USING HANDS-ON ACTIVITIES TO LEARN ABOUT

LINEAR FUNCTIONS

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by
Chris G. Dell
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ABSTRACT

USING HANDS-ON ACTIVITIES TO LEARN ABOUT LINEAR FUNCTIONS

by

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Master of Science in Mathematics Education

California State University, Chico

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There is a need for lessons that combine rigor and relevance. During the first few years I taught at Enterprise High School, we created of a new course, “Hands-on Algebra 1,” for those students with low academic scores in mathematics. The goal of this class was to increase student confidence and develop a deeper understanding of the concepts at the students’ pace by presenting Algebra in a concrete manner. My project was to develop and evaluate five hands-on lessons about linear functions for this course. These lessons used an approach to allow students the opportunity to engage in mathematical reasoning and problem solving. Each of the lessons was embedded in the units about linear functions. The lessons are based on parts of six California Algebra 1 standards (6.0, 8.0, 9.0, 15.0, 17.0 and 18.0), which focus on linear functions and function notation. Each lesson contains a lesson plan outlining the goals, the objectives
and the California Algebra 1 Mathematics content standard(s) being taught, notes to the teacher, a timeline, detailed directions, assessment instruments, a list of required manipulatives and appropriate rubrics. The project will be helpful to other teachers who are looking for curriculum resources that have standard-based concrete activities and examples. Prior to the start of the first of three units on linear functions, the teachers involved in the evaluation of the project administered a linear functions assessment. The same assessment was administered again at the end of the third of three units on linear functions. Another analysis involving a student survey and a teacher survey based on interval variables was administered at the end of each hands-on activity. The results of the assessment tools and the hands-on algebra lessons are included in the project.
CHAPTER I

INTRODUCTION

From the very first classroom, mathematics educators have been trying to find the best way to help students learn. No single method has been elected as the ultimate pedagogical technique. To name a few methods, there is the *traditional passive approach*: teaching math with students seated in rows while the instructor is at the board giving a lecture about the content and how to do the math, then assigning a plethora of similar practice problems for the student to work on independently. Students normally categorize this as a painful but effective process of learning mathematical rules and procedures. Stanford professor, Jo Boaler, makes it clear in her book, *What’s Math Got to Do with It*, students taught with the traditional passive approach “do not engage in sense making, reasoning, or thought” (2008, p. 41). There is the *discovery approach* that allows students the opportunity to collect information and use teacher provided tools in order to create the knowledge. This method has led to peer teaching and is commonly known as *group work*. The discovery approach is *hands-on*, practical and concrete motivating students by giving them real world scenarios where math is used and is important. The fact that math educators have tried so many ways to reach students is proof that they are passionate about their job, and are trying to find that one method that can be elected as the ultimate pedagogy.
Pedagogy is not the only current high priority topic. The selection of courses to be taught in high schools is also being discussed and debated by math educators. To increase accountability of schools and to establish a minimum student performance level, the California Department of Education established mathematics content standards for grades K – 12 in 1997. In 2003, they began administering the California Standards Tests (CST), and established Algebra 1 as a graduation requirement. The expectation of the state is all students will be enrolled in Algebra 1 or higher in grade 9. If a student is in a course lower than Algebra 1, such as Algebra 1A, pre-Algebra or Basic Math, the school’s API (Academic Performance Index) will suffer.

The changes to state requirements have caused secondary schools to limit their course offerings in mathematics. I am a mathematics teacher and I have had the opportunity to connect with other teachers throughout the state. It was becoming more and more apparent that high schools were eliminating all courses lower than Algebra 1. My school did just that. My goal was to create a course that could reach the student who struggled with Algebra concepts while simultaneously teaching them all the CA Algebra content standards. My project was to develop and evaluate five hands-on lessons about linear functions for this course. These lessons used the discovery approach to allow students the opportunity to engage in mathematical reasoning and problem solving.

