussion (see resources). Explain the process of thinking first, then sharing with a partner, and finally sharing whole class. Tell students that they will have about 10-15 minutes to think independently. They will get wrap-up reminders before moving on to the next step in the process.

- Display Problem #2
  “Today, my customer, Mrs. McCrown ordered 4 muffins. She wants me to decorate the muffins with candied pecans. She wants 3 candied pecans on each muffin. How many candied pecans do I need to get from the freezer before I begin baking?”

Independent: Real World Problem Solving

- Pass out paper and make manipulatives available (see resources).
- Give the students a 10-15 minutes to solve the problem.
- Monitor: Walk around and monitor students as they work. Use the question frames that promote mathematical understanding (see resources) to provide scaffolding as needed. Remember to maintain the integrity of the high-level task.
- When most students are finished, give the students a 3 minute “wrap up” reminder, and remind them that they will be explaining their method to a partner in a few minutes.

Peer Discussion: Real World Problem Solving and Math Discourse

- Discourse Partners: The teacher should have the math discourse partners prepared in advance. Partners should be of mixed ability. Teachers can use the pre-assessment information and the math discourse partner page to create groups (see resources). If using the pre-assessment to make math discourse partners, review confidence Q#3 and confidence Q#4. Mix students who answered somewhat confident with students that answered very confident, or not confident. Take into consideration personality and students who work well together. If personalities match well, the teacher might consider matching two students who
stated that they were very confident. However, avoid putting a very confident student with a student that stated they were not confident.

**Explain:** Tell students that it is time to work with their partner. Tell students not to worry if they aren’t finished. Tell students they can explain what they have so far. The teacher will choose the discussion partners today.

- Review the process of discussing math methods and mathematical thinking with a partner by reviewing the poster titled, How to Have a Math Discussion, Part 1, and the questions frames that promote reasoning and evidence and the statement frames that promote connections and comparisons (see resources).
- Put up the partner work posters (see resources). Ask for students to read through the partner posters first. Then refer to them as you explain how to work with a partner.
- **Model:** The teacher will model how to have a peer math discussion using the partner work posters. The teacher should have a method to model that includes a mistake. Call a volunteer to the front of the class to model how to work with a partner. First, the teacher will explain his/her method and mathematical thinking. The volunteer partner will use the question and statement frames to discuss the method. The teacher and the volunteer student will switch roles. The teacher should model how to make changes with colored pencil. The teacher will think out loud. “Do I need to make any changes to my original method? Yes, I will change...Model how to make a change using colored pencil.
- Tell students who their partner will be. Give students a few minutes to rearrange their seats to sit with their new partner.
- Give students about 5 minutes to share their methods and answer questions. When most students are finished, give a 1 minute wrap up signal.
- Ask students to switch partner roles. Give students another 5 minutes to switch roles. When most students are finished, give a 1 minute wrap up signal. Then give students a few minutes to make any changes with colored pencil. Give a 1 minute wrap up signal.
- **Monitor:** While monitoring students, use the teacher’s math discourse rubric (see resources). Stop at various partner groups and listen to the peer discussion. Circle or highlight the various math discourse practices and levels that are apparent in each conversation (see advice and example in the formative assessment section of this lesson).

---

**Research Based Advice**

**Advice:** The teacher will not be able to observe all partners at this point in the lesson. Try to observe students at various levels of math ability. The information you gain from these discussions will not be comprehensive, but it will provide insight into the level of discussions that are currently occurring in the classroom. This information can be used to make instructional decisions in future lessons (see advice in the formative assessment section of this lesson).

**Research:** Chapin, O’Connor, and Anderson (2013) explain that when students participate in math discussions, teachers can see more clearly what students do and don’t understand. This helps teacher adjust their teaching. This process is referred to as formative assessment.
Whole Group Discussion: Real World Problem Solving

- Ask for 1-3 partner volunteers. When the partners come to the front, they should take turns explaining three things...
  1. Explain your method
  2. Explain your thinking
  3. Tell the questions that your partner asked.
  4. Explain any changes you made in colored pencil
  5. Explain any new methods you tried in colored pencil.
- If no one has volunteered who has an example of using colored pencil to make changes or try a new method, ask for a volunteer.
- If no one has volunteered who has an example of multiplication with the correct use of the first and second factors, ask for a volunteer to come to the front to model how to use multiplication and factors to correctly solve problem #2.

Apply What You’ve Learned: Mathematical Thinking and Math Discourse

- Pass out the multiplication and baking worksheet (see resources)
- Students will work on their own to complete the worksheet.
- When they finish, they will have an opportunity to work with a partner of their choice. They will engage in a math discussion with this partner. They will follow the math discourse partner procedures (see resources).
- Monitor: The teacher will walk around the classroom to informally assess how well each student is understanding multiplication. While monitoring students, avoid giving too much support. The students will get support from their discourse partner.
- Discourse Partners: Give students time to work with a discourse partner of their choice. Any changes should be recorded in colored pencil.

Extension Activity: Real World Problem Solving Game

- If students finish the worksheet early, it is a good idea to provide an extension activity.
- Today, if students finish early, they can play the Cookies and Stars Decoration Game (see resources and example).

Example of the Cookies and Stars Decoration Game

<table>
<thead>
<tr>
<th>Cookies and Star Decorations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baking and Multiplication Game</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>1. This game can be played with 1-3 players.</td>
</tr>
<tr>
<td>2. Get a scratch paper and divide it into three or four sections.</td>
</tr>
<tr>
<td>3. If you are playing on your own, you will complete all the sections on your own. If you are playing with other players, you will share the scratch paper.</td>
</tr>
<tr>
<td>4. Roll the dice to determine how many cookies you will be baking today (Player 1).</td>
</tr>
</tbody>
</table>
5. Use scratch paper to draw the number of number of cookies (Player 1).
6. Roll the dice again to determine how many star decorations your customer wants on each cookie (player 2).
7. Use the same scratch paper to draw the number of star decorations (player 2).
8. Record the total number of stars. Try to write a multiplication equation to match the picture (player 3, or player 1).

Source: Adapted from: Burns, M. (2007) About Teaching Mathematics, Circles and Stars

Formative Assessment

- This lesson does not contain a formal assessment piece. However, formative assessment will be used in two ways.
- The teacher will use the math discourse rubric for teachers to assess student’s math discussions. The information can be used to plan questioning strategies in future lessons.

Example of Teacher’s Math Discourse Rubric

<table>
<thead>
<tr>
<th>Math Discourse Overview</th>
<th>Score 1</th>
<th>Score 2</th>
<th>Score 3</th>
<th>Score 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>*did not uncover or analyze errors</td>
<td>During the math discussion students...</td>
<td>*did not uncover or analyze errors</td>
<td>During the math discussion students...</td>
<td>*uncovered and/or analyzed errors as needed</td>
</tr>
<tr>
<td>*did not make connections between strategies or ideas of others only compared answers.</td>
<td>*made one connection between strategies and compared different strategies when applicable.</td>
<td>*made a few connections between and compared strategies when applicable.</td>
<td>made all possible connections between strategies and compared strategies when applicable.</td>
<td></td>
</tr>
<tr>
<td>*did not ask/provide follow-up questions/statements to promote math explanations, reasoning, and evidence.</td>
<td>*asked/provided 1 or 2 follow-up questions/statements to promote math explanations, reasoning, and evidence.</td>
<td>*asked/provided a few follow-up questions/statements to promote math explanations, reasoning, and evidence.</td>
<td>*asked/provided follow-up questions/statements to promote math explanations, reasoning, and evidence.</td>
<td></td>
</tr>
<tr>
<td>*did not attempt to ask for provide conjectures or predictions.</td>
<td>*made one attempt to ask for provide conjectures or predictions, but it didn’t fully make sense.</td>
<td>*ask for/provided one or two conjectures or predictions.</td>
<td>*ask for/provided conjectures and predictions.</td>
<td></td>
</tr>
</tbody>
</table>

Source: Rubric Adapted from Park, J., Nava, I., & Applegate, M., (2011)

Note: The teacher will circle or highlight the math discourse practices that are observed during the peer discussions. At this point, it is not necessary to focus on an overall score of 1,2,3, or 4. Students may not fall completely within one level. For example, a student might make level #3 connections and comparisons. However, that same student may only attempt 1-2 questions/statements that promote reasoning or evidence; thus, giving that student a score of 2 on these math discourse practices. The teacher can use this information to determine which type of questioning strategies will be used in the following lessons. For example, if most students receive a level 3 or level 4 in making connections and comparison, but most students receive a level 2 for explanations, reasoning, and evidence, the teacher can model more question frames that promote reasoning and evidence during the next lesson. Also, a student may provide explanations and reasoning, but may fall short when trying to provide evidence. If this is the case, the teacher can highlight explanations and reasoning, but leave evidence...
blank. By leaving the word evidence blank, the teacher will know to model questions that promote evidence in the following lessons.

**Note:** Many students may not be at the point in which they are providing level 3 and level 4 conjectures or predictions. These math discourse practices have only been introduced, and it has been my experience that these math discourse practices can be challenging for third grade students. However, they should develop over time with continued practice and support.

- Also, the teacher will informally assess the baking worksheet once it is completed. The baking worksheet can be used to assess student’s problem solving ability as well as their ability to use math discourse to improve their problem solving methods.

**Teacher Notes for Lesson 2:**
Multiplication in the World Around Us

Lesson 3– Similarities and Differences, Sorting Math Problems

Lesson 3 Goals

Problem-Based Learning Goals
- Students will read math problems and look for patterns.
- Students will solve real world math problems including repeated addition problems, multiplication problems, and basic addition problems.
- Students will learn that some real world math problems require more than one math process.

Math Discourse Goals
- Students will review math vocabulary words.
- Students will sort and discuss math problems based on patterns that are noticed.
- Students will review and discuss the differences between basic addition problems, multiplication problems, and repeated addition problems.

Writing in Math Goals
- Students will write their own real world math problems.

Math Standards Addressed in Lesson 3

Standard for Mathematical Practice (SMP)

SMP #1 - Make sense of problems and persevere in solving them.
SMP #3 - Construct viable arguments and critique the reasoning of others.
SMP #6 - Attend to precision.

Operations and Algebraic Thinking, Grade 3 (OAT)

OAT Standard #1 – In this lesson students will interpret products of whole numbers

OAT Standard #3 - In this lesson students will use multiplication within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities.
Lesson 3 – Similarities and Differences, Sorting Math Problems

Research Based Rationale

Lesson 3 is based on the premise that sorting and discussing math problems can support students as they develop math concepts, problem solving skills, and the ability to engage in meaningful math discourse. Fuchs, Fuchs, Prentice, Hamlett, Finelli, and Courer, (2004) found that students can learn to see math patterns by sorting and discussing math problems. Structuring lessons to allow students to seek and notice patterns can help students make connections between math problems. Sorting problems can help students begin to make connections between concepts as they are presented in math problems (Fuchs, et al., 2004). In lesson 3 students engage in math discourse while sorting math problems in both small group and whole class settings. The sorting activities are designed so that students can further develop multiplication concepts. Specifically, by sorting math problems, students will develop a deeper understanding of multiplication, repeated addition, basic addition, and multistep math problems. While sorting math problems and developing math concepts, students will engage in math discourse to further develop their understanding of multiplication concepts. Mueller and Maher’s (2009) research showed that in settings where learners were encouraged to engage in math discourse and use one another as a resource, students learn to extend initial math ideas to create deeper forms of reasoning. Chapin et al. (2009) support this idea when they state that math discussions can play a critical part in helping students improve their ability to reason mathematically.

Lesson 3 continues to present multiplication concepts through a problem-based instructional approach. Students are allowed to learn multiplication concepts and develop their understanding of multiplication by working through real world problem situations. When solving such real world math problems, students are exposed to multiple methods. Brendefur, Thiede, Strother, Bunning, and Peck, (2013) explain that when students solve meaningful math problems and examine multiple approaches, they can become mathematical thinkers, and these critical thinking skills can be applied inside and outside the classroom.

In lesson 3 students write their own math problems. The assignment in which students are asked to write their own math problems, is structured in a manner so that students are able to exhibit their understanding of the math lesson by writing their own math problems. The student-produced math problems are used as a formative assessment. Borosi and Rose (1989) found that through evaluation of math writing, teachers become aware of the individual needs of students, and teachers can plan instruction in response to students’ level of understanding.
Lesson 3: Similarities and Differences, Sorting Math Problems

Focus: Using Problem-Based Instruction founded on Mathematical Discourse to Learn About Multiplication in the World Around Us

Materials:
* Four practice problems to cut and sort (see resources)
* Extra copy of four practice problems (see resources, copy and pre-cut before the lesson)
* List of math word problems for students to cut and sort (see resources)
* Extra copy of math word problems for students (see resources, copy and pre-cut before the lesson)
* Vocabulary worksheets or journals (from lesson 1 should be in student’s desks)
* Document camera, projector, or large screen for displaying problems
* 4-3x5 cards with a basic + on one, a X on another, and repeated + written on one card, and a X and + written on one card (see resources)
* 3-3x5 cards with a basic + on one, a X on another, and a X and + on one card (see resources)
* Procedures for sorting math problems with a partner
* Chart paper with driving question written at the top (previous lessons)
* Chart paper to use for modeling how to write real world math problems
* Guidelines for writing today’s math problems lesson 3 (see resources)
* Math discourse rubric information from lesson 2 (use information from the rubric to choose question and statement frames that will further student’s ability to engage in math discourse)
* Question frames that promote math explanations, reasoning and evidence (should be in student desks, make copies as needed, see attachments)

Set the Stage: Review Multiplication Concepts and Vocabulary

- Ask the students to review their vocabulary words from lesson 1 (worksheets or journals)
- Give students a few minutes to read over their vocabulary worksheets or journals.
- Practice the kinesthetic math vocabulary activities from lesson 1 for the vocabulary words repeated addition, multiplication, first factor and second factor.
  * The teacher makes the addition sign and says “Teach repeated addition.” The students make the sign and practice saying the corresponding words with the corresponding gestures.
  * The teacher makes a multiplication sign and says “Teach multiplication.” The students make the multiplication sign and practice saying the corresponding words with the corresponding gestures.
  * The teacher puts up one finger and says, “Teach first factor.” The students put up one finger for the first factor and practicing saying, “The first factor represents the number of equal groups,” while making the corresponding gestures.
  * The teacher puts up two fingers and says, “Teach second factor.” The students put one two fingers for second factor and practice saying, “The second factor represents how many in each equal group.” while making the corresponding gestures.

Explain: Explain that today the class will be reading math problems and sorting them into groups. Students will work with a partner to sort the math problems. The math problems can be sorted into groups based on the methods used to solve each problem. For example, problems can be sorted into groups such as, repeated addition, multiplication, basic addition, or a mixture of
multiplication and basic addition.

- Ask the students questions about repeated addition, multiplication, and basic addition to engage their thinking. The concept that repeated addition and multiplication are similar that was established in lesson 1 should be reviewed.

Possible Wording: “What do you think are the similarities between repeated addition and multiplication?”

(Possible Response: “When a math problem has equal groups you can multiply or you can use repeated addition. When you use repeated addition the groups have to be equal, and when you multiply the groups have to be equal.” Use revoicing and/or restating frames)

Possible Wording: “What are the similarities between basic addition and repeated addition?”

(Possible Response: “You are adding groups together, or you are joining groups to find how many in all.” Use revoicing and/or restating frames)

Possible Wording: “What do you think are the differences between basic addition and repeated addition?”

(Possible Response: “Repeated addition means the groups are equal and you repeated the same number by adding it over and over again, but when you are adding the groups don’t always have to be the same. For example, you can add 6 and 4 and get 10. That is basic addition.” Use revoicing and/or restating frames)

- Students can put the vocabulary word worksheets or journals back in their desks for use in later lessons.

### Set the Stage: Sorting Real World Math Problems

**Explain:** Today students will be sorting math problems into categories. Explain that one of the categories will be problems that require both multiplication and basic addition.

- Ask questions to see if the students can think of situations in which they might need to multiply and add.

- Use revoicing frames to enhance the discussion (see resources and example)

### Example of Revoicing Techniques

<table>
<thead>
<tr>
<th>Revoicing Techniques</th>
<th>Help students access other’s thinking and provide clarification.</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>... pointed out that... Do you agree or disagree?</em></td>
<td></td>
</tr>
<tr>
<td><em>So you are saying that...</em></td>
<td></td>
</tr>
<tr>
<td><em>Can ...say what ...said so I can be sure I understand.</em></td>
<td></td>
</tr>
<tr>
<td><em>Who can put that into their own words?</em></td>
<td></td>
</tr>
<tr>
<td><em>Let me see if I understand. Are you thinking...?</em></td>
<td></td>
</tr>
<tr>
<td><em>Who can repeat what ...said?</em></td>
<td></td>
</tr>
</tbody>
</table>

*Source: Adapted from Chapin et al., 2009 and Chapin et al., 2013*

### Note: If students don’t think of correct ways in which multiplication and addition may occur together within a math problem, honor their thinking, and explain that they will experience at least one math problem today in which multiplication and basic addition are needed to solve the problem. Advise them to look for such a problem.

**Note:** If students can’t think of any ways in which multiplication and addition may be used to solve a math problem, and they don’t offer any ideas, tell them not to worry. Explain that in today’s lesson the students should discover at least one problem in which multiplication and basic addition are used
to solve the problem.

**Pose the Problem: Sorting Real World Math Problems**

- Display the four practice math problems (see resources)

<table>
<thead>
<tr>
<th>Research Based Advice</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Advice:</strong> Teachers are welcome to use the math problems provided as part of this curriculum handbook. However, if an adopted math program has math word problems that are preferable, please feel free to substitute math problems as needed. However, please be aware that the problems contained in this handbook include different styles of multiplication problems. Some of the problems can be referred to as repeated addition style multiplication and some of the math problems can be referred to as rate style multiplication. It is important for students to be exposed to both multiplication styles. If using other math problems, it would be advisable to include both repeated addition and rate style problems.</td>
</tr>
<tr>
<td><strong>Research:</strong> Van de Walle and Lovin (2006) state that both repeated addition style multiplication problems and rate style multiplication problems represent the largest percentage of multiplication problems in the real world. There is a subtle difference between the math problems that can be termed repeated addition and the math problems that can be termed rate.</td>
</tr>
</tbody>
</table>

**Explain:** Tell the students that today they will be solving more real world math problems. Explain that today’s math problems are not necessarily problems that would occur in a restaurant because math is all around us. Explain that some of the problems will require multiplication or repeated addition, some will require basic addition, and at least one problem will require the students to multiply and add. Tell the students that they will need to listen carefully to each problem. Explain that as the students listen, they should try to determine what method they would use to solve each problem. Tell students that they can use scratch paper to record their methods.

- Pass out scratch paper and ask the students to divide the scratch paper into 4 sections.
- Write these four options on the board: +, ×, repeated +, or + and ×.
- Tell students they can use these symbols to record the method they would use for each problem by writing one symbol in each section of their scratch paper as the teacher reads each math problem.
- Read each problem. Pause after each problem and prompt the students to record their chosen method on the scratch paper.
- After each question has been read and students have recorded their methods, cut out the four practice problems.

**Explain:** Tell the students that now, as a group, you will sort each math problem into a category.

- Put the 4 x 5 cards labeled +, ×, repeated +, and + and × on the front board. Set them out so that the math problems can be sorted into categories under the 3x5 cards.
- Reread each problem one at a time.
- After each problem, pause and ask for a volunteer to come to the front and put the math problem in the student’s chosen category (students can refer to their scratch paper for help).
• Ask the student volunteer a few questions using the question frames that promote reasoning and evidence (see resources and example). This will help reveal the student’s thinking to the rest of the class.