Purpose of the Project

There is a need for lessons that combine rigor and relevance. Teachers are inundated with standards from both the state and national board of educators. The achievement of students is measured and used in the Academic Performance Index (API)
to rate schools. Therefore, large demands have been placed on the teachers to ensure all student score well on the CSTs. The standards were probably designed to raise academic achievement, but instead have resulted in teachers who “teach to the tests” and use a more traditional passive approach. The traditional passive approach works for some of the students, but not all the students are engaged and motivated, which leads to low class performance. I am idealistically trying to find a way to reach all students.

During the first few years I taught at Enterprise High School, classes such as Prep Math (content aligned to the CA standards grade 6 & 7), Algebra 1A and Algebra 1B (CA Algebra 1 standards taught over a two-year period) were eliminated. It was apparent the state was “raising the bar” with math education and EHS was willing to take the challenge. The objective was that all of the students in grade 9 should be exposed to, and gain an understanding of the Algebra 1 standards. We were also aware that not all students learn the same way and not all students have a history of success with learning mathematics. Therefore, we created of a new course, “Hands-on Algebra 1”, for those students with low academic scores in mathematics. This course had a “hands-on” or discovery approach to teaching California Algebra 1 Standards in an attempt to better reach all students.

The students who were directed to this course had a history of low academic success in mathematics and were likely to have a lack of self-confidence in their ability as math students. One of the goals of this class was to increase student confidence by presenting Algebra 1 concepts in a concrete manner. Also, because the class filled two consecutive class periods (110 minutes), the teacher was able develop a deeper understanding of the concepts at the students’ pace. This time frame was conducive to
scaffolding the material and allowed teachers to give students the individual attention that many of them needed.

Creating the Hands-on Algebra class accomplished two things:

1) Better served the low-performing students’ needs

2) Avoided the negative affect to the school’s API

Scope (Description) of the Project

Hands-on Algebra 1 was a new course to EHS, so the mathematics department created the structure and organized the curriculum. I was given the role of lead teacher to facilitate the development of the curriculum. Each teacher was given the task of organizing a unit of Algebra 1 instruction using the current textbook and supplementing it with a collection of projects and labs. The projects and labs were designed to either concretely introduce the algebra standards or to support them and show the real-world application of the standards. I developed units of instruction focused on linear functions.

This project contains five lessons that are either hands-on activity-based or project-based. (Lessons are included in Appendix G.) Each of the lessons was embedded in the units about linear functions. The lessons are based on parts of six Algebra 1 standards (6.0, 8.0, 9.0, 15.0, 17.0 and 18.0), which focus on linear functions and function notation.

Each lesson contains:

- A lesson plan outlining the goals, the objectives and the California Algebra 1 Mathematics content standard(s) being taught, notes to the teacher, a timeline, detailed directions, assessment instruments, and appropriate rubrics
All required handouts in both paper and digital form

All required transparencies

A list of required manipulatives and/or the manipulatives themselves

The recipients of the above materials were eight teachers who were required to teach the Hands-on Algebra class, to approximately 180 students enrolled in the classes.

Significance of the Project

“Algebra curriculum should be centered on the concept of the function” (Chambers, 1994). The activities, projects and labs are interlaced throughout the unit to introduce or emphasize the characteristics of linear functions. Low performing students need to explore algebra in a way that is different from the usual practice of just memorizing rules and procedures (Kieran, 1989). The exploratory components of the unit will also serve as a “hook” to learning Algebra.

The significance of this project is that it will determine if hands-on lessons are an effective way to teach students about linear functions. Based on the analysis of student achievement of the objectives established for each activity, the teachers of EHS can appropriately change or remove activities for the following school year. My research is focused only on the units about linear functions.

If the hands-on approach is successful, similar activities can be included in other units in the course. The project will be helpful to other teachers who are looking for curriculum resources that have standard-based concrete activities and examples. The
Message from the State Board of Education and the State Superintendent of Public Instruction (Larsen & Eastin, 2001), states:

Standards describe what to teach not how to teach it. Standards-based education maintains California’s tradition of respect for local control of schools. To help students achieve at high levels, local school officials and teachers – with the full support of cooperation of families, businesses, and community partners – are encouraged to take these standards and design the specific curricular and instructional strategies that best deliver the content to their students. (p. 2)

There are many types of activities that could be used to teach linear functions. It was my goal to find or create, then implement and evaluate hands-on activities, labs and projects in the linear functions unit.

Limitations of the Project

One of the limitations to the project was that EHS is the only test site. This study could not be accurately generalized to other schools. The other limitation was the variance among the teachers involved in this study. Even with the same material, teachers have different styles and attitudes that could affect the students’ understanding of the concepts.

However, even with the above limitations to this study I anticipated positive results, based on Thompson’s observation. Concrete representations of algebra increase student understanding and motivate students to learn the concepts being addressed by the activity (Thompson, 1998).
Definition of Terms

**Academic Performance Index (API)**

A measurement of academic performance and progress of individual schools in California. It is one of the main components of the Public Schools Accountability Act passed by the California legislature in 1999. API scores ranges from 200 to 1000.

**California State Standards in Algebra 1 Targeting Linear Functions**

- Standard 6.0: Students graph a linear equation and compute the x- and y-intercepts (e.g., graph 2x + 6y = 4). They are also able to sketch the region defined by linear inequality (e.g., they sketch the region defined by 2x + 6y < 4).
- Standard 8.0: Students understand the concepts of parallel lines and perpendicular lines and how those slopes are related. Students are able to find the equation of a line perpendicular to a given line that passes through a given point.
- Standard 9.0: Students solve a system of two linear equations in two variables algebraically and are able to interpret the answer graphically. Students are able to solve a system of two linear inequalities in two variables and to sketch the solution sets.
- Standard 15.0: Students apply algebraic techniques to solve rate problems, work problems, and percent mixture problems.
- Standard 17.0: Students determine the domain of independent variables and the range of dependent variables defined by a graph, a set of ordered pairs, or a symbolic expression.
- Standard 18.0: Students determine whether a relation defined by a graph, a set of ordered pairs, or a symbolic expression is a function and justify the conclusion.
California Standards Test (CST)

An annual test designed to determine students' progress toward achieving California's state-adopted academic content standards in English–language arts (ELA), mathematics, science, and history–social science.

Hands-on Algebra 1

A course designed for students with low achievement scores in mathematics. The course covers the CA Algebra 1 content standards using activities, labs and teacher directed lessons. The class is designed to meet daily for 110 minutes.
CHAPTER II

REVIEW OF RELATED LITERATURE

My goal in reviewing the literature was to answer the following questions.

1. Why are functions important?
2. What kind of problems do students have?
3. Are there hands-on activity based curricula focused on functions?

Significance of Functions to Learning Algebra

When a student asks a teacher why learning algebra is an important part of their educational journey, the most common response is that learning algebra increases the student’s problem solving skills, gives them the ability to make symbolic representations and improves their logical reasoning skills. A function is a rule that assigns to each element of a set $A$ a unique element of a set $B$ (where $B$ may or may not equal $A$). The function is a key component within each of those skills, but plays the largest role in strengthening students’ reasoning skills (Kriegler, 1999). The study of patterns (sequences) can be used as the foundation for exploring functional relationships and creates an environment for inductive reasoning.

According to the California Education Round Table (1997), high school graduates should be able to represent functions numerically, symbolically, graphically, and verbally. Students should also be able to recognize characteristics of a function in its
different representations, transfer information about a function from one representation to another, and apply their knowledge of functions to help understand the world around them.

Understanding functions is crucial to learning algebra. Roger Day explains the significance of the concept of function to mathematics:

Functional relationships offer fertile ground for making mathematical connections. As a unifying idea in mathematics, the function concept helps students connect different mathematical ideas and procedures. Functional relationships also provide connections to other content areas and a perspective from which to view real-world phenomena. (1995, p. 54)

Using functions as the unifying theme in mathematics and specifically in the learning of algebra gives the students an important connection to prior knowledge.

Most textbooks introduce algebra by solving an equation for an unknown using an algorithmic approach. From the idea of solving for the unknown, other algebraic concepts are constructed using more rules and algorithms. This memorization approach to learning algebra proves to be faulty when students are asked to justify or explain their solutions (Confrey, 1998). Focusing on the development of a function or equation in order to find the unknown gives students an opportunity to learn from the relationships between quantities. This function-based approach to the instruction of algebra helps students see algebra in the world around them (Chazan, 2000).