**Research Based Advice**

*Advice:* During this math discussion, the teacher can use information gained from the using the math discourse rubric for teacher to choose questions. For example, if the teacher noticed that many students were providing explanations and reasons, but struggling to provide evidence, the teacher might focus on questions that promote evidence such as, "Can you prove to me that ... makes sense?" Or "What convinced you that was the right answer?"

*Research:* See research on formative assessment, Lesson 2

Example of question frames that promote explanations, reasoning, and evidence

<table>
<thead>
<tr>
<th>Questions Frames - Promote Math Explanations, Reasoning and Evidence</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>Why did you ....?</em></td>
</tr>
<tr>
<td><em>Why did you think that strategy would work?</em></td>
</tr>
<tr>
<td><em>How did you know to ....?</em></td>
</tr>
<tr>
<td><em>Can you prove to me that ... makes sense.</em></td>
</tr>
<tr>
<td><em>Can you explain why you...?</em></td>
</tr>
<tr>
<td><em>What convinced you that was the right answer?</em></td>
</tr>
</tbody>
</table>

*Source:* Adapted From Small et al., 2004, Smith and Stein, 2011, Chapin et al., 2013

• After a student volunteer has put a math problem into a category, ask the rest of the class for a thumbs up if they agree. If the teacher sees a student who does not have a thumbs up, ask the student if they used a different method.

• **If there are different methods shared,** have the student come to the front and move the math problem to the new category. The teacher should ask the student a few questions using the question frames that promote math explanations, reasoning, and evidence.

**Note:** At this point it is possible that some problems (problem 1 and 3) will fit into two categories. Use questioning to help students think through the fact that some problems can be in both the repeated addition category and the multiplication category.

**Possible Wording:** "--- says that problem 1 belongs in the X category, and --- says that problem 1 belongs in the repeated + category. Can anyone explain why they are both correct?"

• Sort and discuss all four practice problems.

• Sometime during the math discussion put the extra copy of problems 1 and 3 up on the board so that students can see that these problems belong in both the multiplication and repeated addition categories.

**Note:** If students don’t share multiple methods for problems #1 and #3 during the sorting process, discuss these problems before moving on. Use questioning strategies to ensure that the students understand that some problems can be solved using both repeated addition and multiplication.
Possible Wording: "____ said that problem 1 belonged in the repeated + category. I agree with her/him, but originally, I thought it belonged in the multiplication category. Can anyone explain why problem 1 can be placed in both categories?" Pause for a discussion. Then ask, "Are there any other problems that could be placed in more than one category?"

• After sorting and discussing each practice question, discuss question #2. Use question frames that promote reasoning so that all students understand that two math processes (multiplication and addition) are used to solve this problem.

Note: Problem #2 can be referred to as a multi-step problem. Third grade students can struggle when presented with this type of math problem. Chapin, O’Connor, and Anderson (2013) explain that talking about the different stages of the problem-solving process can help students understand and solve a wide variety of problems including multi-step problems.

Possible questions to promote reasoning: “Can you prove to me that if Shayla made 6 regular baskets and each regular basic is worth 2 points that you could multiply? (pause and discuss, and use revoicing and restating strategies as needed) Can you explain why you would add 3 more points for the three point shot? (pause and discuss, and use revoicing and restating strategies as needed) Why wouldn’t you multiply if she made one 3 point shot?" (discuss and use revoicing and restating strategies as needed)

Research Based Advice
Advice: While questioning students, record their thinking on the front board so that all students can refer to the math thinking of their classmates. The teacher can choose to record student thinking in words or in symbols. Also, the teacher should consider using revoicing and restating strategies while he/she records a student’s mathematical thinking on the board. Revoicing and restating strategies will help connect to the math discussion and deepen their understanding of problem #2.

Research: Chapin, O’Connor, and Andersen (2013) discuss the need to help students externalize their thinking so that others can respond and connect to it.

• Once the teacher has asked questions that promote reasoning, ask a few questions that promote predictions, conjectures, and justification. Teachers can use the question frames that promote predictions, conjectures, and justification (see resources and example).

Possible questions to promote justification: “How do you know for sure that you must add the 3 for the three point shot? Would we still add 3 for the three point shot if Shayla made two 3 point shots? Why not? Can you think of another situation in which you might add after you multiply?"

Example of question frames that promote predictions, conjectures, and justification

<table>
<thead>
<tr>
<th>Question Frame: Promote Predictions, Conjectures, and Justification</th>
</tr>
</thead>
<tbody>
<tr>
<td>*Can you state a general rule you have discovered?</td>
</tr>
<tr>
<td>*Would this still be true if you change…?</td>
</tr>
</tbody>
</table>
Peer Discussion: Sorting Real World Math Problems

**Explain:** Tell students they will be working with a partner to sort math problems. Explain that the teacher will choose the partners today. Explain that the students will cut out math problems and sort them into three categories. The categories will include multiplication or repeated addition, basic addition, or multiplication and addition.

**Note:** The teacher can decide how to group the peer discussion partners. The teacher might decide to keep the same partners that were used in lesson #2. However, if some partner groups did not work well, adjustments can be made. Remember to refer to the math discourse partner page (see resources) to create pairs of mixed confidence levels while taking into consideration personality (see on discourse partners in lesson 2 for details on grouping peer discussion partners).

- Tell students who their partners will be and ask them to move around so that they are sitting with their partner.
- Put up the procedures for sorting math problems with a partner (see resources). Read the procedures together.

**Explain:** Reinforce that students should read each math problem together. The students should not read the problems separately. The students should decide together which category each math problem belongs in. Tell the students that if they disagree with their partner on the method, leave the problem to the side. Explain that if they finish early, each student should choose one problem from each category to solve.

- Pass out the math problems (see resources or use your own) and ask the students to begin.
- **Monitor:** The teacher should monitor the students as use question frames that promote explanations, reasoning, and evidence (see resources) as needed. When choosing questions from the question frames remember to use information gained from the rubric used in lesson 2. Also, look for good discussions and common errors that can be shared during the whole class discussion.
- Give a 3 minute wrap up notice when most students are finished discussing and sorting the problems with a partner.

Whole Group Discussion: Sorting Real World Math Problems

- The teacher should have a pre-cut copy of the eight math problems (see resources). The teacher should have the 3 3x5 cards (see resources) on the front board so that the math problems can be sorted into categories.
- Ask student to wrap of their conversations.
**Explain:** Tell students that it is time to share their mathematical thinking. Explain that if they aren’t finished sorting the eight problems that is perfectly fine. Tell the students that they will have the opportunity to finish when sorting the problems together as a group.

- Ask a student to read question #1 aloud
- Ask for a volunteer to come to the front to place math problem #1 in the correct category. The volunteer will do this by placing the math problem on the front board under the correct 3x5 card.
- Once the student volunteer has placed the math problem on the front board, use question frames that promote reasoning and evidence (see resources).
- While monitoring the peer discussion, if the teacher heard a good discussion and/or a common error that should be shared, call on these students to explain their solution. If not, call on a student volunteer to explain how they solved, or would solve, math problem #1.
- As the student explains his/her method and solution, the teacher will record the method on the front board for everyone to see.
- Use question frames that promote reasoning and evidence as the student explains the method and solution. By using questioning strategies, one student’s thinking can be revealed to the entire class.
- Repeat this process for each of the eight math problems.
- When all eight math problems have been sorted and solved, have a discussion about the problems. Ask students if there are any new types of problems to add to the chart titled, how is Multiplication Used in the World around Us?
- Display the chart titled, how is Multiplication Used in the World Around Us?
- Brainstorm possible real world situations in which one might multiply or multiply and add. Add these ideas to the chart with the driving question, written at the top.

**Possible Ideas:** sporting events, restaurants, packaging, eating, party preparation, buying things…many more

**Set the Stage: Writing in Math / Writing a Real World Math Problem**

**Explain:** Tell students that today they will write their own math word problems. They will be writing two math problems. One problem should involve basic multiplication. One problem should involve both multiplication and addition. Introduce the guidelines for writing today’s real world math problems (see resources). Read the guidelines together.

- Model how to use the ideas on the chart with the driving question at the top to write a real world math word problem that requires multiplication. Do this as a shared writing experience using think alouds and questioning strategies.

**Research Based Advice**

**Advice:** When modeling how to write a real world math problem, use questioning techniques and think aloud strategies rather than a direct instruction model. By using questions and think alouds, the students will be exposed to the thoughtful process of writing and solving math problems. If taught through a direct instruction approach, students might mistakenly believe that there is one way to create and write a math problem.
Research: Chapin, O'Connor, and Anderson (2013) explain that telling students how to do something through the direct instruction method, is not always sufficient. Students need to experience and interact with math ideas to understand and remember concepts (Chapin, et al., 2013).

- Model how to use the ideas from the chart to write a real world math problem that requires both multiplication and addition. Use a shared writing technique involving think alouds and questioning strategies

Writing in Math: Write a Real World Math Problem

**Explain:** Tell students that they can use any ideas from the chart to write two math problems. Explain that they should solve both problems once they are finished.
- Pass out paper, give students time to write and solve two real world math problems
- Collect the finished math problems
- When students are finished, ask them to spend some time at home thinking about other real world situations in which one might multiply.

**Explain:** Tell the students that later the class will want to add more ideas to the chart titled, *how is multiplication used in the world around us?*

- **Search for Equal Groups in Real Life:** Ask the students to think about things that come in equal groups. Explain that if they find things that come in equal groups, they will be on their way to finding real world situations in which one can use multiplication. Tell the students that they can ask their parents or sibling for ideas as well. Ask students to bring 2-3 ideas of things that come in equal groups to share with the class during the next math lesson.

  *Source:* Adapted from Burns (2007). Things That Come in Groups

Formative Assessment:

- Both the peer and whole group discussion will provide insight as to how well students are understanding the concepts of multiplication, repeated addition, and basic addition.
- As the teacher monitors students during the peer sorting activity, the teacher will have the opportunity to notice which students are able to sort the math problems correctly.
- The teacher will collect the student’s written math problems and assess the problems for two things. First, the teacher will be able to assess how well the students understand the concepts of multiplication and basic addition by how well they are able to create a real world math problem for these two concepts. Secondly, the teacher will be able to assess how well students are able to solve real world multiplication and addition problems by assessing the student’s solutions.

Teacher Notes for Lesson 3:
Multiplication in the World Around Us

Lesson 4 – Self-Evaluation and Math Discourse

Lesson 4 Goals

Problem-Based Learning Goals
- Students will sort and solve real world math problems involving multiplication and addition.

Math Discourse Goals
- The students will record their peer math discussions.
- The students will use a rubric to evaluate their math discussions.
- Students will evaluate their math discussions in a manner that allows them reflect upon how to improve their level of math discourse.

Math Standards Addressed in Lesson 4

Standard for Mathematical Practice (SMP)
- SMP #1 - Make sense of problems and persevere in solving them.
- SMP #3 - Construct viable arguments and critique the reasoning of others.
- SMP #6 - Attend to precision. Students will use clear definitions in discussions with others.

Operations and Algebraic Thinking, Grade 3 (OAT)
- OAT Standard #1 – In this lesson students will interpret products of whole numbers,
- OAT Standard #3- In this lesson students will use multiplication within to solve word problems in situations involving equal groups, arrays, and/or measurement quantities.
Lesson 4 –Self Evaluation and Math Discourse
Research Based Rationale

Although lesson 4 continues to utilize a problem-based approach to instruction founded on math discourse principles, the focus of lesson 4 changes from teaching students how to participate in math discussion while solving real-world math problems to teaching students how to evaluate and self-monitor the problem solving and math discussion processes. Papanikolaou and Boubouka (2010) note that metacognitive skills, such as self-monitoring, are necessary when students learn through the problem or project-based instructional approach. Chapin, O’Connor, and Anderson (2013) connect math discourse to metacognition by explaining that properly implemented math discussions can help students realize what they do or do not understand. Becoming aware of what they do and do not know, allows students to adjust their reasoning and improve their metacognitive abilities (Chapin, et al., 2013). In lesson 4, instruction is designed to support students’ metacognitive abilities.

In lesson 4, students are presented with two standards for mathematical practice (SMPs) stated in student-friendly terms, students are taught to use a math discourse rubric to evaluate and improve math discussions, and students learn to use recorded math discussions as a learning tool. First, SMP #1 and #3 are presented to students in student-friendly terms. By presenting students with a clear definition of SMP #1 and #3, students have the opportunity to self-monitor their progress toward achieving each standard. Second, students learn to use a math discourse rubric to evaluate and improve their math discussions. By evaluating math discussions, student’s metacognitive abilities can be enhanced. In addition, the recorded math discussions support students’ metacognitive abilities. The recorded discussions are evaluated by the students. By listening to their own recorded conversations and the recorded conversations of their peers, students are able to learn to self-evaluate and self-monitor their math discussions. Chapin, O’Connor, and Anderson (2013) found that the process of explaining one’s own thinking aloud helps students to clarify their own ideas and correct their own mistakes. Math discussions help students learn to ask questions that will help them modify and deepen their own mathematical understanding. In short, math discussions support students as they learn how to self-monitor their mathematical thinking and understanding (Chapin, et al., 2013). The recorded math discussions provide students with the opportunity to explain their thinking aloud and to return to the discussion for the purposes of improving math discourse and deepening mathematical understanding.

As in lesson 3, in lesson 4 students are asked to sort and discuss math problems. Teachers can substituted the provided math problems for math problems from an adopted math program. When choosing math problems, the goal is to provide problems that encourage reasoning and communication. Yeatts, Battista, Mayberry, Thompson, and Zawojewski (2004) state that good math problems encourage students to communicate with one another. They introduce students to new concepts, and they provide a meaningful context for applying strategies and skills. The math problems provided in lesson 4 have been designed to meet these goals.
Lesson 4: Self-Evaluation and Math Discourse

**Focus:** Using Problem-Based Instruction founded on Mathematical Discourse to Learn About Multiplication in the World Around Us

**Materials:**
* Three student copies of the math discourse rubric for students (see resources). One copy can be single sided, and the other copy should be double sided.
* Chart with driving question at the top, *how is multiplication used in the world around us* (from previous lessons)
* Copy of SMP #1 and SMP #3 to display (see resources)
* Poster, How to Have a Math Discussion, Part 1, Part 2, and Part 3 to display (see resources)
* Student copies of How to Have a Math Discussion Part 2 and Part 3 (see resources)
* Document camera or projector, large display screen
* Example problem to use for modeling (see resources)
* At least 10 recording Devices (any recording device can be used, but Ipad apps provide students with the opportunity to record both their discussion and their work, see notes in lesson 4)
* Procedures for sorting math problems with a partner (see resources)
* Respect Math Discussion Poster (see resources)
* 3 – 5x5 sorting cards X and repeated +, basic +, and X and + (see resources)
* Multiplication word problems (use word problems from the existing math program, or supplement with the attached math problems)
* Copy of two writing prompts to display (see resources)
* Copies of the math discourse rubric for teachers (see resources)
* Student copies of How to Have a Math Discussion, Part 1, question frames that promote explanations, reasoning, and evidence, question and statement frames that promote connections and comparisons (these should be in student desks, make copies as needed)
* Copies of the Cookies and Decoration game for early finishers (see resources)

**Set the Stage: Math Discourse**

**Explain:** Tell students that they will be sorting and solving math problems. Once they have sorted and solved each problem with a partner, they will choose one math problem to use for recording purposes. Explain that each student will use a recording device to record their math discussion.

- Engage students in a discussion to review math discourse practices such as explaining your reasoning, providing evidence, making connections between methods or problems, comparing methods or problems, and actively listening to your partner. Students may discuss the practices of making conjectures or predictions. However, these practices have only been introduced so a deep understanding should not be expected.
- Once the students have shared their ideas, put up a copy of SMP #1 and SMP #3 (see resources), and read them together.

**Explain:** Reinforce that SMP #3 illustrates that all third graders should learn how to construct good mathematical arguments and learn how to critique the math ideas of others in a constructive way that promotes learning. Reinforce that SMP #1 illustrates how students can persevere when solving challenging math problems. Make connections between these SMPs and the math problem solving, math discussions, and math discourse practices that have been used in class.
- Display the How to Have a Math Discussion poster Part 1, Part 2, and Part 3 (see resources and example). How to Have a Math Discussion poster Part 1 was introduced during lesson 1 and practiced during lesson 2. How to Have a Math Discussion part 2 and part 3 put everything that was taught and practiced in lesson 1 and lesson 2 into one poster.

**Explain:** The goal of the How to Have a Math discussion posters are to guide students as they begin to fully engage with SMP #3. Explain that good math discussions also help students become strong math problem solvers so that we can attain the goals of SMP #1.

**Example of How to Have a Math Discussion Posters**

<table>
<thead>
<tr>
<th>How to Have a Math Discussion</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Part 1: Explain Your Method</strong></td>
</tr>
<tr>
<td>1. <strong>Explain how</strong> you solved the problem. Include <strong>math reasons</strong> that tell <strong>why</strong> you solved it this way.</td>
</tr>
<tr>
<td>Possible sentence starters...</td>
</tr>
<tr>
<td>“First I _______ because _______.”,</td>
</tr>
<tr>
<td>“Then I _______ because _______.”,</td>
</tr>
<tr>
<td>“Also, I _______ because _______.”</td>
</tr>
<tr>
<td>2. <strong>Provide evidence</strong> that shows why your method makes sense.</td>
</tr>
<tr>
<td>Possible sentence starter...</td>
</tr>
<tr>
<td>“______ makes sense because _______”</td>
</tr>
<tr>
<td>“This strategy works because _______”</td>
</tr>
<tr>
<td>3. <strong>Provide connections and comparisons.</strong> Does this math problem or method remind you of another? How are they similar or different?</td>
</tr>
<tr>
<td>Possible sentence starters...</td>
</tr>
<tr>
<td>“I’ve used this method before. I used this method when I _______”</td>
</tr>
<tr>
<td>“I used a similar method when I ______, but his time I ______”</td>
</tr>
<tr>
<td>“This problem reminds me of _______.”</td>
</tr>
<tr>
<td>“This problem reminds me of _______, but it is a little different because _______.”</td>
</tr>
<tr>
<td><strong>Part 2: Have a Math Discussion</strong></td>
</tr>
<tr>
<td>1. Math partners should always <strong>actively and carefully listen.</strong></td>
</tr>
<tr>
<td>2. Ask follow-up questions that promote <strong>explanations, reasoning, and evidence.</strong></td>
</tr>
<tr>
<td>Possible Sentence starters...</td>
</tr>
<tr>
<td><em>Why did you...?</em></td>
</tr>
<tr>
<td><em>Why did you think that strategy would work?</em></td>
</tr>
<tr>
<td><em>How did you know...?</em></td>
</tr>
<tr>
<td>*Can you prove to me that... makes sense. *</td>
</tr>
<tr>
<td><em>Can you explain why you...?</em></td>
</tr>
<tr>
<td><em>What convinced you that was the right answer?</em></td>
</tr>
<tr>
<td>3. Make statements or ask questions that promote <strong>connections and comparisons.</strong></td>
</tr>
<tr>
<td>I used a different method...</td>
</tr>
<tr>
<td>“I... and you...”, but I understand how they both make sense.</td>
</tr>
<tr>
<td>“I... and you...”, I am having a hard time understanding your method.</td>
</tr>
<tr>
<td>“I... and you... I like how you... that thinking is new to me.</td>
</tr>
<tr>
<td>We used the same method...</td>
</tr>
<tr>
<td>“You... and I...”</td>
</tr>
<tr>
<td>My method is similar to your method.</td>
</tr>
<tr>
<td>“You and I both... but I didn’t...”</td>
</tr>
<tr>
<td>“You and I both... but I...”</td>
</tr>
<tr>
<td>“I... but I didn’t...”</td>
</tr>
<tr>
<td><strong>Questions that promote Connections and Comparisons</strong></td>
</tr>
<tr>
<td><em>Does this math problem remind you of another problem?</em></td>
</tr>
</tbody>
</table>
*Have you solved a math problem like this before?
*Have you used this math method before?
* Can you create another problem that can be solved the same way?

**Part 3: Look for patterns and make predictions and conjectures.**

1. Once you have shared your math methods and explanations with a partner, look for **patterns** and have a discussion about the patterns you observed.
2. Try to make **predictions** or **conjectures** based on the patterns you observed.
3. Provide **justification** that your predictions or conjectures make sense.

Possible sentence starters...
- I notice a pattern. It is __________.
- I think I noticed a math rule. It is __________.
- If I changed __________ the pattern would still be true.
- If I changed __________ it would change the pattern because __________.
- Let me tell you another example of the math rule I discovered.

- Discuss the poster together. Explain that today students will focus on part 1 and part 2.
- Students should have part 1 in their desks, pass out How to Have a Math Discussion Part 2 and Part 3 (see resources).
- **Provide choice:** Tell students that if they want to use part 3 to attempt to make predictions or conjectures they can, but the teacher won’t expect everyone to make predictions and conjectures just yet. Also, tell students that if they prefer to use question frames that promote explanations, reasoning, and evidence and statement and question frames that promote connections and comparing (see resources) instead of Part 2 that is their choice. Tell students that these are tools that are at their disposal.

**Set the Stage: Math Discourse and Real World Problem Solving**

- Display and review the chart with the driving question written at the top, **how is multiplication used in the world around us?**
- Ask students if they have new situations to add to the chart.
- Write down any new real world math situations (possible ideas: homework, practicing …). If students haven’t added practicing to the chart, write it down and tell students that today the first practice problem has to do with practicing.

**Explain:** Tell students that today they sort and solve real world math problems, and they will choose math problem to use for recording purposes. Explain that the recorded math discussions will be evaluated. Tell students that you will solve, record, and evaluate the practice problem together as a class as an example.

**Note:** Spend time to find the best recording device for your teaching situation. I DO NOT recommend using any one particular recording device. All classrooms are different, and teachers must be free to use recording materials that best fit their teaching situation. However, if you have access to student ipads, there are a few apps that allow students to use a sketchpad while discussing and recording their work. I have used Educreations and Doceri Ipad apps in the past, and, in my experience, they both work well for recording math discussions.

- **Post today’s example problem** (see resources): Marisol plays the flute. She practices regularly. Marisol always begins her practice sessions with a finger warm-up. She did a finger warm-up 5 days last week. Each warm up took her 6 minutes. How many minutes did she spend doing finger warm-ups last week?
• **Model:** The teacher will model how to record the solution method. The teacher will model how to use the chosen recording devise as he/she explains a solution method for the example problem. While recording, the teacher should use an Ipad sketch pad or a scratch paper to display the method.
  *While explaining the solution method, the teacher should refer to How to Have a Discussion, Part 1 to reinforce the expectations of a good math discussion.
  *While discussion the solution method, the teacher should model how to use a picture or diagram to solve the math problem. The teacher should also model how to write a multiplication equation to represent the picture or diagram.
  *Display How to Have a Math Discussion, Part 2 (see resources).
  *Ask a student volunteer to choose 2-3 follow-up questions using the display of How to Have a Math Discussion, Part 2. Record the follow up questions using the recording devise.
  *If the student volunteer is more comfortable with the question and statement frames, allow the student to use the question frames that promote explanations, reasoning, and evidence and the statement frames that promote connections and comparisons (see resources).

**Note:** During the modeling process, the teacher can use **formative assessments** from previous lessons to guide instructional choices. For example, if the information generated form the teacher’s math discourse rubric showed that students were struggling to provide evidence during their math discussions, the teacher can model the math discourse practice of providing evidence. Also, if the teacher noticed that students have not been writing multiplication equations that correctly represent math problems, this should be modeled and discussed. For example, the teacher can ask, ‘*Why did I write the equation 5x6=30? What does the 5 represent in this equation? What does the 6 represent in this equation? How does 5x6=30 represent the diagram that I drew?*’

• Display the student math discourse rubric (see resources and example) and read it together.

**Explain:** Tell the students that they will listen to the recorded example of a math discussion. Explain that they will use the rubric to evaluate the math discussion. They will evaluate both the explanation of the solution method and the discussion with the partner.
• Pass out the single sided copy of the student math discourse rubric.
• Play the recording of the example problem and discussion.
• As a whole class evaluate the recorded discussion using the student’s math discourse rubric.
• First, go through each sub-component of each section of the rubric together one at a time.
  *Begin with the sub-component that discusses connections and read all the explanations score 1 - score 4. Circle or highlight the asterisk next to the subcomponent that best describes the example discussion. Ask the students, *What questions or statements helped so that the math discussion included connections?* If the conversation did not include connections ask the students, *What questions or statements could have been used so that the math discussion included connections?*
  *Then move on to the sub-component that discusses comparisons and read all the explanations score 1 - score 4. Follow the same procedure of circling or highlighting an asterisk and asking questions about the math discussion.*
• Next, with input from the students, give the recorded math discussion an overall score for both sections. There should be one overall score for the section titled, Connect Your Math Ideas to Those of Others. There should be another score for the section titled, Math Rigor.

Example of Math Discourse Rubric for Students

<table>
<thead>
<tr>
<th>Overall Score 2</th>
<th>Score 1</th>
<th>Score 2</th>
<th>Score 3</th>
<th>Score 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students Discourse</td>
<td>Connect Your Math Ideas to Those of Others</td>
<td>* I did not connect my ideas, strategies, or answers to those of others.</td>
<td>* I connected my ideas, strategies, or answers to those of others at least once.</td>
<td>* I connected my ideas, strategies, or answers to those of others at least three times.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>* And I did not compare my strategies or ideas, with those of others. I only compared my answers with those of others.</td>
<td>* I used these connections to compare strategies, ideas, or answers with those of others.</td>
<td>* I made several conjectures or predictions based on what I have learned and discussed.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>* I did not make a conjecture or a prediction based on what I have learned or discussed.</td>
<td>* I attempted to make one conjecture or prediction based on what I have learned and discussed.</td>
<td>* I conveyed procedural understanding, understanding, and I explained some of the procedures by explaining why.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Overall Score 3</th>
<th>Score 1</th>
<th>Score 2</th>
<th>Score 3</th>
<th>Score 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students Discourse</td>
<td>Math Rigor</td>
<td>* I did not share procedural understanding, I only told my answer.</td>
<td>* I did not provide reasoning to explain the math behind my strategies.</td>
<td>* I conveyed procedural understanding, but I did not explain my procedures.</td>
</tr>
<tr>
<td></td>
<td>Procedural Understanding</td>
<td>* My conversation did not generate any new math ideas.</td>
<td>* I also provided some reasoning to explain the math behind my strategy.</td>
<td>* I conveyed procedural understanding, and I explained some of the procedures by explaining why.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>* I provided some justification and evidence to show that my strategy makes sense.</td>
<td>* I conveyed procedural understanding, and I explained each procedure by explaining why.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>* I justified my solution and provided evidence that my strategy makes sense.</td>
<td>* I also provided reasoning to explain the math behind my strategy.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>* I provided some justification and evidence to show that my strategy makes sense.</td>
</tr>
</tbody>
</table>

Source: Rubric Adapted from Park, J., Nava, I., & Applegate, M., (2011)

**Explain:** Convey the fact that today’s score will be a starting place. It is important for students to understand that the rubric is a tool that can be used to improve math discussions.

• Collect the rubrics

**Peer Discussion: Sorting and Solving Real World Math Problems**

• Display and review the procedures for sorting math problems with a partner (see resources)

**Explain:** Tell students that today the goal is to sort and solve all the problems by the end of the lesson. Remind students that they can use pictures or diagrams to solve the math problems, but they should also show how to use a multiplication or addition equation to solve the math problem. Remind the students to use the first and second factors correctly when writing a multiplication equation. Tell students that they will choose one problem for recording purposes, and their chosen math problem should be different than their partner’s.

• Ask students to sit with their math partners.

• Pass out Lesson 4’s real world problems to cut, sort, and solve (see resources)

• Pass out the 3 3x5 sorting cards X, basic +, and X and + (see resources)
• Give the students time to sort and discuss the math problems. When most students are finished, give them a 3 minute wrap up signal.

**Explain:** After about 3 minutes, tell students they should be finished sorting and discussing the math problems, and they should move on to solving the problems. Explain that if they don’t agree on how to solve a problem, put it to the side. Remind students to follow the guidelines for having a respectful math discussion, and refer to the Respect Math Discussions as needed (see resources)

• Give students time to solve the math problems with their partner.

• **Monitor:** The teacher should monitor the students and ask questions that promote understanding (see resources) as needed. If students have moved a question to the side due to the fact that they could not agree on how to solve it, use questioning strategies to help the students work through the math problem.

• When most students are finished, give them a 3 minute wrap up signal.

• When most students are finished, give a 1 minute wrap-up signal, and ask students to choose one math problem each for recording purposes

**Explain:** Tell students that it is now time to record their math discussions.

• Display and review the How to Have a Math Discussion Posters, Part 1, 2, and 3.

**Explain:** Remind students that they will not be expected to use How to Have a Math Discussion, Part 3 today, but if they would like to try using Part 3 this will be good practice for future lessons. Explain that you are allowing the students to choose their tools, and encourage the students to choose the tools that help them have a good math discussion.

• Give one recording device to each group

• Give students about 5-10 minutes to record their math discussions.

• Give a 1 minute wrap-up signal, and remind students that they should record both part 1, the explanation of the method, and part 2 the follow-up questions.

• Pass out the double sided copies of the student math discourse rubric (see resources)

• Ask the students to take a few minutes to listen to the recorded math discussion and use the rubric to evaluate the discussion.

**Explain:** Tell students that both students can evaluate the recording. They can discuss the evaluation and scores, but they don’t have to agree on a score.

• **Monitor:** The teacher should monitor the students as they record their math discussions and evaluate the discussion using the student math discourse rubric. As the teacher monitors, **select** 2-3 math discussion that can later be used as part of a whole class discussion. The teacher should select discussions that provide positive examples of one or two of the subcomponents, or math discourse practices, from the rubric. For example, the teacher should select a recorded math discussion that exemplifies connections between strategies or ideas. The teacher could select another math discussion that exemplifies how to provide reasoning and evidence to support math choices or math ideas. By choosing math discussions that provide one or two positive examples of the math discourse practices that are contained in the rubric, all students will have an opportunity to be exposed to and learn from positive examples of math discourse practices. While monitoring, attempt to **select** a recorded discussion of problem #4 or problem #8. These are challenging math problems, and the
independent practice problem is similar to these two problems. Students will benefit from being exposed to their fellow students’ mathematical thinking and solution methods.

**Note:** Since today is the first day that student work is being evaluated using the rubric, sharing one’s work could be intimidating. Therefore, when selecting work, the teacher should let the student know that his/her work has been selected, but the teacher should ask the students if they would be willing to share their work and have it evaluated using the rubric. If a student declines, look for similar work to select.

- Ask students to switch partners.
- Give 5-10 minutes for the second partner to record the new math discussion.
- Give a 1 minute wrap-up signal, and remind students that they should record both part 1, the explanation of the method, and part 2 the follow-up questions.
- Ask students to take a few minutes to listen to the recorded math discussions and use the rubric to evaluate the discussion. Continue to monitor and select as before.

**Note:** If a classroom does not have enough recording devices, the teacher can allow students to begin recording as soon as they finish sorting the math problems and solving their chosen problem. Once one group is finished recording and evaluating the discussions, they can hand the recording device to another group. The finished group can solve the remainder of the math problems as they wait for others to finish recording and evaluating their discussions. The teacher can also ask students to play the Cookies and Decorations Game from lesson 2 while they wait for a recording device.

**Whole Group Discussion: Real World Problem Solving and Math Discourse**

**Explain:** Ensure that students understand that the class is evaluating the discussion, not the person. It might be helpful to use the term, hard on content, soft on people. Explain that the students will use positive comments while evaluating the math discussions.

- Display the student math discourse rubric.
- Call students to the front who were selected by the teacher and who agreed to share their recorded discussions with the class (you will be evaluating 2-3 math discussions).
- Once a volunteer has shared his/her recorded discussion, refer to the student rubric on the document camera/projector.
- Don’t evaluate the discussion using the whole rubric. Instead, the teacher should select one or two math discourse practices, or sub-components of the rubric, to focus on.
- First, the teacher will read all 4 levels of the chosen component. For example, the teacher might read levels 1-4 that describe how to use connections to compare math strategies and ideas. Then discuss the positive elements of this particular math discussion. Specifically show how this discussion utilized the math discourse practice in a positive manner.
- After the teacher has pointed out how the discussion used the chosen math discourse practice, and the teacher has made a connection between the recorded math discussion and the rubric, ask students for positive feedback. Today’s discussion should focus on ONLY the positive aspects of the math discussions.
• Collect the math discourse rubrics, recorded discussions, and today’s math problems.

**Note:** If you are using an Ipad app as a recording device, it is possible that the students can email the recorded discussion to your teacher email account. If this is possible, take time to learn the Ipad app and teach the students how to safely send an email to the teacher. These recordings can be used as a formative assessment.

**Independent Problem Solving: Real World Problem Solving**

• Students will have the opportunity to apply problem solving strategies to a new, but similar problem.
• Students will have time to reflect on math discourse practices that they experienced today.
• Pass out the practice problem and writing prompt (see resources). Read the directions together. Ensure that students understand the assignment. Then let them work independently to complete the practice problem and writing prompt.

**Formative Assessment:**

• The teacher will evaluate the recorded student discussions using the teacher rubric (see resources). Each student’s current level of math discourse will be assessed.
• The teacher will informally evaluated the student discourse rubrics. By determining each student’s own score, the teacher can gain insight into how each student views his/her own level of math discourse. This information can be used in future lessons to build confidence and to provide scaffolding to students so that they can learn to use their scores to continue to improve.
• The student’s written reflections of question #2 of the independent problem solving worksheet will give insight as to which discourse practices students feel confident with and which discourse practices students would like to work on. This information can be used to guide questioning strategies utilized by the teacher.
• Today’s math problems can be used to determine how well students understand multiplication concepts.

Teacher’s Notes for Lesson 4:
Multiplication in the World Around Us

Lesson 5 – Multiplication and the Array Model

Source: office.com royalty free clip art

Lesson 5 Goals

Problem-Based Learning Goals
- Students will use arrays to solve real world multiplication problems.
- Students will play a multiplication game to practice applying understanding of arrays to real world situations.
- Students will complete a garden design using multiplication and arrays.

Math Discourse Goals
- Students will review math vocabulary learned in lesson 1 for the purpose of applying this vocabulary to the array model of multiplication.
- Students will use math vocabulary accurately and with precision.

Math Standards Addressed in Lesson 5

Standard for Mathematical Practice (SMP)
- SMP #1 - Make sense of problems and persevere in solving them.
- SMP #3 - Construct viable arguments and critique the reasoning of others.
- SMP #6 - Attend to precision.

Operations and Algebraic Thinking, Grade 3 (OAT)
- OAT Standard #1 – In this lesson students will interpret products of whole numbers.
- OAT Standard #3- In this lesson students will use multiplication within to solve word problems in situations involving equal groups, arrays, and/or measurement quantities.
Lesson 5—Multiplication and the Array Model
Research Based Rationale

Van de Walle and Lovin (2006) explain that an array is an important and widely used model for multiplication. Lesson 5 uses problem-based instruction strategies and math discourse practices in a manner that helps student develop an understanding of how arrays can be used as a model for multiplication. Lesson 5 adheres to problem-based learning practices by providing opportunities for students to use arrays to invent their own methods and solutions. First, students are introduced to multiplication arrays as a tool. Students learn that arrays can be used to represent, or model, multiplication problems. Once students have been introduced to multiplication arrays, they are presented with a real world math problem (practice problem #2). When working through practice problem #2, students are able to use their understanding of arrays in a manner in which the array becomes a tool that can be used to make sense of the math problem. Practice problem #2 gives student the opportunity to use arrays as a tool to support thinking and problem solving while students develop their own method and solution. Hiebert, Carpenter, Fennema, Fuson, Wearne, Murray, Oliver, and Human (1997) explain that in problem-centered learning, students should be allowed to invent methods and discuss, critique, and explain their methodologies and solutions. Therefore, once students have time to grapple with practice problem #2 and use arrays to develop methods and solutions, students are given the opportunity to share their methods and their thinking with the whole class.

In this lesson, math discourse practices are used to enhance the whole class discussion and allow students to reflect on their own thinking and the mathematical thinking of their peers. Hiebert et al., (1997) explain that by reflecting on one’s own methods and the methods of others and discussion methods with peers, students can develop an advanced concept of number and value. Therefore, in lesson 5, while students discuss their methods with the whole class, the teacher uses questioning strategies to highlight the relationships between factors and products with the goal of developing advanced understandings of number and value. When discussing arrays, students will learn that changing one factor produces a change in the array and changing one factor can produce a change in the other factor when the product remains the same.

Yeatts et al., (2004) explain that change is an important math idea. By thinking through changes in quantities, students develop flexibility in their mathematical reasoning abilities. Also, by analyzing array models, students can begin to recognize and analyze patterns within math problems (Yeatts, et al., 2004). Lesson 5 allows students to reflect on changes that occur in arrays with the goal of helping students recognize and analyze patterns while developing a deeper understanding of numbers and values.
Lesson 5: Multiplication and the Array Model

Focus: Using Problem-Based Instruction founded on Mathematical Discourse to Learn About Multiplication in the World Around Us

Materials:
* 1 copy of grid paper to display (see resources)
* Student copies of grip paper, 3 copies for each student (see resources)
* Vocabulary journals or vocabulary worksheet (these should be in students’ desks)
* Copy of lesson 5 vocabulary worksheet to display (see resources)
* Four practice problems (see resources)
* Copy of Multiplication and Arrays Game Procedures (see resources)
* Dice (teacher will supply dice)
* Real World Scenarios for the Multiplication and Array Game (see resources)
* Chart of possible real world math problem situations (from previous lessons)
* One copy of the Build a Garden worksheet to display (see resources)
* Students copies of the Building a Garden worksheet (see resources)

Set the Stage: Math Vocabulary Review

- Review the math vocabulary and matching kinesthetic gestures that were introduced in lesson 1. Review the vocabulary words and gestures for multiplication, equal groups, first factor, and second factor.
- Make the X sign and say, “Teach multiplication.” The students make the corresponding gestures while saying, “When you multiply there must be equal groups.”
- Put up one finger and say, “Teach first factor.” The students make the corresponding gestures while saying, “The first factor represents the number of equal groups.”
- Puts up two fingers and say, “Teach second factor.” The students make the corresponding gestures while saying, “The second factor represents how many in each equal group.”
- Ask students to get out their math vocabulary journals or worksheets from lesson 1

Example of Math Vocabulary Journals or Worksheets

| Repeated Addition | Repeated addition means there are equal groups and you add the groups together | **** **** ****
|-------------------|--------------------------------------------------------------------------------|---------------
|                   | 4 + 4 = 8                                                                       | 4 + 4 = 8     |

- Ask students to add three more sections to their math vocabulary journals or pass out the vocabulary worksheet for lesson 5 (see resources)
- Display the example of the vocabulary worksheet (see resources).
- Ask the students to add the vocabulary words first factor and second factor to their own vocabulary journals/worksheets, and tell students to leave the third section blank for now.

Example of Today’s New Vocabulary

<table>
<thead>
<tr>
<th>First Factor</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Second Factor</td>
<td></td>
</tr>
<tr>
<td>Product</td>
<td>8</td>
</tr>
</tbody>
</table>
• Ask students to write a definition in their own words for the two vocabulary words first factor and second factor. Ask students to draw a picture that shows the meaning of each word.

• Give students a few minutes to finish. Then ask for student volunteers to come to the front to display their work using the document camera.

• Tell students that they can make changes as needed while listening to other’s definitions.

• Ask students to keep out their vocabulary journals or worksheets, but the teacher can take the example from the document camera.