The significance of function-based approach is that students are not focusing on the algebraic manipulations, but rather on finding the function. However, the concept of function has proven to have difficult components that cause students to struggle. In the following section those difficulties will be explored.
Students’ Difficulties with the Concept of Functions

In order to create an effective unit of algebra with the focus being the concept of functions, we need to consider which concepts will be difficult for students. There are many aspects of functions that students might find challenging. Sheryl Stump of Ball State University has spent time researching student and teacher understanding of the concept of slope and its connection to real-life applications.

Research has documented difficulties that students have with the concept of slope. There are misconceptions associated with the calculation of slope and with the interpretation of linear functions and their graphs. Students also have difficulty relating graphs to linear equations and to the notion of rate of change. They have difficulty considering slope as a ratio and connecting the ratio to a physical model. (1999, p. 126)

Also, the vocabulary and notation used with functions is new and can be confusing. One of the best ways to verify that students understand notation is to describe functions using common everyday language. For example, the words input and output in place of domain and range can give the student a better approach to grasping the concept of the function (Pierce, Wright & Roland, 1997). There are two common ways to represent functions. Writing \( f(a) = b \) is called \( f(x) \) or \( f( ) \) notation; \( f: a \rightarrow b \), called the arrow or mapping notation (Usiskin, Peressini, Marchisotto, & Stanley, 2003). When students use the \( f(x) \) notation, they often perceive the \( ( ) \) as an indicator to multiply.

Another problem area for students is classifying relations as functions or non-functions (Markovits, Eylon, & Bruckheimer, 1988). If a student sees a relation represented as a graph, he/she is able to successfully use the vertical line test to determine if it is a function or not. However, this geometric understanding does not always transfer
to algebraic understanding. But it is clear that the introductory stages of learning about functions should involve graphical representation before algebraic representation.

Students often have difficulties classifying specific types of functions (Markovits et al., 1988). A common misconception is that all functions are linear. Showing students the variety of types of functions and explaining why they are functions can be overwhelming for students. Examples of particular functions with which students struggle are the constant function and piecewise-defined functions. The most commonly taught functions in the beginning algebra are linear, quadratics, cubics and exponentials. Once students see both the algebraic and graphical representation of each function, it is easier to compare and contrast characteristics.

Lastly, a teacher’s understanding and knowledge of the concept directly affect student understanding (Nathan & Koedinger, 2000). Unfortunately, most teachers cannot explain how to apply algebra in the world around them. If teachers do not have a conceptual understanding of algebra, can we expect students to obtain such an understanding (Chazan, 2000)? Using a hands-on curriculum design requires that teachers be trained in how to facilitate the lessons. It cannot be expected that a teacher will be able to use a lesson without proper guidance and examples to learn from.

The difficulties described above must be addressed if a new algebra unit is to be effective. The next section details suggested curriculum from the literature that deals with them.
Suggested Curriculum Design from the Literature

A single individual did not discover functions, nor were they discovered at a particular time. Rather, the concept has evolved over a period of several centuries. Function notation was introduced during the 18th century. Leonhard Euler was the man to introduce the notation of \( f(x) \) for a function of a variable quantity \( x \). Some of the other mathematicians who helped develop the concept of function were Johann Bernoulli, d’Alembert, Fourier, and Dirichlet (Usiskin et al., 2003).

Teaching functions using pattern-based problems requires the student to use manipulatives to discover and extend patterns. Once the pattern can be articulated in words, student must define the pattern using function notation. Some common titles of this type of problem are Toothpick Squares, Towering Numbers, Paper Folding, Painted Cubes, Matchstick Rectangles, Triangles, Arithmagons Extensions, etc. (Driscoll, 1999). Another source of pattern-based problems comes from Virginia Polytechnic Institute and State University (n.d.). Some of these problems are Slicing Cheese, Bunny in a Field, Building a Staircase, Pascal’s Triangle, etc. Using the idea of a function machine to model how the input value produced an output value is an effective way to help students understand the algebraic form of a function (Pierce et al., 1997).

Lastly, the curriculum gives attention to graphing functions. Analyzing the graphical representation of functions is fundamental to understanding the characteristics of functions (Saunders & DeBlassio, 1988). Solid understanding of graphs will help address some of the difficulties students have with functions.
Conclusion

From the literature I read, it is apparent that the concept of functions is central to the learning of algebra. Since students have a hard time understanding the concept, they would benefit from a concrete approach to learning about functions. Many math educators have created materials to teach functions from a hands-on discovery approach. A survey of curriculum reveals that many useful activities already exist for teaching the concept of functions in an algebra course. These activities and an awareness of the problems that students face when learning about linear functions became the starting point of my project.
CHAPTER III

METHODOLOGY

To replace our classes that were not at the grade 9 state expectations, eight math teachers collaborated to create the Hands-on Algebra 1 course. Each teacher used the same warm-ups, activities, labs, homework assignments and assessments for each unit. The curriculum is designed to be presented during two consecutive 55 minutes class periods. It was our goal to cover at least 90% of the California State Standards for Algebra 1.

For this project, I created five hands-on activities that were embedded in the three units focused on linear functions and based on the California State Standards 6.0, 8.0, 9.0, 15.0, 17.0 and 18.0 for Algebra 1. The eight teachers taught a total of nine sections. The average class size was approximately 20 students. Altogether approximately 180 students were enrolled in this course. All the above-mentioned teachers used the five activities from my project. Only five teachers (including me) were involved in evaluating the project.

The lessons I used for the project were based on the ideas or similar lessons I had previously seen in workshops, in collaboration with colleagues or from the reviewed literature. I used All Tied Up in Knots, Can Barbie Survive the Bungee, and The Tower of Tumblers as culminating activities at the end of each of the three linear functions units. Predictable Patterns and Intercepts in Real Life were used in the middle of a unit to re-
enforce the concrete applications of the algebra. I created a list of all the activities I had seen or read about that involved the linear functions standards. Then, for each of the lessons, I named the specific concepts and state standards that the activity addressed. From the list, I was able to determine the five activities that best fit my project. I have included the concept chart for the five lessons used in the project (see Appendix A).

Every lesson includes a section titled “Teacher Notes” which contains the goal, objectives, California state standards, the multiple intelligence reference, lesson time period, materials provided and needed list, assessment reference, extension idea(s), directions for implementation, and a closure idea. Every lesson also included a warm-up (with solutions), the activity (if applicable), a student worksheet, a solution guide to the activity and a quiz with its solutions. The full layout of each lesson is included in Appendix G. Each lesson was created using Microsoft Word so that after the lesson had been field tested, it could be edited to fit an individual teaching style. I asked each teacher not to make changes to the lessons until after completing the project.

Before the curriculum was used in the classrooms, I conducted an in-service training for the teachers. I wanted to ensure that all the teachers involved in presenting the material were confident about their process of instruction. I showed the teachers how to teach and guide the activities, in order to eliminate some of the possible teaching differences that could skew the results of the study. The objective for each activity was explained in order to help the teachers guide their students through the purpose of each activity.

Vocabulary, objectives and standards were presented to the students before the activities or labs. A graphic organizer, familiar to the students because of its use in
prior units, was used to present the vocabulary. Students were informed of the California state standards for Algebra 1 that were addressed in each activity. This was intended to help students see how the activities fit in the framework of the Algebra 1 content. Most importantly, the objectives for the activities were clearly presented to all students. Knowing the objectives helped the students focus on what was to be learned and later assessed.

Prior to the start of the first of three units on linear functions, the teachers involved in the evaluation of the project administered a linear functions assessment (Appendix B). This assessment focused on the concepts of the CA Algebra 1 standards 6.0, 8.0, 9.0, 15.0, 17.0, and 18.0. It consists of twenty multiple-choice questions, each designed to assess a learning objective, a state standard or a combination of both. In order to deter students from guessing, every question has a choice that reads, “I don’t know.” The teachers involved in the project evaluation encouraged their students to answer if they did not know the correct answer and avoid random guessing. The same assessment was administered again at the end of the third of three units on linear functions.