• Display the example and non-example of using the first factor and second factor correctly (see resources). Discuss the example and non-example to ensure that students understand how to apply the meaning of the definitions first factor and second factor to a real world problem. Allow students to refer back to their definitions of first factor and second factor as needed. Answer the question at the bottom of the example and non-example.

• Ask students what the 8 refers to in the example problem.

• Introduce the vocabulary word, product.

**Explain:** Explain that when multiplying, the product is the number that represents the total or how many in all. Tell students that in the example problem the 8 is called the product because it represent the total number of decorations.

• Have the students write the vocabulary word, product in the last section of their vocabulary journal. Ask them to write their own definition and a picture to represent the meaning of the vocabulary word product.

• Ask for volunteers to come to the front to display their work.

• Tell students they are allowed to add on or change their definitions and pictures while listening to other’s definitions.

**Set the Stage: Real World Problem Solving and the Array Model**

**Explain:** Reinforce the idea that multiplication is used in many different real world situations. Today I am going to use multiplication to plan my garden. I am going to use grid paper to plan my garden.

• Display the grid paper (see resources) on the document camera/projector.

**Pose Practice Problem 1:** I am a gardener. I have been growing seedlings in my greenhouse, and now it is time to plant them in my garden. I want to place my carrot plants equal distance from each other, so I will plant them in equal rows. I will plant 3 rows of carrots. I will put 6 carrot plants in each row. How many carrot plants will I have in my garden?

**Model** Show how to draw 3 rows of 6 carrots using the grid paper.

**Explain:** Once the teacher has drawn 3 rows of 6 on the grid paper, explain that this type of multiplication model is referred to as an array. An array can be used to organize and solve many different types of multiplication problems.

• Ask the students if they can use the array to determine how many total carrots you will plant.

• Ask the students if they can write a multiplication equation that represents this math problem.
• Give students time to think independently about a possible equation.
• Ask students to share their multiplication equation with a shoulder partner.
• Ask the students which factor should be written first? “Should I write 3x6=18 or 6x3=18.”
• Give student time to discuss this with their shoulder partner.
• Reinforce the idea that the 3 represents that number of groups, so it should be the first factor. Reinforce the idea that 6 represents the number of carrots in each group, so it should be the second factor. Connect the concepts of first and second factor to the array.
• Ask what the number 18 is called. This will reinforce the vocabulary word, product.
• Ask what 18 represents to reinforce the concepts that 18 refers to the total number of carrots.
• Refer back to the student’s vocabulary journal or worksheet to reinforce math vocabulary as needed.
• Put vocabulary journal or worksheets in student’s desks.
• Write the multiplication equation 3x6=18 on the grid paper inside or next to the drawing.
• **Pose Practice Problem 2:** I have also been growing onions in my greenhouse. I have 18 onions to plant. I don’t want to arrange my onions exactly like I arranged my carrots, but I do want to plant my onions in equal rows. Is there another way I could arrange my onions in equal rows?
• Give students time to think.
• Ask students to share their ideas with a shoulder partner.
• Ask for 3-4 volunteers to record and explain their thinking using the grid paper. Make sure that different arrays are represented in these examples.
• As students explain their thinking, the teacher will ask questions that promote explanation, reasoning, and evidence (see resources and example) as students explain their thinking.

### Example of Question Frames that Promote Explanations, Reasoning, and Evidence

<table>
<thead>
<tr>
<th>Questions Frames</th>
<th>Promote Math Explanations, Reasoning and Evidence</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>Why did you ....?</em></td>
<td><em>Why did you think that strategy would work?</em></td>
</tr>
<tr>
<td><em>How did you know to ....?</em></td>
<td><em>Can you prove to me that ... makes sense.</em></td>
</tr>
<tr>
<td><em>Can you explain why you ....?</em></td>
<td><em>What convinced you that was the right answer?</em></td>
</tr>
</tbody>
</table>

*Source: Adapted From Small et al., 2004, Smith and Stein, 2011, Chapin et al., 2013*

• Use questions and statements that promote connections and comparisons to help students understand the similarities and differences between the different arrays.
• Reinforce the concept that factors mean different things in the different positions.
• Ask questions that help students understand how the scenarios can possibly have the same product.

**Note:** While making connections and comparisons ensure that students understand that a change in one factor causes a change in the second factor and that two factors can change positions, but the product stays the same.
Possible Questions and Statements: “In the first array __________, but this array __________. Why? I’m having a hard time understanding why these two arrays look different. Can anyone explain the differences between the arrays? Why does the drawing for 2×9 look different than 9×2? Why do the drawings of 9×2 and 9×2 look different? Why can the arrays that represent 9×2 and 6×3 both work in this situation?”

Whole Group Discussion: Real World Problem Solving and the Array Model

- Pass out grid paper (see resources).
- Display and pose the four practice problems one at a time (see resources).
- After posing one practice problem, give students time to create an array to represent the problem.
- Ask for a volunteer to come to the front to display student work.
- After the first volunteer has displayed his/her drawing, the teacher will ask questions that promote explanations, reasoning, and evidence (see resources).
- Ensure that the student shares the multiplication that represents the array.
- Ask follow-up questions using revoicing or restating strategies (see resources) to ensure that all students understand the meaning of 6×2=12.
- Pose practice problem #2 (see resources)
- Give students time to create their arrays.
- Ask for a volunteer to come to the front to display their array for practice problem #2.
- Call on students to ask follow-up questions about the array using question frames that promote explanations, reasoning, and evidence or How to Have a Math Discussion, Part 2 (these should be in student’s desks).
- Ask the student volunteer to write the multiplication equation and explain why the equation correctly matches the problem and the array.
- Discuss problem #1 and problem #2. Take time to discuss the similarities and differences between Mr. Green’s and Mr. Brown’s gardens. Through questioning, reinforce the concept that the arrays are different, yet the products are the same.
- Continue the same process for practice problem #3 and #4.
- Pose the Problem: Are there any other ways in which Mrs. Green or Mrs. Brown could create a garden pathway using 24 tiles?
- Give students time to create garden pathways using their grid paper.
- Give students time to have an informal peer discussion with a shoulder partner to discuss their method for creating a garden pathway using 24 tiles.
- Ask for volunteers to display the garden paths in front of class.
- Ask follow-up questions that promote explanation, reasoning, evidence, connections and comparisons using the question and statement frames (see resources).
- Discuss the relationship between the various factors. Help student understand that the change in one factor can change the other factor if the product stays the same.
- Collect student’s work and grid paper.
Apply What You’ve Learned: Real World Problem Solving and the Array Model

**Explain:** Tell students that they will play a dice game today to practice using the array model of multiplication. They will play the game in groups of 2. Tell students that they should complete at least 4 arrays.
- Post the Multiplication and Arrays Game Procedures (see resources).
- Read the procedures together.
- Ask students to use How to Have a Math Discussion, Part 1 or questions that promote explanations, reasoning, and evidence, and statements and questions that promote connections and comparisons (these should be in student desks).
- Pass out dice and gardening cards
- Pass out grid paper
- Give students 15 – 20 minutes to play the multiplication array game

Example of Multiplication and Arrays Game Procedures

<table>
<thead>
<tr>
<th>Multiplication and Arrays Game Procedures</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Today you are planting a garden. You will plant your vegetables in equal rows.</td>
</tr>
<tr>
<td>2. Player 1 pulls a gardening card to determine what type of vegetable you will be planting.</td>
</tr>
<tr>
<td>3. Player 1 rolls the dice to determine how many rows will be planted.</td>
</tr>
<tr>
<td>4. Player 2 rolls the dice to determine how many vegetables will be planted in each row.</td>
</tr>
<tr>
<td>5. Both players draw use grid paper to draw an array and write an equation to match the math problem.</td>
</tr>
<tr>
<td>6. Player 1 explains his/her math method using How to Have a Math Discussion, Part 1.</td>
</tr>
<tr>
<td>7. Player 2 asks follow-up questions or makes follow-up statements using the question and statement frames that promote connections and comparisons.</td>
</tr>
<tr>
<td>8. Both players have a chance to make changes to their array or equation if needed.</td>
</tr>
<tr>
<td>9. Players switch roles so player 1 now becomes player 2, and player 2 becomes player 1.</td>
</tr>
</tbody>
</table>


Independent Practice: Real World Problem Solving and the Array Model

- Display the chart of possible real world math problem situations that has the driving question at the top, *How is multiplication used in the world around us.*
- Add gardening to the list of real world problem situations. Ask student if they have any other ideas that should be added to the chart.
- **Model:** Display a copy of the worksheet titled, ‘Building a Garden’ (see resources). Read the directions and model how to complete the worksheet.
- Pass out the worksheet titled, ‘Building a Garden’ and the grid paper (see resources)
- Ask students to complete the worksheet independently
- **Monitor:** While monitoring use the question frames that promote understanding as needed (see resources). Remember to *maintain the high-level task* by using questioning techniques
that help students think through the problem. Remember to use the formative assessments from lesson 4 to guide instructional choices when providing support. The information gained from the student rubrics and from the written response, can be used to determine which questioning styles to use with students of various levels and abilities.

**Formative Assessment**

- Use the worksheets titled, Building a Garden to determine how well students are able to correctly use arrays to solve real world math problems.
- While assessing the worksheets, choose a few that can be used in lesson 6 to review the array model of multiplication and connect arrays to the concept of area.

**Teacher Notes for Lesson 5:**
Multiplication in the World Around Us

Lesson 6– Arrays and Finding the Area of an Object

Lesson 6 Goals

Problem-Based Learning Goals
- Students will determine the area of various two dimensional shapes using a variety of methods.
- Students will write the area of various shapes uses square units.
- Students will find the total area of three shapes.

Math Discourse Goals
- Students will participate in math discussions that help them build on their understanding of multiplication arrays and connect arrays to the concept of area.

Math Standards Addressed in Lesson 6

Standard for Mathematical Practice (SMP)
- SMP #1 - Make sense of problems and persevere in solving them.
- SMP #3 - Construct viable arguments and critique the reasoning of others.
- SMP #6 - Attend to precision.

Operations and Algebraic Thinking, Grade 3 (OAT)
- OAT Standard #1 – In this lesson students will interpret products of whole numbers,
- OAT Standard #3 - In this lesson students will use multiplication within to solve word problems in situations involving equal groups, arrays, and/or measurement quantities.

Geometric Measurement, Grade 3 (MG)
- MG Standard #7 – In this lesson students will relate area to the operations of multiplication and addition.
Lesson 6—Arrays and Finding the Area of an Object

Research Based Rationale

Lesson 6 is designed to support students as they learn to relate the array model of multiplication to the concept of area. Masingila, Lester, Raymond (2002) explain that it is valuable for students to be able to see multiplication in an array model because array multiplication is connected to finding the area of an object. This lesson allows students to make connections between arrays and finding the area of an object. In the first two sections of lesson 6, the teacher uses a whole class discussion format. During the whole class discussion, questioning strategies are used to cultivate students' understanding of area. The teacher facilitates a discussion that promotes connections between multiplication arrays and area. The questioning strategies prompt students to think about math in a manner that allows them to build upon their current math ideas while creating their own understanding of the concept of area.

Van de Walle and Lovin (2006) encourage teachers to use student’s existing ideas to build new understandings. They explain that teachers should engage students by associating new learning with existing ideas in a meaningful network so that students can create and develop understanding and make sense of mathematics (Van de Walle & Lovin, 2006). The whole group discussion and questioning is designed to help students connect new concepts with existing ideas and to further develop their understanding of how multiplication and area are related.

In lesson 6 the handouts titled, How to Have a Math Discussion Part 1 and Part 2 are used to facilitate math discourse. Since these math discussion procedures have been previously taught and practiced, the teacher can quickly review the procedures and allow students to engage in and use math discussion to improve their mathematical understanding. The focus is no longer how to have a math discussion. The focus has become using math discussions to develop and deepen math understanding. Brendefur and Frykholm (2002) explain that math discourse can provide an opportunity for students to develop reasoning skills and a deeper understanding of math concepts.

Once students have engaged in math discussions using How to Have a Math Discussion Part 1 and Part 2, Part 3 is reintroduced. How to Have a Math Discussion, Part 3 prompts students to make predictions and conjectures and to justify their mathematical thinking. The common core standards (www.corestandards.org) state that mathematically proficient students “make conjectures and build a logical progression of statements to explore the truth of their conjectures. They justify their conclusions; communicate them to others, and respond to the arguments of others” (p.5). Lesson 6 allows students to practice making prediction and conjectures and justifying their math arguments.
Lesson 6: Arrays and Finding the Area of an Object

Focus: Using Problem-Based Instruction founded on Mathematical Discourse to Learn About Multiplication in the World Around Us

Materials:
* Construction paper cut into 1 foot x 1 foot squares. Cut enough squares for groups of 4 students to share 8-16 squares.
* Selected Building a Garden worksheets from lesson 5
* Square math tiles (use manipulatives, or see attachments)
* Zoo employee badge (create before the lesson)
* Animal pen worksheets #1, #2 and #3 (see attachments)
* Definition of area to display (see attachments)
* How to have a math discussion, Part 1, Part 2, and Part 3 (see attachments)
* Question frames that promote explanations, reasoning, and evidence (see attachments)
* Question frames that promote predictions, conjectures, and justification (see attachments)
* Statement and question frames that promote connections and comparisons (see attachments)
* Directions and game cards for Find the Area Dice Game (see attachments)
* Student copies of grid paper and writing prompt (see attachments) copied back to back
* Writing prompt (see attachments)
* Blank chart paper for writing down general rules for determining area

Set the Stage: Review the Area of an Object and Real World Problem Solving

Objective: Students review concepts from lesson 5 and build on these concepts while developing an understanding of the area of an object.

- Share a few Building a Garden worksheets from lesson 5. Each Build a Garden worksheet should be carefully selected by the teacher with the specific purpose of meeting the objectives of this lesson. When sharing the worksheets, there are two goals. One goal is to review the math concepts that were taught in lesson 5 such as the relationship between factors, products, and arrays. The other goal is to help students understand how multiplication arrays are related to area.

- To help students attain the first goal, the teacher should select worksheets that can be used to show the relationship between factors and products. Therefore, the teacher should select worksheets that contain arrays that have one common factor, two common factors but in reverse order, or common products but different factors. For example, 6x7 = 42 shares one factor with 6x4 = 24. The two arrays that represent these equations can be used to help students understand what happens if you increase one factor while the other factor remains the same. The arrays that represent 4x3 = 12 and 3x4 = 12 can be used to help students understand the commutative property of multiplication and the fact that equal products can
look differently when arranged in an array. The arrays 2x6=12 and 3x4=12 can be used to help students understand that different factors can produce equal products.

- Once the teacher has asked questions that help students attain the first goal. The teacher should then ask questions about the garden pathways. By discussing the garden pathway arrays, the teacher can help students attain the second goal. Students should begin to understand how multiplication arrays are related to area.
- Display one Building a Garden worksheet from lesson 5. Use questioning strategies to compare the garden pathway to one of the arrays that represent plants. In comparing the two arrays, the teacher should help students understand that the pathway is covering an area, and the array can be used to measure the area of the garden path as well as the number of tiles used to create the garden path.

<table>
<thead>
<tr>
<th>Research Based Advice</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Advice:</strong> At this time students may not be able to figure out the area covered by the square tiles. Students might come up with partial answers such as “the path takes up the space of ___ square blocks. If students don’t answer, or come up with partial answers, allow them to grapple with real world problems to develop their own understanding of math concepts.</td>
</tr>
<tr>
<td><strong>Research:</strong> Melville et al. (2013) stated that students learn mathematics most effectively when they are given opportunities to investigate ideas and concepts through problem solving.</td>
</tr>
</tbody>
</table>

### Set the Stage: The Area of an Object and Real World Problem Solving

<table>
<thead>
<tr>
<th>Objective: Students utilize information from the discussion/review to make sense of and develop their own understanding of the concept of area.</th>
</tr>
</thead>
<tbody>
<tr>
<td>- Hold up a 1 foot x 1 foot square made from construction paper and tell the students that this square represents one square tile that can be used to create a garden path.</td>
</tr>
<tr>
<td>- It is important that the students understand the square construction paper is 1 foot square.</td>
</tr>
<tr>
<td>- Arrange students in groups of 4. Ask students to spread out around the room, so they can build their garden paths on the floor around the classroom.</td>
</tr>
<tr>
<td>- Give each group 8-15 construction paper tiles.</td>
</tr>
<tr>
<td>- Ask students to work as a team to create a garden path using all tiles.</td>
</tr>
<tr>
<td>- Give students a few minutes to create garden paths.</td>
</tr>
<tr>
<td>- Ask students to stop building and take a walk around the classroom to observe the different garden paths that were created. Ask the students to note the similarities and differences that they notice between the different garden paths.</td>
</tr>
<tr>
<td>- Give students a few minutes to walk around the classroom and review the variety of garden paths.</td>
</tr>
<tr>
<td>- Ask students to share their observations. Ask students to explain the similarities and differences between the paths.</td>
</tr>
</tbody>
</table>
• Ask students questions that link the discussion to multiplication concepts such as factors, multiplication equations, products, and area.

**Possible Wording:** “said that this array had ___ rows with ___ in each row. How many total tiles did this group use? If each tile is 1 square foot, how many square feet are covered by this garden path?"

**Explain:** Sometimes an array can be used to present a total amount or a product. Provide students with an example, such as the total number of plants in a garden bed. However, sometimes an array can be used to determine the total surface or area covered by an object. Provide an example of the garden path.

• Hold up a ruler that shows one foot, and hold up an example of a square foot tile that was used to create the garden paths.

**Explain:** Tell students that when measuring area, we use square units such as square feet.

• Pose a question and ask student to think about square units.

**Possible Wording:** “Why did we use square units, in this case square feet instead of feet, to measure the area of our garden paths? Why didn’t we use regular feet instead of square feet to measure the garden paths?"

• Clean up the construction paper pathways, and prepare students for another real world situation in which they might use multiplication to determine the area of a surface or space.

**Put it Into Practice: The Area of an Object and Real World Problem Solving**

• The teacher will put on a zoo employee badge and act like a zoo employee who designs and builds animal pens.

**Explain:** Tell students that designing and building animals pens can be very complex. To build animal pens, it is important to know a lot about math.

• Ask the students, “Do you think all animal pens are designed and built the same? How do you think they might be different?”

**Explain:** Tell students that today you need their help to determine how the animal pens are different mathematically.

• Ask students to sit in groups of two. They do not have to sit with their designated math partner. They can choose their own partner.

• Pass out animal pen drawings #1 (see resources) and square tiles (use math tiles, or make tiles using the attached black line master)

**Explain:** Ask the students to look at the pens and decide how they are different. Tell students that they can use square tiles to measure the pens if they would like. Tell students they can discuss their thinking with their chosen partner.

• Give students time to determine how the animal pens are different.

• **Monitor:** While monitoring, allow students to explore. Use questions to promote understanding only if a student is not moving forward with their thinking. Also, use the math discourse rubric for teachers (see resources and example) while listening to peer math
discussions. The teacher can circle or highlight the math discourse practices that are observed during the peer discussions.

Example Teacher’s Math Discourse Rubric

<table>
<thead>
<tr>
<th>Score 1</th>
<th>Score 2</th>
<th>Score 3</th>
<th>Score 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Math Discourse Overview</td>
<td>During the math discussion students…</td>
<td>During the math discussion students…</td>
<td>During the math discussion students…</td>
</tr>
<tr>
<td>*did not uncover or analyze errors</td>
<td>*did not uncover or analyze errors</td>
<td>*uncovered and/or analyzed errors as needed</td>
<td>*uncovered and analyzed errors as needed</td>
</tr>
<tr>
<td>*did not make connections between strategies or ideas of others only compared answers.</td>
<td>*made one connection between strategies and compared different strategies when applicable.</td>
<td>*made a few connections between and compared strategies when applicable.</td>
<td>*made all possible connections between strategies and compared strategies when applicable.</td>
</tr>
<tr>
<td>*did not ask/provide follow-up questions/statements to promote math explanations, reasoning, and evidence.</td>
<td>*asked/provided one or two follow-up questions/statements to promote math explanations, reasoning, and evidence.</td>
<td>*asked/provided a few follow-up questions/statements to promote math explanations, reasoning, and evidence.</td>
<td>*asked/provided follow-up questions/statements to promote math explanations, reasoning, and evidence.</td>
</tr>
<tr>
<td>*did not attempt to ask for/provide conjectures or predictions.</td>
<td>*made one attempt to ask for/provided conjectures or predictions, but it didn’t fully make sense.</td>
<td>*asked for/provided one or two conjectures or predictions.</td>
<td>*asked for/provided conjectures and predictions.</td>
</tr>
</tbody>
</table>

Source: Rubric Adapted from Park, J., Nava, I., & Applegate, M., (2011)

• After a few minutes, ask students to stop, put down the tiles and share some preliminary thinking.

Possible Answers: “Some pens are bigger than others. The pens have different shapes. I can fit two tiles inside this pen, but I can fit 6 tiles in this pen. The giraffes have much more space than the monkeys.”

Explain: Tell students that one of your jobs as an animal pen designer is to determine the area of each pen.

• Ask if anyone has already determined the area of one of the animal pens or if anyone can explain what the area of the pen refers to or means.

Research Based Advice

Advice: Use questioning strategies to further students’ thinking, but avoid merely telling students the answers or showing students how to determine the area. Remember to create a math experience that allows students to treat the problem as something they need to think about rather than a prescription they need to follow (Heibert at al., 1997).

• If students have a hard time explaining what the word area refers to in this situation, use questioning strategies such as revoicing or restating (see resources) to develop an understanding.

Possible Wording: If a student mentioned that the giraffes have more space, ask a follow up question such as, “When did ______ say this pen had more space, did ______ mean the inside of the pen is larger? How do you know the pen has more space? Can you measure the inside of the giraffe pen and compare it to the inside of the monkey pen to find out for sure?”

• If students already know that area refers to the measurement of the inside of the animal pens, then use questioning strategies to build on this thinking and develop the concept that area is
measured in square units and area refers to the surface of the pen, not the edge of the pen and not the depth or height of the pen.

- Once the students are on their way to creating their own understanding of area, display the definitions of area (see resources), and read the various definitions together.
- Display the drawings of animal pens #1 and ask for a volunteer to come to the front to use math tiles to show the area of the giraffe pen.
- Ask the student volunteer to tell the class, how many tiles it took to cover the surface of the giraffe pen.
- Ask the students if the tile is a regular inch or a square inch. Discuss this concept of square units as needed.
- Once it has been established that the math tile is a square inch, ask the class how to properly write the area of the giraffe pen.

**Note:** Since student might share incorrect answers, remember to treat mistakes as a natural part of the learning process. The teacher can highlight the learning opportunity provided by an incorrect answers that are shared.

- Ask for a volunteer to come to the front of the class to use math tiles to show and write the area of the monkey pen.
- Give students time to work with their partner to determine the area of the panda pen and the penguin pen. Ask students to write the area next to each pen.
- Ask for volunteers to come to the front to explain the area of the panda and penguin pens.
- Remind student volunteers to use How to Have a Math Discussion, Part 1 (see resources) when explaining their method of finding the area of each animal pen.
- Once students have explained their math method, call on students to ask follow up questions using question and statement frames that promote explanations, reasoning, and evidence, connections, and comparisons or How to Have a Math Discussion, Part 2 (see resources).
- When both the panda pen and penguin pen have been discussed, use question frames that promote predictions, conjectures, and justification or How to Have a Math Discussion, Part 3 (see resources).

**Possible Wording:** “If I changed ______ would the area still be the same? How do you know?” Pause to allow students to think and discuss. “Are there changes I could make to the pens that would not cause the area to change?” Pause to allow student to think and discuss. “How do you know?” Discuss and use revoicing and restating techniques to enhance the math discussion. “What if I only knew the measurements of the sides of the pens, could I still find the area?” Pause and discuss. “Have you discovered any general rules about area?”

- If students discover or share general rules, record them on a chart paper for others to see.
- If students share general rules that are not correct, ask the class to test the rule. This provides an opportunity for students to explore examples and non-examples

**Independent Problem Solving: The Area of an Object and Real World Problem Solving**
• Display the animal pens worksheet #2 with the bird aviary, lemur pen, and lion pen (see resources).

**Explain:** Tell students that some of the animal pens are regular shapes and some are irregular shapes. Tell students that they will determine the area of each pen. Write Independent Thinking to Peer Discussion to Whole Group Discussion on the board. Explain to the students that they will be given time to think independently first. Then they will have a chance to share their thinking with a partner. Later, they will share their methods with the class.

• Pass out animal pen worksheets #2 (see resources) and give students time to work independently to determine the area of each pen.

### Peer Discussion: The Area of an Object and Real World Problem Solving

• Ask students to sit with their math partner.
• Ask students to use How to Have a Math Discussion, Part 1 and Part 2 (see resources/student desks) to have a math discussion with their partner.
• Tell students to take turns discussing each animal pen.
• Monitor students’ math discussion. Use questioning strategies that promote mathematical understanding (see resources) as needed.

**Select:** As you monitor student discussions, select student work that can be used during the whole group discussion.

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**Research Based Advise**

**Advice:** While the teacher is selecting work, select at least one basic pen first. Look for examples of the lemur pen or the bird aviary to display and discuss first. By starting with a math discussion about the basic pens, the teacher increases the opportunities for students to learn from each other. For example, students who have a basic understanding of area can use the discussion to review the concept of area, and students who have a stronger understanding of area can use the discussion to gain access to multiple methods used to determine the area of the pen. After discussing one or two basic pens, display and discuss the more challenging area problems presented by the zebra pen or the lion pen.

**Research:** Hiebert et. al., (1997) emphasizes the fact that it is each student’s responsibility to share methods that he or she understands. In a healthy math classroom, there will be a good deal of give and take so that students can learn from each other.

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### Whole Group Discussion: The Area of an Object and Real World Problem Solving

• Ask one of the selected student groups to display either the bird aviary or the lemur pen.
• Ask the selected students to use How to Have a Math Discussion, Part 1 (see resources) to explain their thinking.
• As the students explain their thinking, the teacher can use revoicing, restating techniques and question frames that promote exposure and connection amongst students as needed (see resources).
• Make sure students say and write their answers using square units such as square inches. Reinforce the concept that area is measured in square units, and when you say or write the answer, it must be written in square units.
• Ask the selected students to call on volunteers to use How to Have a Math Discussion, Part 2 (see resources/student desks) to ask follow up questions or make connections and comparisons between methods used.

Research Based Advice

Advice: The zebra pen and the lion pen require students to apply their understanding of area to a new type of problem. Therefore, there may be many methods used to find the area of these two pens. Therefore, the teacher might want to remind the students that there can be many ways to solve a math problem.

Research: Hiebert et al., (1997) explain that when students work together they are likely to discover a variety of methods. This allows students to view and approach math problems in a new ways. Van de Walle and Lovin (2006) state, “learning math is about coming to understand the ideas of the mathematical community” (p. 6). Reminding students that multiple methods are valued might motivate students to learn and try new methods.

• Ask one of the selected students to display an example of the zebra or lion pen.
• Ask the selected students to use How to Have a Math Discussion, Part 1 (see resources) to explain their thinking.
• As the students explain their thinking, the teacher can use revoicing, restating techniques and question frames that promote exposure and connection amongst students as needed (see resources).
• Ask the selected students to call on volunteers to use How to Have a Math Discussion, Part 2 (see resources) to ask follow up questions or make connection and comparisons between methods used.
• Display SMP #3 titled, Construct Viable Arguments and Critique the Reasoning of Others and read it together.

Explain: Tell students that now they will practice making predictions and conjectures. Tell students that they will practice justifying their predictions and conjectures.
• Display How to Have a Math Discussion, Part 3. Read the poster as a class.
• The teacher will model how to use the poster to make predictions and conjectures and to justify these claims.

Possible Wording: “_________ and ________ used different methods to find the area of the _____ pen. However, I noticed a pattern in their methods. It is _________. “ Pause to explain that by looking for patterns, you can make conjectures. Use revoicing and restating techniques (see resources) to enhance the math discussion. “Let me give you another example of this pattern that I noticed.”
• Ask students if they see any other patterns.
• Repeat this discussion process for the next pen. This should be either the zebra or lion pen. First, ask the selected students to display and discuss their method of finding the area of the animal pen. Then ask the selected students to call on volunteers to ask follow up questions and make connections and comparisons. Finally, the teacher can call on students to use How to Have a Math Discussion, Part 3 to make predictions, conjectures, and justifications.
• Display animal pens worksheet #3 (see resources).
• Ask students if they notice anything different about these pens (they have numbers along the sides of the pen).

**Explain:** Tell the students that the numbers are measurements. Each number represents the number of feet on each side of the pen.
• Ask the students if there is any way to use the measurements to find the area.
• Give students a few minutes to think about how they might find the area by using the measurements.
• Have a whole class discussion about using the measurements of the sides of the pen to find the area.
• Call on volunteers to come to the front to explain their method using How to Have a Math Discussion, Part 1.
• The teacher can use revoicing and restating techniques and question that promote connections and exposure amongst students to enhance the math discussion.

**Apply What You’ve Learned: The Area of an Object and Real World Problem Solving**

**Explain:** Today the students will play a dice game to find the area of various animal pens. Tell students that they will have an opportunity to use graph paper to check their work during the game.
• Display the procedures for playing the dice game called, Find the Area (see resources and example).

**Example of Find the Area Dice Game**

<table>
<thead>
<tr>
<th>Find the Area Dice Game</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Game Procedures</strong></td>
</tr>
<tr>
<td>1. New animals will be coming to the zoo soon. You must build new animal pens for these animals.</td>
</tr>
<tr>
<td>2. Player 1 pulls an animal card to determine the type of animal that is coming to the zoo soon.</td>
</tr>
<tr>
<td>3. Player 1 rolls the dice to determine how long (up and down) the pen will be.</td>
</tr>
<tr>
<td>4. Player 2 rolls the dice to determine how wide (across or side to side) the pen will be.</td>
</tr>
<tr>
<td>5. Player 1 uses a grid to create the animal pen with the correct length and width. Player 1 writes the area of the pen using square units.</td>
</tr>
<tr>
<td>6. Player 1 explains his/her math method for determining the area of the animal pen using How to Have a Math Discussion, Part 1.</td>
</tr>
<tr>
<td>7. Player 2 asks follow-up questions or makes follow-up statements using the question and statement frames that promote connections and comparisons.</td>
</tr>
<tr>
<td>8. Players always have a chance to make changes to their answer if needed</td>
</tr>
<tr>
<td>9. Players switch roles so player 1 now becomes player 2, and player 2 becomes player 1.</td>
</tr>
</tbody>
</table>
10. When all the animal cards have been used, each player should determine the total area of all the animal pens on their grid paper.


- Read the procedures together as a class and ensure that students understand how to play the game. Model how to play if needed.
- Pass out grid paper. The grid paper should have the writing prompt (see resources) copied onto the backside of the paper.

Writing in Math: Reflect on Real World Problem Solving

- When students have completed the Find the Area Dice Game, ask them to respond to the writing prompt (see resources) on the back of the grid paper.
- Reinforce the fact that there are no right or wrong answers to the writing prompt.
- Ask students to think first, discuss ideas with a friend if needed, and write independently.
- Once students have had a chance to write a response to the prompt, allow them to share their response with 1-3 other students.
- Once students have listened to a few responses, allows them to make changes to their own response if they would like.

Formative Assessment

- The information gained from the teacher’s math discourse rubric can be used as a formative assessment in two ways. First, the highlighted sections will inform the teacher as to which math discourse practice are being used by the students. Secondly, the rubrics and highlighted sections can be compared to the rubrics collected in lesson 2. By comparing the rubrics from previous lessons, the teacher can ascertain if student’s math discourse is improving.
- The grids created during the Find the Area Dice Game can be used to determine if students understand the concept of area and if students are able to accurately determine the area of an object.

Teacher Notes for Lesson 6
Multiplication in the World Around Us
Lesson 7– Self-Evaluation and Math Discourse #2

Lesson 7 Goals

Problem-Based Learning Goals
- Students will sort and solve real world math problems involving multiplication, addition, and subtraction.

Math Discourse Goals
- The students will record their math discussions with a peer.
- The students will use a rubric to evaluate their math discourse.

Math Standards Addressed in Lesson 7

Standard for Mathematical Practice (SMP)
- SMP #1 - Make sense of problems and persevere in solving them.
- SMP #3- Construct viable arguments and critique the reasoning of others.

Operations and Algebraic Thinking, Grade 3 (OAT)
- OAT Standard #1 – In this lesson students will interpret products of whole numbers,
- OAT Standard #3- In this lesson students will use multiplication within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities.
Lesson 7 - Self-Evaluation and Math Discourse #2
Research Based Rationale

Lesson 7 is designed in a similar format to lesson 4. Lesson 7 provides students with the opportunity to sort and discuss problems, and students have the chance to record and evaluate their math discussions. During lesson 7, students will revisit the student discourse rubric to further reinforce metacognition and self-evaluation for the purpose of improvement. While solving and discussing real world math problems, students will use a discourse rubric to self-monitor the problem solving and math discussion processes just as they did in lesson 4. The recorded math discussions will be used to help students analyze their math discourse as well as their understanding of the math problems. Math discourse will be used as a means of improving student’s understanding of the math problems contained in the lesson. The process of explaining one’s own thinking aloud helps students to clarify their own ideas and correct their own mistakes (Chapin, et al., 2013). Recorded math discussions will be used to help students learn to improve their questioning skills so that questioning can be used to deepen their own mathematical understanding. By analyzing the recorded discussions, students can realize what they do or do not understand, and they can analyze and fix errors. Becoming aware of what they do and do not know, allows students to adjust their reasoning and improve their metacognitive abilities (Chapin, et al., 2013). See lesson 4 for the research based rationale behind the design of the lesson.

As in lesson 3 and 4, in lesson 7 students are asked to sort and discuss math problems. If teachers prefer to use math problems from an adopted math program, the math problems contained in this lesson can be substituted by other math problems. However, it is important to remember that sequencing math problems in a manner that allows students to build on previous knowledge is essential. The math problems contained in this handbook build on previously learned information by taking what has been learned about the relationship between addition and multiplication and applying that to new situations that include subtraction.
Lesson 7: Self-Evaluation and Math Discourse #2

Focus: Using Problem-Based Instruction founded on Mathematical Discourse to Learn About Multiplication in the World Around Us

Materials:
* Three student copies of the math discourse rubric for students (see resources). One copy can be double sided, and one copy can be single sided.
* Chart of possible real world math problem situations (from previous lessons)
* Copy of SMP #1 and SMP #3 to display (see resources)
* Poster, How to Have a Math Discussion, Parts 1, 2, and 3 to display (see resources)
* Example problems #1 and #2 to use for modeling (see resources)
* At least 10 recording devices (any recording device can be used, but Ipad apps provide students with the opportunity to record both their discussion and their work, see notes in lesson 4)
* Procedures for sorting math problems with a partner (see resources)

Set the Stage: Math Discourse and Real World Problem Solving

Explain: Tell students that they will be sorting and solving math problems. Once they have sorted and solved each problem with a partner, they will choose one problem to use for recording purposes. Explain that each student will use a recording device to record their math discussion.

- Engage students in a discussion about math discourse practices. For example, discuss explaining your reasoning, providing evidence, making connections between methods or problems, and comparing methods or problems.
- Once the students have shared their ideas, put up a copy of SMP #1 and SMP #3 (see resources). Read and review the SMPs together.
- Post the How to Have a Math Discussion Poster Part 1, Part 2, and Part 3 (see resources).
- Read and review the posters together.
- Post today’s practice problem #1 (see resources): Marisol enjoys working in the garden. She planted 8 rows of corn. She planted 6 seedlings in each row. All the plants survived. How many plants did Marisol harvest?
- Ask for a volunteer to come to the front to model how to record today’s math discussion. Remind the volunteer to actively listen to the explanation so they can ask good follow-up questions.

Model: The teacher will model how to use the chosen recording devise as he/she models how to explain a solution method for the example problem using How to Have a Math Discussion, Part 1 (see resources).

- The teacher should model how to use a picture, or diagram, and a multiplication equation to represent the problem.
- Display How to Have a Math Discussion, Part 2 (see resources).
- Continue to record as the student volunteer chooses 2-3 follow-up questions using the display of How to Have a Math Discussion, Part 2 (see resources).

- Pass out the student math discourse rubric (see resources) and read it together.
- Ask students to sit with their partner.
- Play the recording of the example problem and discussion.
• Ask the students to use the rubric to evaluate the recorded math discussion with their partner. 

**Explain:** Tell students that they can discuss the rubric with their partner. Tell them to read each subcomponent of the rubric. If both partners agree that a subcomponent describes the math discussion, they can highlight the asterisks next to the description (see example). Tell students that if they disagree, leave the asterisks blank. They can revisit the blanks during the whole group discussion.

• Give students time to fill-in the rubric with their partner.

• Have a whole group discussion. With input from the class, highlight the subcomponents that describe the math discussion and give the recorded math discussion an overall score for both sections. There should be one overall score for the section titled, Connect Your Math Ideas to Those of Others, and one overall score for the section titled, Math Rigor.

**Example of Math Discourse Rubric for Students**

<table>
<thead>
<tr>
<th>Students Discuss</th>
<th>Score 1</th>
<th>Score 2</th>
<th>Score 3</th>
<th>Score 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Connect Your Math Ideas to Those of Others</td>
<td>* I did not connect my ideas or strategies to those of others.</td>
<td>* I connected my ideas, strategies, or answers to those of others at least once.</td>
<td>* I connected my ideas, strategies, or answers to those of others at least three times.</td>
<td>* I connected my ideas, strategies, or answers to those of others at least three times.</td>
</tr>
<tr>
<td><strong>Overall Score 2</strong></td>
<td>* And I did not compare my strategies or ideas, with those of others. I only compared my answers with those of others.</td>
<td>* But I did use these connections to compare strategies, ideas, or answers with those of others.</td>
<td>* I used these connections to compare strategies, ideas and answers to those of others.</td>
<td>* I used these connections to compare and evaluate ideas, strategies, and answers to those of others.</td>
</tr>
<tr>
<td><strong>Overall Score 3</strong></td>
<td>* I did not make a conjecture or a prediction based on what I have learned or discussed.</td>
<td>* I attempted to make one conjecture or prediction based on what I have learned and discussed.</td>
<td>* I made a few conjectures or predictions based on what I have learned and discussed.</td>
<td>* I made conjectures and predictions based on what I have learned and discussed.</td>
</tr>
<tr>
<td>Math Rigor: Procedural Understanding: <strong>why</strong> you solved the problem</td>
<td>* I did not share procedural understanding, but I did not explain my procedures.</td>
<td>* I conveyed procedural understanding, but I did not explain my procedures.</td>
<td>* I conveyed procedural understanding, and I explained some of the procedures by explaining why.</td>
<td>* I conveyed procedural understanding, and I explained with procedure by explaining why.</td>
</tr>
<tr>
<td>Explain: <strong>tell why you used a strategy</strong></td>
<td>* My conversation did not generate any new math ideas.</td>
<td>* I did not provide reasoning to explain the math behind my strategies.</td>
<td>* I also provided some reasoning to explain the math behind my strategy.</td>
<td>* I also provided reasoning to explain the math behind my strategy.</td>
</tr>
<tr>
<td>Justify: <strong>provide evidence that your math strategy makes sense.</strong></td>
<td>* I attempted to justify my solutions and provide evidence that my strategy makes sense.</td>
<td>* I provided some justification and evidence to show that my strategy makes sense.</td>
<td>* I provided justification and evidence that my strategy makes sense.</td>
<td>* I provided justification and evidence that my strategy makes sense.</td>
</tr>
</tbody>
</table>

**Source:** Rubric Adapted from Park, J., Nava, I., & Applegate, M., (2011)

• Discuss the scores as a whole class.
  * Ensure that students understand that the rubric is a tool that can be used to improve math discussions.
  * Ask the students for constructive advice, *How could we improve this math discussion? What questions or statements helped you understand the math problem? What questions or statements could have been added to the math discussion to help you better understand the math problem? What questions or statements could have been added to the math discussion to improve the depth of the discussion?*
• **Post today’s practice problem #2** (see resources): Marisol also planted tomato plants in her garden. She planted 4 rows of tomato plants. She put 8 seedlings in each row. Marisol did not water her plants enough and 3 plants died. How many plants survived?