The assessments were administered by each participating teacher, then given to me to correct and aggregate the data. The pretest and posttest data were compiled into a Microsoft Excel spreadsheet. The student names were coded using their initials, their teacher’s initials and the class period number. Pretest and posttest results were paired. Student responses to individual items were recorded as well as their total scores. The pretest and posttest results will be discussed in the following chapter.

The control group for this project was made up of students enrolled in Algebra 1 during the year before the hands-on lessons were implemented. The textbook and
pacing of the Algebra 1 classes for the students in the control group were similar to the Hands-on Algebra 1 classes. The students in the control group were not taught the CA standards using the hands-on activities presented in this project. The control students were also given both the pretest and posttest found in Appendix B. The results were collected and aggregated similarly to the process done with the treatment group.

Another analysis involving a student survey (Appendix D) and a teacher survey (Appendix E) based on interval variables was administered at the end of each hands-on activity. This questionnaire had a structured response format with answer choices of “strongly agree, agree, disagree, strongly disagree and undecided.” Both questionnaires ask questions about how students enjoyed the activity, the difficulty level, the value, the fairness of the activity, attentiveness of the students, the clarity of directions and whether it is an activity that could be recommended for others to use. The teacher specific questionnaire includes questions pertaining to the connections to the California standards and multiple learning style accommodations. Every student and teacher was asked to give honest feedback. In order to allow individuals to express thoughts or concerns that were not addressed in the interval response questions, one open response question was included at the end of each survey asking for comments and suggestions.

In the next chapter, the results from the pretest and posttest for both the control and treatment groups will be discussed. The statistical tests used on the data include a paired t-test by group to determine if student post knowledge is greater than pre-knowledge, and an independent t-test for control group versus treatment group difference. Individual analyses of the responses to key questions from pretest to posttest
were considered. Lastly, the survey responses to each activity were addressed and analyzed.
CHAPTER IV

RESULTS

When I began this project, I wanted to discover if the hands-on activities embedded in the Algebra 1 curriculum increased student understanding of Algebra. The results and data collected from the pretest and posttest, along with the activity surveys strongly suggest they do.

I used a paired t-test by group to determine if student post knowledge is greater than pre-knowledge of linear functions, and an independent t-test for the control group difference versus the treatment group difference. The test results and individual item analysis of each key question will be discussed. Lastly, the analysis of the student and teacher surveys will by presented.

For the control group, the average difference was approximately 4.4 points on the 20-point assessment (1 point for each question). For the treatment group, the average difference was approximately 6.4 points. The paired-t analysis produced a 95% confidence interval for the increase of approximately 3.7 to 5.1 points from the pretest to the posttest for the control group. The data collected from the treatment group has a 95% confidence interval of approximately 5.4 points to 7.4 points from the pretest to posttest.

An independent t-test was done to determine if the increase in scores for the treatment group was significantly different than the increase for the control group. The results of that test determined with 99.96% confidence that the treatment group had a
greater increase in pretest to posttest scores, on average, than the control group. The 95% confidence interval for this test was 0.8 to 3.2 points. The statistical significance of these tests supports the conclusion that the hands-on lessons were more effective than the traditional teaching style.

I went further to study some of the key individual questions asked on the pretest/posttest. The full item analysis of each question is included in Appendix C. Look closely at the result of question three of the assessment since each of the five lessons addressed the initial condition of the problem as the y-intercept of the linear function. The question asks:

3. In a linear function the initial condition is often referred as which of the following? a) Slope, b) Range, c) y-intercept, d) domain, e) x-intercept, f) I do not know.

Figure 1 shows us the percent of treatment students who were able to answer the question correctly on the posttest (33%) was higher than those in the control group (16%). The numbers show us that the hands-on lessons may have contributed to this higher percentage of students knowing the correct response (C). Still, only having 33% with the ability to answer it correctly is not something to brag about. It would benefit the participating teachers to re-teach that concept and consider putting more emphasis on it when teaching linear functions to future classes.