• Discuss Problem #2. How is it similar/different to practice problem #1?

**Explain:** Tell students that they will have time to independently solve practice problem #2. Then the teacher will ask for volunteers to come to the front to model how to discuss and record the math problem.

• Give students time to solve practice problem #2.

• Ask for two volunteers to come to the front.

• The students will model how to use the recording device while one volunteer uses How to Have a Math Discussion, Part 1 (see resources) to explain how they solved the problem. The second volunteer with use How to Have a Math Discussion, Part 2 (see resources) to ask questions that promote explanations, reasoning, and evidence and promote a discussion that includes connections and comparisons.

• Display the student discourse rubric (see resources and example).

• Replay the discussion, and as a whole class evaluate the discuss using the rubric.

• When evaluating the math discussion, tell students to begin with specific positive comments. Go through the rubric and note the components that were covered in the discussion.

• Students can highlight the asterisk next to the components that were used during the math discussion on their copy of the rubric.

• Once the positives have been discussed, ask for constructive advice. The advice should focus on how the discussion could be improved. Refer to the rubric to discuss ways in which the discussion could be improved.

• Display How to Have a Math Discussion, Part 3 (see resources). As a whole class practice using the sentence starters that promote predictions, conjectures, and justification to discuss practice problem #2.

**Peer Discussion: Sorting and Solving Real World Math Problems**

• Ensure that students are sitting with their partners.

• Post and review the procedures for sorting math problems with a partner (see resources)

**Explain:** Remind that students that they should sort all the problems first before solving them. When they finish sorting the problems with their partner, they can attempt to discuss the problems and solve them together. If they disagree on a solution, put the card to the side.

• Pass out the 4 3x5 sorting cards X, basic +, X and +, or X and - (see resources). Read the cards together and ensure that the students realize that there is a new category, X and -.

• Pass out Lesson 7’s real world problems to cut, sort, and solve (see resources)

• Give the students time to sort and discuss the math problems. When most students are finished, give them a 3 minute wrap up signal. Early finishers can start solving the problems.

• After a few minutes ask all students to stop for a minute to discuss the next step, solving the math problems.
**Explain:** Remind students that when solving each problem, they can use pictures or diagrams, and they should show how to use an equations to solve the math problem. Remind students that if they don’t agree on how to solve a problem, they should put it to the side. Remind students about having a respectful math discussion and refer to the Respectful Math Discussions poster as needed (see resources). Finally, remind the students that they will choose one problem for recording purposes, and their chosen math problem should be different than their partners. Give students time to solve the math problems with their partner.

- **Monitor:** The teacher should monitor the students and ask questions that promote understanding (see resources) as needed. If students have moved a math problem to the side due to the fact that they could not agree on how to solve it, use questioning strategies to help the students work through the math problem.
- When most students are finished, give a 3 minute wrap up signal.
- Ask students to choose one math problem each for recording purposes. Remind students that their math problem should be different than their partner’s math problem. Ask students to choose math problems from the X, X and +, or X and – categories. Tell students that they cannot choose a basic + problem for recording purposes.

**Explain:** Tell students that today they should use How to Have a Math Discussion, Part 1, 2, and 3 to have a math discussion. Emphasize that they should ask at least a few questions that promote predictions, conjectures, or justification.

- Give one recording device to each group. Remind them to take turns explaining and asking questions. Remind students that when recording their math discussion they should record both the explanation and the follow-up questions.
- Give students time to record their math discussions. Then give a 1 minute wrap-up signal.
- Ask the students to take a few minutes to listen to the recorded math discussion and use the rubric to evaluate the discussion.

- **Monitor:** As the teacher monitors, select 2-3 math discussion that can later be used as part of a whole class discussion. The teacher should select discussions that provide positive examples of one or two of the sub-components from the rubric. Also, select a recorded discussion of problem #3 or problem #7. These are challenging math problems. Students will benefit from being exposed to their fellow students’ mathematical thinking and solution methods.

**Whole Group Discussion: Real World Problems**

- Display the student discourse rubric (see resources) on the large screen.
- Call selected partners to the front to play their recorded discussion.
- Once the recording has been played, ask the student volunteers to tell which components they highlighted on their rubric. This provides an opportunity for the students to evaluate the positive aspects of their math discussion.
- Ask the class if they would like to add any positive comments about the discussion.
- Asked the class to share constructive advice.
- While students share constructive advice, the teacher should ensure that the discussion includes conjectures, predictions, and justifications. If these math discourse practices do not come up in the student conversation, the teacher should remind students to include these practices in their math discussions.
- Select 1 or 2 more recording to listen to and evaluate.

**Independent Problem Solving: Real World Problem Solving**

- Students will have the opportunity to apply problem solving strategies to a new, but similar problem.
- Students will have time to reflect on math problem solving and math discourse practices.
- Pass out practice problem and writing prompt (see resources). Read the directions together. Ensure that students understand the assignment.

**Writing in Math: Reflecting on Learning**

- Once the students are finished, ask them to share their method for solving problem #1 and their written reflection with a partner.
- By sharing their methods and written reflection, students will have an opportunity to learn multiple methods and to reflect upon their own mathematical thinking and the mathematical thinking of others.

**Formative Assessment:**

- The teacher will evaluate the recorded student discussions using the teacher rubric (see resources). Each student’s current level of math discourse will be assessed.
- The current scores can be compared to the scores that were gathered informally during lessons 2 and 6 to determine if student’s math discourse is improving.
- The teacher can assess the independent problem solving worksheet. The scores will provide information as to how well each student was able to solve real world math problems.
- The scores from the independent problem solving worksheet can be compared to the scores from lesson 4 to determine if each student’s problem solving ability has improved.

Teacher Notes Lesson 7:
Multiplication in the World Around Us

Lesson 8 – PBL, Demonstration of Learning

Lesson 8 Goals

Problem-Based Learning Goals

- Students will create a project to show how multiplication is used in the world outside the classroom.
- Students will present their project to an audience.

Math Standards Addressed in Lesson 8

Operations and Algebraic Thinking, Grade 3 (OAT)

- **OAT Standard #1** – In this lesson students will interpret products of whole numbers.
- **OAT Standard #3** – In this lesson students will use multiplication within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities.
Lesson 8 – PBL, Demonstration of Learning

Research-Based Rationale

This lesson allows students to address the original question, *how is multiplication used in the world around us?* This driving question has been used throughout the curriculum handbook to engage students in inquiry and to make learning more meaningful. Through the curriculum handbook the driving questions have helped students connect multiplication lessons to the world outside the classroom. Now students are given the chance to answer this driving question and in doing so demonstrate what they have learned about the concept of multiplication. By demonstrating how multiplication can be used in the world outside the classroom, students have the opportunity to extend their learning experience to the real world. Boss (2013) touches on the need for students to showcase what they have learned when she explains that that demonstrations of learning are a key component of problem- and project-based learning, and when engaging in inquiry, students should be given an opportunity to demonstrate what they have learned. In this lesson students will not only have the opportunity to demonstrate what they have learned through the creation of a project, but they will present their project to an authentic audience.

By allowing students to demonstrate what they have learned through the creation of a project, students are able to widen their scope of learning, and the learning experience itself is broadened through this process. When applying understanding to a project, learning no longer has a narrow emphasis. Learning is no longer about finding the right answer, but learning becomes focused on inquiry and application. Estes, Mintz, and Gunter cite this as one of the positive aspects of problem-based learning. They explain that in problem-based learning, the focus is no longer about filling in the blank with the correct answer. Learning is about critically thinking about and studying different aspects of an interesting problem (Estes, Mintz, & Gunter, 2011). When students learn through a problem-based approach to instruction, they learn by solving problems, they are able to construct and analyze information, and they create products that reflect their understanding (Papanikolaou & Boubouka, 2010). Lesson 8 provides students with these opportunities. In lesson 8 students are given the opportunity to construct a project that reflects their understanding of multiplication.
Lesson 8: PBL, Demonstration of Learning

Focus: Using Problem-Based Instruction founded on Mathematical Discourse to Learn About Multiplication in the World Around Us

Materials:
* Chart of possible real world math problem situations (saved from previous lessons)
* Division in the World Around Us, PBL Project direction Page (see resources)
* PBL project supplies such as: poster board, writing paper, access to google presentations, cardboard boxes for dioramas... (as needed).

Set the Stage: Real World Connections to Multiplication

- Post the driving question, *how is multiplication used in the world around us?* (see resources)
- Discuss answers to this question.
- Post the chart of real world math problem situations
- Read the chart together
- Ask students if they have any other real world situations to add to the chart

Explain: Tell students that they will be using information from the chart to create a presentation that shows how multiplication is used in the world outside the classroom.
- Show a video of multiplication in the real world. There are many great videos on Ted Talks Math in the Real World Series. Choose a video that is engaging to your students.
  

Set the Stage: Introduce the Demonstration of Learning

- Display the Multiplication in the World Around Us, PBL Project direction page (see resources and example) to introduce the project.

Example of PBL project direction page

<table>
<thead>
<tr>
<th>Multiplication in the World Around Us</th>
</tr>
</thead>
<tbody>
<tr>
<td>PBL Project</td>
</tr>
<tr>
<td>Directions:</td>
</tr>
<tr>
<td>1. Through this project, you will show how multiplication can be used in the world outside the classroom.</td>
</tr>
<tr>
<td>2. You will choose to work independently or in a group of two or three.</td>
</tr>
<tr>
<td>3. You will create a real world situation, and each group member will create two multiplication math problems that match that real world situation.</td>
</tr>
<tr>
<td>4. You will write a rough draft of each math problem including the solution, and you will write a final copy of each math problem including its solution. The final copies must be handed in before they are displayed or used in the project.</td>
</tr>
<tr>
<td>5. A final copy of each math problem, but without the solution, will be displayed or used as a part of your final project presentation.</td>
</tr>
<tr>
<td>6. You should be prepared to present the project to an audience. During the presentation you will ask the audience to solve the multiplication problems that you have created.</td>
</tr>
</tbody>
</table>

Create a Real World Situation

Circle one: I will make create my project (independently, with a partner, in a group of three)

My partner(s): __________________________

Circle one main dish project:

1. Create a *google presentation*. The google presentation should include an example of how multiplication is used in the real world. This can be accomplished by using a picture or typing a scenario. A final copy of all multiplication problems should be included.

2. Create a *poster* to show how math is used in the world outside the classroom. The poster should include a drawing or picture of the real world setting, and a final copy of all multiplication problems should be included.
3. Create a diagram. Create a 3 dimension diagram that depicts a real world setting in which multiplication can be used to solve math problems. A final copy of all multiplication problems should be included in the diagram.
4. Create a play or skit. Write a play or skit that shows how multiplication is used in a real world setting. Throughout the skit, the multiplication math problems should be posed as part of the plot of the play or skit.

Demonstration of Learning: Work on PBL projects

**Explain:** Each project should answer the original driving question, *how is multiplication used in the world around us?*
- Provide students with time to work on project that address this driving question.
- This is the opportunity for students to demonstrate what they have learned about multiplication in the world outside the classroom.
- Students should be given a deadline for turning in both the rough draft and final copies of their multiplication problems. The deadlines will help students stay on task and meet the goals of the project.

**Demonstration of Learning: Presentations**

**Explain:** Remind students that you will be inviting another class and/or parents to view the projects. Let the students know who their audience will be.
- When students have completed their projects, give them time to think about how they will present their project to an audience. Remind students that part of their presentation should include asking others to solve the multiplication problems that they have created.
- Invite an audience and allow the students to present their projects.
- Presentations can be done in different ways. Each student can be given time to formally present their project to the audience, or the project can be presented in an informal museum walk format. In this format, the projects are set-up around the classroom and the audience moves from one project to the next. Students present each time a new audience arrives at their project.

**Formative Assessment:**

- The PBL projects and presentations are not graded or assessed.
- The student’s math problems can be assessed to determine how well they accurately depict real world situations and for correct application of multiplication concepts.
- The post-assessment can be given as a formative or summative assessment.

Teacher Notes Lesson 8:
LESSON RESOURCES

Curriculum Handbook: Multiplication in the World Around Us

Problem-Based Math Lesson Founded on Math Discourse Principles

Level: Third Grade
By Signe Miller
California State University, Chico
Fall 2014
Name: ____________________

**Sharing Stickers with your Classmates** (Pre-Assessment)

**Directions:** For each math problem, show your work and write your answer with a label.

**Problem 1:** The friends in your class like to exchange stickers. You decide to give each classmate three stickers. You have 19 classmates. How many stickers will you need?

<table>
<thead>
<tr>
<th>Your Work:</th>
<th>Changes to your answer. Explain.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Your Answer: ______________________

**Problem 2:** Your friend decides to give each classmate 4 stickers. There are still 19 classmates. How many stickers will your best friend need?

<table>
<thead>
<tr>
<th>Your Work:</th>
<th>Changes to your answer. Explain.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Your Answer: ______________________

**Problem 3:** Your younger brother wants to give two stickers to each of his classmates. Explain to him how to figure out how many stickers he needs.

<table>
<thead>
<tr>
<th>Your Explanation:</th>
<th>Changes to your explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>
1. How confident are you that you understand how to solve these three math problems?
   Circle one response.
   a. very confident
   b. somewhat confident
   c. not confident at all

2. Which math problem was the most difficult to solve? You can circle more than one response.
   a. Problem 1
   b. Problem 2
   c. Problem 3
   d. None of the problems were difficult

3. Do you think you are good at solving math word problems?
   a. I am very good at solving math word problems
   b. I am good at solving math word problems
   c. I am just o.k. at solving math word problems

**Directions:** Now, discuss the problems with a partner. Discuss how you solved each problem. Find out if you solved the problems the same way or in different ways. Find out if you got the same answer or different answers. You can change your answers if you would like, but use color pencil to show the changes that you make. Also, explain why you change your answer.

4. Did your discussion with a partner help you gain confidence in your solutions?
   a. Yes, I felt more confident
   b. I felt somewhat more confident
   c. The discussion did not make me feel more confident.

5. Did you enjoy solving these math problems on your own?
   a. Yes
   b. No
   c. A little

6. Did you enjoy discussing these math problems with your partner?
   a. Yes
   b. No
   c. A little

7. In one or two sentences, tell why you enjoyed discussing the math problems with your partner or why you didn’t enjoy solving the math problems with your partner.

*Source:* Pre-Assessment Math Problems Adapted From: Problem of the Month: Friends You Can Count On (c) Noyce Foundation 2005
Visiting the Museum to See the Dinosaur Exhibit (Post-Assessment)

Directions: Solve all parts of the math problem. Show your work, and write your answer with the correct label.

Math Problem: You and your sister want to go to the museum to see the dinosaur exhibit. The museum has three different plans to pay for going to see the dinosaur exhibit.

Dinosaur Exhibit Museum Rates andPlans

<table>
<thead>
<tr>
<th>Plan A:</th>
<th>Pay $3.00 per person to visit the museum.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plan B:</td>
<td>Pay $8.00 for a monthly membership for one person, but you can go as many times as you want for one month.</td>
</tr>
<tr>
<td>Plan C:</td>
<td>Pay $17.00 for a monthly membership for the whole family. Everyone in your family can go as often as they like for one month.</td>
</tr>
</tbody>
</table>

Part 1: You and your sister want to go see the dinosaur exhibit three times this month. How much will it cost if you use Plan A?

Your Work: 

Your Answer: 

Changes to your answer. Explain.

Part 2: If you and your sister want to go see the dinosaur exhibit three times this month, which plan should you buy to save the most money?

Your Work: 

Your Answer: 

Changes to your answer. Explain.

Part 3: Your sister doesn’t understand why you aren’t going to buy plan C. Do your best to explain why plan C is not the best option.

Your Explanation: 

Changes to your explanation.
1. How confident are you that you understand how to solve these three math problems? Circle one response.
   a. very confident
   b. somewhat confident
   c. not confident at all

2. Which part of the math problem was the most difficult to solve? You can circle more than one response.
   a. Part 1
   b. Part 2
   c. Part 3
   d. None of the parts of this problem were difficult

3. Do you think you are good at solving math word problems?
   a. I am very good at solving math word problems
   b. I am good at solving math word problems
   c. I am just o.k. at solving math word problems

**Directions:** Now, discuss the problems with a partner. Discuss how you solved each problem. Find out if you solved the problems the same way or in different ways. Find out if you got the same answer or different answers. You can change your answers if you would like, but use color pencil to show the changes that you make. Also, explain why you change your answer.

4. Did your discussion with a partner help you gain confidence in your solutions?
   d. Yes, I felt more confident
   e. I felt somewhat more confident
   f. The discussion did not make me feel more confident.

5. Did you enjoy solving these math problems on your own?
   d. Yes
   e. No
   f. A little

6. Did you enjoy discussing these math problems with your partner?
   d. Yes
   e. No
   f. A little

7. In one or two sentences, tell why you enjoyed discussing the math problems with your partner or why you didn’t enjoy solving the math problems with your partner.

**Source:** Adapted From: Problem of the Month: Museum Rate Plans (c) Noyce Foundation 2005
<table>
<thead>
<tr>
<th>Student Name</th>
<th>Problem Solving (PS)</th>
<th>PS Confidence Q1</th>
<th>PS Confidence Q2</th>
<th>Math Discourse Any corrections in colored pencil?</th>
<th>Math Discourse Q41</th>
<th>Math Discourse Q46</th>
</tr>
</thead>
<tbody>
<tr>
<td>Example</td>
<td>2 out of 3</td>
<td>3</td>
<td>2</td>
<td>Yes</td>
<td>4</td>
<td>Yes</td>
</tr>
</tbody>
</table>
Math Discourse Partners: Use the pre-assessment or formative assessments to create math discourse partners

<table>
<thead>
<tr>
<th>Student Name</th>
<th>PS Confidence Q#1</th>
<th>Math Discourse Q#4</th>
<th>Discourse Partner</th>
</tr>
</thead>
<tbody>
<tr>
<td>Example</td>
<td>very confident</td>
<td>confident</td>
<td>Drew</td>
</tr>
</tbody>
</table>


### Evaluating Math Discourse - Assessment Rubric for Teachers

Circle the asterisk (*) and the score that best describes the math conversation

<table>
<thead>
<tr>
<th>Score 1</th>
<th>Score 2</th>
<th>Score 3</th>
<th>Score 4</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Math Discourse Overview</strong></td>
<td>During the math discussion students...&lt;br&gt;*did not uncover or analyze errors&lt;br&gt;*did not make connections between strategies or ideas of others only compared answers.&lt;br&gt;*did not ask/provide follow-up questions/statements to promote math explanations, reasoning, and evidence.&lt;br&gt;*did not attempt to ask for/or provide conjectures or predictions.</td>
<td>During the math discussion students...&lt;br&gt;*did not uncover or analyze errors&lt;br&gt;*made one connection between strategies and compared different strategies when applicable.&lt;br&gt;*asked/provided one or two follow-up questions/statements to promote math explanations, reasoning, and evidence.&lt;br&gt;*made one attempt to ask for/or provide conjectures or predictions, but it didn’t fully make sense.</td>
<td>During the math discussion the students...&lt;br&gt;*uncovered and/or analyzed errors as needed&lt;br&gt;*made a few connections between and compared strategies when applicable.&lt;br&gt;*asked/provided a few follow-up questions/statements to promote math explanations, reasoning, and evidence.&lt;br&gt;*asked for/provided one or two conjectures or predictions.</td>
</tr>
</tbody>
</table>

Source: Adapted from Park, J., Nava, I., & Applegate, M. (2011) *Observation Rubric for Secondary Mathematics: UCLA Teacher Education Rubric*
<table>
<thead>
<tr>
<th>Score 1</th>
<th>Score 2</th>
<th>Score 3</th>
<th>Score 4</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>I did not connect my ideas or strategies to those of others.</em></td>
<td><em>I connected my ideas, strategies, or answers to those of others at least once.</em></td>
<td><em>I connected my ideas, strategies, or answers to those of others at least three times.</em></td>
<td><em>I connected my ideas, strategies, or answers to those of others at least three times.</em></td>
</tr>
<tr>
<td><em>And I did not compare my strategies or ideas, with those of others. I only compared my answers with those of others.</em></td>
<td><em>But I did not use these connections to compare strategies, ideas, or answers with those of others.</em></td>
<td><em>I used these connections to compare and evaluate ideas, strategies, and answers to those of others.</em></td>
<td><em>I used these connections to compare and evaluate ideas, strategies, and answers to those of others.</em></td>
</tr>
<tr>
<td><em>I did not make a conjecture or a prediction based on what I have learned or discussed.</em></td>
<td><em>I attempted to make one conjecture or prediction based on what I have learned and discussed.</em></td>
<td><em>I made a few conjectures or predictions based on what I have learned and discussed.</em></td>
<td><em>I made conjectures and predictions based on what I have learned and discussed.</em></td>
</tr>
</tbody>
</table>

**Students Discourse**

**Math Rigor**

**Procedural Understanding:** how you solved the problem

**Explain:** tell **why** you used a strategy

**Reasoning:** explain the math behind your strategy

**Justify:** provide evidence that your math strategy makes sense.

---

**Evaluating Math Discussions – Math Discussion Rubric for Students**

Circle or color the asterisks (*) and the score that best describes the conversation.

How to Have a Math Discussion, Part 1

Part 1: Explain Your Math Method

4. Explain **how** you solved the problem. Include **math reasons** that tell **why** you solved it this way.

   Possible sentence starters...
   “First I ______ because ________.”
   “Then I ______ because ________.”
   “Also, I ______ because ________.”

5. Provide **evidence** that shows why your method makes sense.

   Possible sentence starter...
   “______ makes sense because ________”
   “This strategy works because ________”

6. Provide **connections** and **comparisons**. Does this math problem or method remind you of another? How are they similar or different?

   Possible sentence starters...
   “I used a similar method when I ______.”
   “I used a similar method when I ______, but his time I ______”
   “This problem reminds me of ________.”
   “This problem reminds me of________, but it is a little different because ________.”
   “Honestly, I don’t think I’ve ever solved a math problem like this before.”
Question Frames for Students

**Questions Frames** – That promote math **Explanations**, **Reasoning** and **Evidence**

*Why did you ....?*
*Why did you think that strategy would work?*
*What is your evidence?*
*How did you know to....?*
*Can you prove to me that … makes sense.*
*Can you explain why you…?*
*What convinced you that was the right answer?*

(Source: Adapted from Small et al., 2004, Smith and Stein, 2011, Chapin et al., 2013)
<table>
<thead>
<tr>
<th>Statement and Question Frames: Promote Connections and Comparing</th>
</tr>
</thead>
<tbody>
<tr>
<td>I used a different method.</td>
</tr>
<tr>
<td>*I ... and you..., but I understand how they both make sense.</td>
</tr>
<tr>
<td>*I ...and you..., I am having a hard time understanding your method.</td>
</tr>
<tr>
<td>*I ...and you... I like how you... that thinking is new to me.</td>
</tr>
<tr>
<td>We used the same method.</td>
</tr>
<tr>
<td>*You...and I ....</td>
</tr>
<tr>
<td>My method is similar to your method.</td>
</tr>
<tr>
<td>*You and I both...but I didn’t...</td>
</tr>
<tr>
<td>*You and I both...but I ...</td>
</tr>
<tr>
<td>* I ..., but I didn’t....</td>
</tr>
<tr>
<td>Questions that promote Connections and Comparison</td>
</tr>
<tr>
<td>*Does this math problem remind you of another problem?</td>
</tr>
<tr>
<td>*Have you solved a math problem like this before?</td>
</tr>
<tr>
<td>* Have you used this method before?</td>
</tr>
<tr>
<td>* Can you create another problem that can be solved the same way?</td>
</tr>
</tbody>
</table>

Source: Adapted from Small et al., 2004 and Smith and Stein, 2011
### Questioning and Statement Frames for Teachers

<table>
<thead>
<tr>
<th>Possible Questions: Promote <strong>Mathematical Understanding</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>* What do you know about this problem?</td>
</tr>
<tr>
<td>* What do we need to find out?</td>
</tr>
<tr>
<td>* What information is necessary for you to solve this problem?</td>
</tr>
<tr>
<td>* How can you use what you know to solve this problem?</td>
</tr>
<tr>
<td>* Finally – What could be your first step? Try taking a first step and go from there.</td>
</tr>
<tr>
<td>* Source: Adapted from Small et al., 2004</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Restating Techniques</strong>: These promote student <strong>reflection</strong> and improve <strong>metacognition</strong>.</th>
</tr>
</thead>
<tbody>
<tr>
<td>* What did you do first? Why?</td>
</tr>
<tr>
<td>* What were you thinking when you wrote this?</td>
</tr>
<tr>
<td>* How did you know to …?</td>
</tr>
<tr>
<td>* Can you tell me what you did?</td>
</tr>
<tr>
<td>* Source: Adapted from: Chapin et al., 2009 and Smith and Stein, 2011</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Revoicing Techniques</strong>: Help students access other’s thinking and provide <strong>clarification</strong>.</th>
</tr>
</thead>
<tbody>
<tr>
<td>* …pointed out that….do you agree or disagree?</td>
</tr>
<tr>
<td>* So you are saying that…</td>
</tr>
<tr>
<td>* Can …say what …said so I can be sure I understand.</td>
</tr>
<tr>
<td>* Who can put that into their own words?</td>
</tr>
<tr>
<td>* Let me see if I understand. Are you thinking…?</td>
</tr>
<tr>
<td>* Who can repeat what ….said?</td>
</tr>
<tr>
<td>* Source: Adapted from: Chapin et al., 2009 and Chapin et al., 2013</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Question Frames</strong>: Promote <strong>Exposure and Connections</strong> amongst students</th>
</tr>
</thead>
<tbody>
<tr>
<td>* Can you model what you were thinking?</td>
</tr>
<tr>
<td>* How do you know you can do that?</td>
</tr>
<tr>
<td>* How can we represent that with symbols or with a formula?</td>
</tr>
<tr>
<td>* Who can add on?</td>
</tr>
<tr>
<td>* Does this method make sense to you? Why?</td>
</tr>
<tr>
<td>* Does anyone want to contribute more evidence to explain why this method works?</td>
</tr>
<tr>
<td>* Does anyone have a different way to explain this?</td>
</tr>
<tr>
<td>* Source: Adapted from Small et al., 2004, Smith and Stein, 2011, and Chapin et al., 2013</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Question Frame</strong>: Questions that Promote <strong>Predictions, Conjectures, and Justification</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>* Can you state a general rule you have discovered?</td>
</tr>
<tr>
<td>* Would this still be true if you change…?</td>
</tr>
<tr>
<td>* How do we know this for sure?</td>
</tr>
<tr>
<td>* Can you give another example of your thinking?</td>
</tr>
<tr>
<td>* Source: Adapted From: Small et al., 2004 and Smith and Stein, 2011</td>
</tr>
</tbody>
</table>
Part 2: Have a Math Discussion

11. Math partners should always **actively** and **carefully listen**.
12. Ask follow-up **questions** that promote **explanations**, **reasoning**, and **evidence**.
   
   Possible sentence starters…
   *Why did you ....?*  
   *Why did you think that strategy would work?*  
   *How did you know to....?*  
   *Can you prove to me that ... makes sense.*  
   *Can you explain why you...?*  
   *What convinced you that was the right answer?*

3. Make statements or ask questions that promote **connections** and **comparisons**
   
   I used a different method.  
   *I ... and you..., but I understand how they both make sense.*  
   *I ...and you..., I am having a hard time understanding your method.*  
   *I ...and you... I like how you.... that thinking is new to me.*  
   *We used the same method.*  
   *You...and I ....*  
   *My method is similar to your method.*  
   *You and I both...but I didn’t...*  
   *You and I both...but I ...*  
   *I ...., but I didn’t....*  
   *Questions that promote Connections and Comparison*
   *Does this math problem remind you of another problem?*  
   *Have you solved a math problem like this before?*  
   *Have you used this math method before?*  
   *Can you create another problem that can be solved the same way?*
Part 3: Look for patterns and make predictions and conjectures.

4. Once you have shared your math methods and explanations with a partner, look for **patterns** and have a discussion about the patterns you observed.

5. Try to make **predictions** or **conjectures** based on the patterns that you observed.

6. Provide **justification** that your predictions or conjectures make sense.

   Possible sentence starters…

   I notice a pattern. It is ________.

   I think I noticed a math rule. It is ____________.

   If I changed _______ the pattern would still be true.

   If I changed _______ it would change the pattern because__________.

   Let me tell you another example of the math rule I discovered.

   ____________.
Vocabulary Worksheet

<table>
<thead>
<tr>
<th>Repeated Addition</th>
<th></th>
<th></th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Multiplication</th>
<th></th>
<th></th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Equal Groups</th>
<th></th>
<th></th>
</tr>
</thead>
</table>
Writing a Math Reflection

Directions

1. Think first
2. Talk with a partner
3. Write
   * What was difficult or easy about the math problem?
   * What helped you understand or become successful with the math problem?
     * Explain what you learned
   * Use at least one vocabulary word
Lesson 1: Evaluation / Assessment

The Fortune Cookie Problem

Source: office.com royalty free clip art

Problem: My whole class went to a restaurant together. All 20 students ate 2 fortune cookies. How many cookies did our class eat?

Show Your Solution Method:

Your Answer: ____________________________
Cookie Manipulatives

Cookies
Cookie Manipulatives
Cookies and Decorations
Muffin Manipulatives

Source: office.com royalty free clip art
### Muffin Manipulatives

<table>
<thead>
<tr>
<th>Pecans</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Pecans" /></td>
</tr>
<tr>
<td><img src="image5.png" alt="Pecans" /></td>
</tr>
<tr>
<td><img src="image9.png" alt="Pecans" /></td>
</tr>
</tbody>
</table>
Partner Work Poster

**Partner 1**

*Explain your math Method.*  
(Use the Poster, How to Have a Math Discussion, Part 1 for Support)

*Listen to your partner’s questions and/or statements.*

*Answer your partner’s questions.*

*Listen to your partner’s method and math thinking.*

*Use at least 3 question frames and/or statement frames.*

*IF you want to make changes, make changes to your original solution in colored pencil. IF you don’t want to make changes, use the colored pencil to write down a second method for solving the problem.*

*Be ready to explain any changes that you chose to make.*
<table>
<thead>
<tr>
<th>Partner Work Poster</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Partner 2</strong></td>
</tr>
<tr>
<td><em>Listen to your partner’s method and math thinking.</em></td>
</tr>
<tr>
<td><em>Use at least 3 questions and/or statement frames.</em></td>
</tr>
<tr>
<td><strong>Explain your math Method.</strong> <em>(Use the Poster, How to Have a Math Discussion, Part 1 for Support)</em></td>
</tr>
<tr>
<td><em>Listen to your partner’s questions and statements.</em></td>
</tr>
<tr>
<td><em>Answer your partner’s questions.</em></td>
</tr>
<tr>
<td><em>IF you want to make changes, make changes to your original solution in colored pencil. IF you don’t want to make changes, use the colored pencil to write down a second method for solving the problem.</em></td>
</tr>
<tr>
<td><em>Be ready to explain any changes that you chose to make.</em></td>
</tr>
</tbody>
</table>
Evaluating Math Discourse - Assessment Rubric for Teachers

Circle the asterisk (*) and the score that best describes the math conversation.

<table>
<thead>
<tr>
<th>Math Discourse Overview</th>
<th>Score 1</th>
<th>Score 2</th>
<th>Score 3</th>
<th>Score 4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>During the math discussion students…</td>
<td>During the math discussion students…</td>
<td>During the math discussion students…</td>
<td>During the math discussion students…</td>
</tr>
<tr>
<td>*did not uncover or analyze errors</td>
<td>*did not uncover or analyze errors</td>
<td>*did not uncover or analyze errors</td>
<td>*uncovered and/or analyzed errors as needed</td>
<td>*uncovered and analyzed errors as needed</td>
</tr>
<tr>
<td>*did not make connections between strategies or ideas of others only compared answers.</td>
<td>*made one connection between strategies and compared different strategies when applicable.</td>
<td>*made a few connections between and compared strategies when applicable.</td>
<td>*asked/provided a few follow-up questions/statements to promote math explanations, reasoning, and evidence.</td>
<td>*asked/provided a few follow-up questions/statements to promote math explanations, reasoning, and evidence.</td>
</tr>
<tr>
<td>*did not ask/provide follow-up questions/statements to promote math explanations, reasoning, and evidence.</td>
<td>*made one attempt to ask for/provided conjectures or predictions, but it didn’t fully make sense.</td>
<td>*asked for/provided one or two conjectures or predictions.</td>
<td>*asked for/provided conjectures and predictions.</td>
<td>*asked for/provided conjectures and predictions.</td>
</tr>
</tbody>
</table>

Name: 

**Multiplication and Baking Math Problems**

**Directions:** First, show your work. Then write your answer with a label. Discuss each problem with a partner. After a partner discussion, make any changes with a colored pencil. If you don’t want to make changes to your original answers, choose one problem and use color pencil to show a different method that could be used to solve the problem.

<table>
<thead>
<tr>
<th>Question 1: Baking Cookies</th>
<th>Question 2: Baking Dinner Rolls</th>
</tr>
</thead>
<tbody>
<tr>
<td>Today the baker is making 7 cookies. He wants to put 3 decorations on each cookie. How many decorations will he need?</td>
<td>The baker is making 6 dinner rolls. She will put 2 olives in each roll. How many olives will she need?</td>
</tr>
<tr>
<td><strong>Solution Method:</strong></td>
<td><strong>Solution Method:</strong></td>
</tr>
</tbody>
</table>

**Answer:**

Changes or new method in colored pencil:

**Answer:**

Changes or new method in colored pencil:
### Question 3: Baking Pies

The baker is making 4 pies. She needs 3 cups of flour for each pie. How many cups of flour will she need?

**Solution Method:**

<table>
<thead>
<tr>
<th>Answer:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Changes or new method in colored pencil:</td>
</tr>
</tbody>
</table>

### Question 4: Baking Cakes

The baker is making 3 cakes. He needs 2 cubes of butter for each cake. How many cubes of butter will he need?

**Solution Method:**

<table>
<thead>
<tr>
<th>Answer:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Changes or new method in colored pencil:</td>
</tr>
</tbody>
</table>

---

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Cookies and Star Decorations
Baking and Multiplication Game

Directions:
1. This game can be played with 1-3 players.
2. Get a scratch paper and divide it into three or four sections.
3. If you are playing on your own, you will complete all the sections on your own. If you are playing with other players, you will share the scratch paper.
4. Roll the dice to determine how many cookies you will be baking today (Player 1).
5. Use scratch paper to draw the number of number of cookies (Player 1).
6. Roll the dice again to determine how many star decorations your customer wants on each cookie (player 2).
7. Use the same scratch paper to draw the number of star decorations (player 2).
8. Record the total number of stars. Try to write a multiplication equation to match the picture (player 3, or player 1).

### Four Practice Problems To Cut and Sort

<table>
<thead>
<tr>
<th>Math Problem #1</th>
<th>Math Problem #2</th>
</tr>
</thead>
<tbody>
<tr>
<td>At the basketball game, each regular basket is worth 2 points. If Shayla made 6 regular baskets, how many points did she earn for the team?</td>
<td>At the baskte ball game, each regular basket is worth 2 points. However, if you shoot from outside the line, it is called a 3 point shot. If you make a 3 point shot, you earn 3 points for the team. Shayla made 6 regular baskets and one 3 point shot. How many points did she make for the team?</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Math Problem #3</th>
<th>Math Problem #4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Four of my friends, Shayla, Barry, Saul, and Jenna all brought grapes for lunch. Each friend ate 6 grapes. How many grapes did my friends eat?</td>
<td>My four friends Shayla, Barry, Saul, and Jenna brought grapes for lunch. Shayla ate 4 grapes. Barry ate 5 grapes, Saul ate 6 grapes, and Jenna only ate 2 grapes. How many grapes did my friends eat?</td>
</tr>
</tbody>
</table>
Real World Math Problems to Cut and Sort

<table>
<thead>
<tr>
<th>Problem #1</th>
<th>Problem #2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jasmin bought 4 jars of apple sauce at the market. Each jar cost $6. How much did the apple sauce cost in all?</td>
<td>Erik was trying to get in shape. He was trying to jog 3 miles each day. He was able to do this for 4 days. How many miles did he run during the four day period?</td>
</tr>
</tbody>
</table>

**Solution Method:**

**Answer:**

<table>
<thead>
<tr>
<th>Problem #3</th>
<th>Problem #4</th>
</tr>
</thead>
<tbody>
<tr>
<td>The baker was making decorative cookies for his customer, Mrs. McCrown. Mrs. McCrown wanted 3 cookies, and she wanted all the cookies to be decorated differently. The baker put 2 chocolate swirls on one cookie, 3 chocolate swirls on another cookie, and 1 chocolate swirl on the last cookie. How many chocolate swirls did the baker make?</td>
<td>Jasmin bought 4 apples at the market. The clerk weighed each apple to find its cost. The first apple cost 5 cents. The second apple cost 6 cents. The third apple cost 10 cents. The last apple cost 8 cents. How much did the apples cost in all?</td>
</tr>
</tbody>
</table>

**Solution Method:**

**Answer:**
## Real World Math Problems to Cut and Sort, Continued

<table>
<thead>
<tr>
<th>Problem #5</th>
<th>Problem #6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Juanita was trying to get in shape. She ran 3 miles a day for 4 days. The next day she ran 5 miles. How many miles did she run? <strong>Solution Method:</strong></td>
<td>The baker was making decorative cookies for his costumer, Mr. McCrown. Mr. McCrown wanted 4 cookies, and he wanted them all to be decorated exactly the same. Mr. McCrown wanted 3 chocolate swirls on each cookie. How many chocolate swirls will the baker make for Mr. McCrown? <strong>Solution Method:</strong></td>
</tr>
<tr>
<td><strong>Answer:</strong></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Problem #7</th>
<th>Problem #8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jerry bought 4 jars of apple sauce at the market. Each jar cost $6. He also bought a jar of peanut butter for $5. How much did Jerry spend? <strong>Solution Method:</strong></td>
<td>Erica was trying to get in shape. One day she ran 3 miles. The second day she ran for 4 miles. On the last day she ran 2 miles. How many miles did she run all together? <strong>Solution Method:</strong></td>
</tr>
<tr>
<td><strong>Answer:</strong></td>
<td></td>
</tr>
</tbody>
</table>
4 - 3 x 5 cards for sorting whole class

<table>
<thead>
<tr>
<th>Basic</th>
<th>Repeated</th>
</tr>
</thead>
<tbody>
<tr>
<td>+</td>
<td>+</td>
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</table>

<p>| | |</p>
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<tbody>
<tr>
<td>X</td>
<td>X</td>
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<tr>
<td></td>
<td>and</td>
</tr>
<tr>
<td></td>
<td>+</td>
</tr>
</tbody>
</table>
3 - 3 x 5 cards for sorting with a partner

<table>
<thead>
<tr>
<th>Basic</th>
<th>+</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>X</td>
</tr>
<tr>
<td></td>
<td>X and</td>
</tr>
<tr>
<td></td>
<td>+</td>
</tr>
</tbody>
</table>
Procedures for Sorting Math Problems with a Partner

- Read each math problem together. You can take turns reading each problem. When one partner is reading, the other one should be following along and thinking.