Another key concept that was addressed in four of the five lessons is interpreting the slope of a linear function as a rate of change. Look closely at the results of question one on the assessment.
1. Which of the following best describes the rate of change within a linear function?
   
a) y-intercept, b) domain, c) x-intercept, d) slope, e) range, f) I do not know

   From Figure 2 there are a few things to observe. First, notice that 83% of the treatment group was able to obtain the correct answer (D) on the posttest versus only 66% of the control group. In my opinion, having 83% correct means the concept was covered well. The hands-on activities seem to be the contributing source of the increased understanding of slope as a rate of change.

   Calculating slope or rate of change from data tables was a key concept addressed in four of the five lessons. On the pretest/posttest, question 11 assesses part of this objective.
11. Determine the slope of the linear function represented by the following table-of-values:

<table>
<thead>
<tr>
<th>n</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>f(n)</td>
<td>4</td>
<td>7</td>
<td>10</td>
<td>13</td>
<td>16</td>
</tr>
</tbody>
</table>

a. 0  
b. 1  
c. 2  
d. 3  
e. 1/3  
f. I do not know
The percentage of students in the treatment group that answered correctly increased from 31% on the pretest to 50% on the posttest, an increase of 19%. For the control group, it was an increase of 16%. This data (Figure 3) does not support the use of hands-on lessons particularly. The purpose of highlighting this question is to point out that in both the control and treatment groups, the concept of “slope” was not mastered. Notice that the second most common answer for both groups was E, the reciprocal of the correct answer. Something I would consider to study further is why 10% of the treatment group answered B.

The last key concept that was assessed is the graphical representation of linear functions. All five lessons addressed this concept in some manner. There are a couple of questions from the pretest/posttest to highlight that assess this concept).
6. Which aspect of a linear function determines the steepness of the graph of its line? a) y-intercept, b) domain, c) x-intercept, d) slope, e) range, f) I do not know (Figure 4).

![Figure 4](image)

**Figure 4.** Question #6 results comparison.

19. Which of the following linear functions is represented by the given graph? (Figure 5)

![Figure 5](image)

Figure 5. Question #19 graph.

a. \( f(x) = \frac{3}{2}x - 1 \)

b. \( f(x) = \frac{2}{3}x - 1 \)

c. \( f(x) = -\frac{3}{2}x - 1 \)

d. \( f(x) = -\frac{2}{3}x - 1 \)

f. I do not know.
From Figure 6, we see that the percentage of students in the treatment group that answered correctly increased from 43% on the pretest to 73% on the posttest, an increase of 30%. For the control group it was an increase of 19%. From Figure 6, the increase in percentage of students in the treatment group that answered correctly went from 18% on the pretest to 69% on the posttest, an increase of 51%. For the control group it was an increase of 32%. This data strongly supports the idea that using hands-on lessons increases student learning about the graphical representation of linear functions. It is necessary to state that in both questions, neither the control nor the treatment group achieved a sufficient percentage of students at a proficiency level. In either case, it would be necessary for the teachers to re-teach and review the concept.

Figure 6. Question #19 results comparison.
Figures 7-11 show the percentage of students who responded to each option of the first seven survey questions. Figure 12 pulls the responses from all five activities.

**Figure 7.** Predictable Patterns survey results.

The first seven survey questions:

1. This activity was enjoyable.
2. This activity was easy.
3. This activity was valuable.
4. This activity was fair.
5. This activity kept my attention.
6. The directions were clear and easy to follow.
7. I would recommend using this activity to other teachers of Algebra 1 students.
Figure 8. The tower of tumblers survey results.

Figure 9. Intercepts in real life survey results.
Figure 10. All tied up in knots survey results.

Figure 11. Can Barbie survive the bungee survey results.
Each of the seven structured-response questions was written so that if the student chose “agree” or “strongly agree” on all or most of the questions, it would suggest that the lesson was effective. The only structured-response question that was written so that “agree” meant the lesson was not effective is the second question: “This activity was easy.” The difficulty level of an activity does not always correlate to its effectiveness. That question could have been substituted with something less ambiguous, such as, “I understood the mathematics in the activity.”