- Decide together how to sort each problem.
  * How do you think we should sort problem ____? I agree.
  * How do you think we should sort problem ____? I disagree…I was thinking…
  * I think we should sort problem ____ into the category _____. What do you think?

- If you disagree about a problem, put it to the side and move on to the next one.

If you finish, try to solve one problem from each category.
Guidelines
For Writing Today’s Real World Math Problems

- Use a real life situation.
- Create a problem situation, or story, in which the reader would have to multiply to solve the problem.
- Ask a question at the end.
- Use digits 0-10.
- Create a problem situation, or story, in which the reader would have to multiply and add to solve the problem.
- Ask a question at the end.
- When you finish, solve both problems.
Standards for Mathematical Practices #1

Make sense of problems and persevere in solving them

Mathematically proficient students…

- Explain the meaning of a problem
- Look for entry points to a problem
- Plan a solution path rather than simply jump into a solution attempt
- Try a special case or a simple form of the original problem to gain insight to a possible solution

Mathematically proficient students…

- Analyze the constraints, the relationships, and the goals involved in the problem
- Monitor your progress and change course if needed.
- Check their answers, and ask, "Does this make sense?"
- Understand the approaches of others and identify connections between different approaches.

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(Section 4.5.7)
Standards for Mathematical Practices #3
Construct Viable Arguments and Critique the Reasoning of Others

Mathematically proficient students…

Construct viable arguments

Respond to the arguments of others

Compare two arguments and distinguish correct reasoning from flawed reasoning

Elementary students can construct arguments using objects, drawing, diagrams, or actions

Make conjectures and explore the truth of their conjectures

Analyze math situations by breaking them into cases, and can recognize and use examples or counterexamples.

Justify their conclusions, and communicate them to others

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Example Problem to Teach the How to Record a Math Discussion

Marisol plays the flute. She practices regularly. Marisol always begins her practice sessions with a finger warm-up. She did a finger warm-up 5 days last week. Each warm up took her 6 minutes. How many minutes did she spend doing finger warm-ups last week?
Real World Problems to Cut, Sort, and Solve

Choose one problem that you will use for recording purposes.

<table>
<thead>
<tr>
<th>Problem #1</th>
<th>Problem #2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Joel bought 4 packages of gum. Each pack of gum contains 8 sticks of gum. How many sticks of gum does he have?</td>
<td>Erica was selling lemonade on Saturday. She sold small cups of lemonade and large cups of lemonade. A small cup of lemonade cost $2, and a large cup of lemonade cost $3. She sold 9 small cups of lemonade. How much money did she make?</td>
</tr>
<tr>
<td><strong>Solution Method:</strong></td>
<td><strong>Solution Method:</strong></td>
</tr>
<tr>
<td>Answer:</td>
<td>Answer:</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Problem #3</th>
<th>Problem #4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bailey was celebrating her birthday at school. She wanted to bring cookies. She bought 2 packs of cookies. There were 8 cookies in each pack. She didn’t think that would be enough, so she bought 6 more single cookies. How many cookies did Bailey purchase?</td>
<td>Erica was selling lemonade on Sunday. She sold small cups of lemonade and large cups of lemonade. A small cup of lemonade cost $2, and a large cup of lemonade cost $3. She sold 6 small cups of lemonade and 2 large cups of lemonade. How much money did she make?</td>
</tr>
<tr>
<td><strong>Solution Method:</strong></td>
<td><strong>Solution Method:</strong></td>
</tr>
<tr>
<td>Answer:</td>
<td>Answer:</td>
</tr>
</tbody>
</table>
Real World Problems to Cut, Sort, and Solve, Continued

<table>
<thead>
<tr>
<th>Problem #5</th>
<th>Problem #6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Caden practiced his spelling words three nights this week. Each night he practiced his spelling words for 10 minutes. How many minutes did Caden spend practicing his spelling words this week? <strong>Solution Method:</strong></td>
<td>Jessica practiced her spelling words three nights this week. One night she practiced for 15 minutes, one night she practiced for 8 minutes, and one night she practiced for 12 minutes. How many minutes did Jessica spend practicing her spelling words this week? <strong>Solution Method:</strong></td>
</tr>
<tr>
<td><strong>Answer:</strong></td>
<td><strong>Answer:</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Problem #7</th>
<th>Problem #8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Darrell ordered 6 boxes of candy bars for the party. Each box contained 8 candy bars. How many candy bars will he have for the party? <strong>Solution Method:</strong></td>
<td>Udonise bought 3 boxes of candy bars for the party. Each box contained 8 candy bars. He also bought 4 packs of cookies for the party, and there were 6 cookies in each package. How many total treats will Udonise have for the party? <strong>Solution Method:</strong></td>
</tr>
<tr>
<td><strong>Answer:</strong></td>
<td><strong>Answer:</strong></td>
</tr>
</tbody>
</table>
Name: ______________________

**Independent Practice**

**Real World Problem Solving**

1. Jerome went to the store to buy party supplies. He bought 8 packs of party hats. Each pack contained 2 hats. Also, he bought 4 packs of noise makers. Each pack contained 4 noise makers. How many total party supplies did he purchase?

   **Solution Method:**

   Answer:

2. Tell about **two** math discourse practices that you used well or that you would like to improve upon next time.
Example Vocabulary Worksheet

<table>
<thead>
<tr>
<th>First Factor</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td></td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Second Factor</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
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<td></td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Product</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Example and Non-Example of First and Second Factor

<table>
<thead>
<tr>
<th>Example 1</th>
<th>Example 2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>My customer Mrs. McCrown ordered 2 cookies. She wanted 4 star style decorations on each cookie. How many total decoration do I need to make today?</strong>&lt;br&gt; <strong>Solution Method:</strong>&lt;br&gt; <img src="image1.png" alt="Image of star decorations" />&lt;br&gt; <strong>2 X 4 = 8</strong>&lt;br&gt; <strong>Answer: 8 star style decorations</strong></td>
<td><strong>My customer Mrs. McCrown ordered 2 cookies. She wanted 4 star style decorations on each cookie. How many total decoration do I need to make today?</strong>&lt;br&gt; <strong>Solution Method:</strong>&lt;br&gt; <img src="image2.png" alt="Image of star decorations" />&lt;br&gt; <strong>4 X 2 = 8</strong>&lt;br&gt; <strong>Answer: 8 star style decorations</strong></td>
</tr>
</tbody>
</table>

Which example correctly uses the first factor and the second factor correctly in the multiplication equation?
Practice Problems for Setting the Stage: Using the Array Model

Practice Problem 1:
I am a gardener. I have been growing seedlings in my greenhouse, and now it is time to plant them in my garden. I want to place my carrot plants equal distance from each other, so I will plant them in equal rows. I will plant 3 rows of carrots. I will put 6 carrot plants in each row. How many carrot plants will I have in my garden?

Practice Problem 2:
I have also been growing onions in my greenhouse. I have 18 onions to plant. I don’t want to arrange my onions exactly like I arranged my carrots, but I do want to plant my onions in equal rows. Is there another way I could arrange my onions in equal rows?
Practice Problems for Whole Group Discussion: Using the Array Model

Practice Problem #1

Mr. Green planted green beans in his garden. The garden manual said to space the beans evenly. Mr. Green only had a long narrow area to plant his beans, so he decided to plant 2 beans in each row. He had enough space for 6 rows of beans. How many beans was Mr. Green able to plant?

Practice Problem #2

Mr. Brown’s garden was the opposite shape of Mr. Green’s garden. He decided to plant his beans in 2 rows, but he put 6 beans in each row. How many beans was Mr. Brown able to plant?

Practice Problem #3

Mrs. Green decided to build a garden pathway. She used square tiles to build her path. She placed the tiles so that there were 8 rows with 3 tiles in each row. How many tiles did she use?

 Practice Problem #4

Mrs. Brown liked Mrs. Green’s garden pathway, but she thought it was too big. She decided that she would only do 6 rows in her garden path. She used the same square tiles that Mrs. Green had used, but she placed the tiles so that there were 6 rows with 4 tiles in each row. How many tiles did she use?

Why do you think Mrs. Brown was surprised by her garden path?
Multiplication and Arrays

Game Procedures

1. Today you are planting a garden. You will plant your vegetables in equal rows.

2. Player 1 pulls a gardening card to determine what type of vegetable you will be planting.

3. Player 1 rolls the dice to determine how many rows will be planted.

4. Player 2 rolls the dice to determine how many vegetables will be planted in each row.

5. Both players use grid paper to draw an array and write an equation to match the math problem.

6. Player 1 explains his/her math method using How to Have a Math Discussion, Part 1.

7. Player 2 asks follow-up questions or makes follow-up statements using the question and statement frames that promote connections and comparisons.

8. Both players have a chance to make changes to their array or equation if needed.

9. Players switch roles so player 1 now becomes player 2, and player 2 becomes player 1.

Gardening Cards

Multiplication and Arrays Game

**Directions:** Cut out each gardening card. Place the cards face down. Follow the procedures for playing the Multiplication and Arrays game.

<table>
<thead>
<tr>
<th>Tomato Plants</th>
<th>Green Peppers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cucumbers</td>
<td>Broccoli Plants</td>
</tr>
<tr>
<td>Lettuce</td>
<td>Onions</td>
</tr>
</tbody>
</table>
Building a Garden

**Directions:** Today you will use grid paper and arrays to design a garden. First, you will choose four plants to grow in your garden. Second, you will decide how many equal rows of each plant you will have in your garden, and you will use grid paper to draw a design of how your plants will be arranged in the garden. Fourth, you will design one garden pathway. Your garden path should be made from square tiles, and it should be designed in equal rows. Lastly, you will determine the number of plants you will need to purchase before planting your garden.

1. List four plants that you will grow in your garden.
   1. ______________________
   2. ______________________
   3. ______________________
   4. ______________________

2. I will grow ___ rows of ________ with ___ plants in each row.
   I will grow ___ rows of ________ with ___ plants in each row.
   I will grow ___ rows of ________ with ___ plants in each row.
   I will grow ___ rows of ________ with ___ plants in each row.

3. Use grid paper to create your garden. For each array that you draw, write the corresponding multiplication equation inside the array and label the array with the plant that it represents. Remember to leave space for one pathway.

4. Review your garden design, and decide where your new path will go. You must use square tiles and design your path in equal rows. Draw a new pathway. Then write the multiplication equation that shows how many square tiles you will use to build the path.

   Multiplication Equation: ______ x ______ = ______

   I will use ______ tiles to build my garden path.

5. Review your garden design. How many total plants will you need to purchase?
Grid Paper
Animal Pens at the Zoo #1

Monkey Pen

Panda Pen

Giraffe Pen

Penguin Pen
Animal Pens at the Zoo #2

Bird Aviary

Lemur

Lion Pen
Animal Pens at the Zoo #2

Bear Pen

Zebra Pen
What is the area of an object?
People describe area in different ways. Basically…

*Area is a measurement.

*Area is the inside of a flat, or two dimensional, object.

*Area is used to measure the surface of a flat object.

*Area is measure in square units.
Zoo Pens #3

3 inches

5 in.

6 in.

2 in.

1 in.

2 in.
1 in. 3 in.

1 in.
Find the Area Dice Game

Game Procedures

1. New animals will be coming to the zoo soon. You must build new animal pens for these animals.
2. Player 1 pulls an animal card to determine the type of animal that is coming to the zoo soon.
3. Player 1 rolls the dice to determine how long (up and down) the pen will be.
4. Player 2 rolls the dice to determine how wide (across or side to side) the pen will be.
5. Player 1 uses a grid to create the animal pen with the correct length and width. Player 1 writes the area of the pen using square units.
6. Player 1 explains his/her math method for determining the area of the animal pen using How to Have a Math Discussion, Part 1.
7. Player 2 asks follow-up questions or makes follow-up statements using the question and statement frames that promote connections and comparisons.
8. Players always have a chance to make changes to their answer if needed
9. Players switch roles so player 1 now becomes player 2, and player 2 becomes player 1.
10. When all the animal cards have been used, each player should determine the total area of all the animal pens on their grid paper.

Find the Area Dice Game

Game Cards

Directions: Cut out each animal card. Place the cards face down. Follow the procedures for playing the *Find the Area* dice game.

<table>
<thead>
<tr>
<th>Shark</th>
<th>Dolphin</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grizzly Bear</td>
<td>Tiger</td>
</tr>
<tr>
<td>Fox</td>
<td>Gorilla</td>
</tr>
</tbody>
</table>
Sometimes when you are measuring area you do not have tiles or squares to help you find the area. If you don’t have tiles or squares, how can you use the measurements of the sides of your animal pens to determine the area of each pen?
Lesson 7 Practice Problem #1

Marisol enjoys working in the garden. She planted 8 rows of corn. She planted 6 seedlings in each row. All the plants survived. How many plants did Marisol harvest?

Lesson 7 Practice Problem #2

Marisol also planted tomato plants in her garden. She planted 4 rows of tomato plants. She put 8 seedlings in each row. Marisol did not water her plants enough and 3 plants died. How many plants survived?
4 - 3 x 5 cards for sorting with a partner

<table>
<thead>
<tr>
<th>Basic</th>
<th>X</th>
</tr>
</thead>
<tbody>
<tr>
<td>+</td>
<td>and</td>
</tr>
<tr>
<td>-</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>X</th>
<th>X</th>
</tr>
</thead>
<tbody>
<tr>
<td>and</td>
<td>+</td>
</tr>
</tbody>
</table>
Real World Problems to Cut, Sort, and Solve

Choose one problem that you will use for recording purposes.

<table>
<thead>
<tr>
<th>Problem #1</th>
<th>Problem #2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jessica built a rectangular garden patio. She used 1 foot square tiles. The patio is 3 tiles wide by 9 tiles long. What is the area of Jessica’s patio?</td>
<td>Joel build a garden patio. He used 1 foot square tiles to build the patio. The patio is 4 tiles long by 4 tiles wide. Joel didn’t think his patio was large enough so he added 2 more rows with 4 tiles in each row. How much area did Joel’s patio cover at first? How much did it cover once he enlarged the patio?</td>
</tr>
<tr>
<td>Solution Method:</td>
<td>Solution Method:</td>
</tr>
<tr>
<td>Answer:</td>
<td>Answer:</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Problem #3</th>
<th>Problem #4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solution Method:</td>
<td>Solution Method:</td>
</tr>
<tr>
<td>Answer:</td>
<td>Answer:</td>
</tr>
<tr>
<td>Problem #5</td>
<td>Problem #6</td>
</tr>
<tr>
<td>----------------------------</td>
<td>---------------------------------</td>
</tr>
<tr>
<td>Kellen practices his</td>
<td>Darnell planted 2 rows of 9</td>
</tr>
<tr>
<td>multiplication facts</td>
<td>tomato plants and 4 rows of 6</td>
</tr>
<tr>
<td>four nights a week. He</td>
<td>broccoli plants. How many</td>
</tr>
<tr>
<td>practices 10 minutes each</td>
<td>plants are growing in Darnell’s</td>
</tr>
<tr>
<td>night. However, this week</td>
<td>garden?</td>
</tr>
<tr>
<td>he decided to spend an</td>
<td>Solution Method:</td>
</tr>
<tr>
<td>additional 15 minutes</td>
<td></td>
</tr>
<tr>
<td>practicing his math</td>
<td></td>
</tr>
<tr>
<td>facts on Sunday before</td>
<td>Answer:</td>
</tr>
<tr>
<td>going back to school. How</td>
<td></td>
</tr>
<tr>
<td>many minutes did he spend</td>
<td></td>
</tr>
<tr>
<td>practicing his math facts</td>
<td></td>
</tr>
<tr>
<td>this week?</td>
<td></td>
</tr>
<tr>
<td>Solution Method:</td>
<td></td>
</tr>
<tr>
<td>Answer:</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Problem #7</th>
<th>Problem #8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shayla’s mom had 3</td>
<td>Joel has a desk, a bed, and a</td>
</tr>
<tr>
<td>packages of cookies in the</td>
<td>dresser in his room. He wants</td>
</tr>
<tr>
<td>pantry. Each package had 8</td>
<td>to find the total area of his</td>
</tr>
<tr>
<td>cookies. Shayla ate 3</td>
<td>furniture, so he measured each</td>
</tr>
<tr>
<td>cookies. How many cookies</td>
<td>piece of furniture. His desk is</td>
</tr>
<tr>
<td>were left?</td>
<td>4 feet by 2 feet. His bed is 6</td>
</tr>
<tr>
<td>Solution Method:</td>
<td>feet by 4 feet. His dresser is</td>
</tr>
<tr>
<td>Answer:</td>
<td>5 feet by 3 feet. How much area</td>
</tr>
<tr>
<td></td>
<td>is covered by his furniture?</td>
</tr>
<tr>
<td></td>
<td>Solution Method:</td>
</tr>
<tr>
<td></td>
<td>Answer:</td>
</tr>
</tbody>
</table>
Name: ______________________

Independent Practice

Real World Problem Solving

1. Udonis went to the store to buy party favors. He bought 4 packs of party favors. Each pack cost $9. He gave the cashier $40. How much change should he receive?

Solution Method:

Answer:

2. Your friend is stuck on problem #1. Your teacher wants you to help you friend. What can you say to your friend to help them understand math problem #1?
How is Multiplication used in the world around us?

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Multiplication in the World Around Us

PBL Project

Directions:

7. Through this project, you will show how multiplication can be used in the world outside the classroom.

8. You will choose to work independently or in a group of two or three.

9. You will create a real world situation, and each group member will create two multiplication math problems that match that real world situation.

10. You will write a rough draft of each math problem including the solution, and you will write a final copy of each math problem including its solution. The final copies must be handed in before they are displayed or used in the project.

11. A final copy of each math problem, but without the solution, will be displayed or used as a part of your final project presentation.

12. You should be prepared to present the project to an audience. During the presentation you will ask the audience to solve the multiplication problems that you have created.

Create a Real World Situation

Circle one: I will make create my project (independently, with a partner, in a group of three)

My partner(s): ____________________________________________

Circle one project.

5. Create a google presentation. The google presentation should include an example of how multiplication is used in the real world. This can be accomplished by using a picture or typing a scenario. A final copy of all multiplication problems should be included.

6. Create a poster to show how math is used in the world outside the classroom. The poster should include a drawing or picture of the real world setting, and a final copy of all multiplication problems should be included.

7. Create a diorama. Create a 3 dimension diorama that depicts a real world setting in which multiplication can be used to solve math problems. A final copy of all multiplication problems should be included in the diorama.

8. Create a play or skit. Write a play or skit that shows how multiplication is used in a real world setting. Throughout the skit, the multiplication math problems should be posed as part of the plot of the play or skit.
References


California Department of Education. [http://cde.ca.gov](http://cde.ca.gov)


Common Core State Standards Initiative. [http://www.corestandards.org/Math/Practice/](http://www.corestandards.org/Math/Practice/)


Illustrative Mathematics Website. https://www.illustrativemathematics.org/illustrations


Noyce Foundation (2005). Problem of the Month: Friends You Can Count On(c)

Noyce Foundation (2005). Problem of the Month: Museum Rate Plans (c)

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repository/images-and-media/math-observation-rubric.