On average, the population of students enrolled in these classes had a low level of confidence in mathematics. The student surveys also investigated attitude change towards mathematics. It was expected that because of the nature of hands-on activities and the ability to connect linear functions to real world situations, student attitudes towards mathematics would be positively affected. From the responses to the first seven...
questions and the comments (see Appendix F) made in question eight, it appears that the attitudes toward math were positively affected for some of the students. Consider question one: “This activity was enjoyable.” For the Predictable Patterns activity a total of 74% of the students either agreed or strongly agreed with this statement. For the Tower of Tumblers, Intercepts in Real Life, All Tied Up in Knots and Can Barbie Survive the Bungee, the percents of agreement are 53, 82, 70 and 88 respectively. Most students enjoyed the lessons except for the Tower of Tumblers activity.

When a student recommends that a math lesson should be used in other classes filled with their peers, that lesson must be good. That is why I want to highlight that the percentage of students who answered either “agree” or “strongly agree” to question 7: “I would recommend using this activity to other teachers of Algebra 1 students.” Results from the greatest to least support are as follows: the Can Barbie Survive the Bungee activity with 92%, the Intercepts in Real Life activity with 89%, the Predictable Patterns activity with 82%, the All Tied Up in Knots activity with 68%, and the Tower of Tumblers activity 55%.

It would also be important to analyze the result of question 6: “The directions were clear and easy to follow.” Each of the lessons had an element of student exploration and problem solving. Therefore, the lesson cannot be considered effective if it was unclear or hard to follow. The survey results show, from the greatest to least, the percentage “agree” plus “strongly agree” to question six are as follows: the Intercepts in Real Life activity with 90%, the Can Barbie Survive the Bungee activity with 79%, the
Predictable Patterns activity with 75%, the All Tied Up in Knots activity with 75%, and the Tower of Tumblers activity 51%.

The Tower of Tumblers activity was less popular than all the other activities. It also received the most negative comments (see Appendix F). It is apparent the activity needs to be re-worked. The two activities that received the most positive feedback were the Can Barbie Survive the Bungee activity and the Intercepts in Real Life activities. They both had the highest “agree” plus “strongly agree” percentages from questions one, six and seven. They also had the most positive comments (see Appendix F). I would recommend these two activities for use in any Algebra 1 course.

Conclusion

My project was to determine if using hands-on activities improve students’ performance, understanding and ability to achieve success in Algebra 1. I hypothesized that hands-on activities could also improve students’ attitude towards algebra and mathematics in general while creating a better climate for learning algebra. My recommendation to all Algebra 1 teachers is be aware of the need for a curriculum that is rigorous and relevant within their Algebra 1 program. Using hands-on lessons that are standards-based with concrete examples can make a significant difference in student attitude and learning.
REFERENCES


<table>
<thead>
<tr>
<th>CONCEPTS</th>
<th>ACTIVITIES</th>
<th>CA Standard</th>
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<td>X</td>
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LINEAR FUNCTIONS TEST

Directions: Your responses to this test will be used in a scientific study. Please answer seriously, and do not just guess. If you do not know the answer to a question, choose option f.

1. Which of the following best describes the rate of change within a linear function?
   a. y-intercept
   b. domain
   c. x-intercept
   d. slope
   e. range
   f. I do not know.

2. Which of the following best describes the following model?
   “When Troy adds 5 marbles to the beaker containing water, the water level is now at 120 mL. When Cathy adds 9 more marbles to the beaker, the level reaches 210 mL.”
   a. A continuous function
   b. A discrete function
   f. I do not know.

3. In a linear function the initial condition is often referred as which of the following?
   a. slope
   b. range
   c. y-intercept
   d. domain
   e. x-intercept
   f. I do not know.
4. After graphing a system of linear functions, the solution is seen as which of the following?

   a. the slope of the lines
   b. the intersection of the lines
   c. the x- & y-intercepts of the lines
   f. I do not know.

5. Given the following three functions, which numbers represent their slopes.

   \[ y = 2 + 3x; \quad f(x) = 4x + 5; \quad F(n) = 6(n - 1) + 7 \]

   a. 2, 4, 6
   b. 3, 5, 7
   c. y, f, F
   d. 3, 4, 6
   f. I do not know.

6. Which aspect of a linear function determines the steepness of the graph of its line?

   a. y-intercept
   b. domain
   c. x-intercept
   d. slope
   e. range
   f. I do not know.

7. Consider the graph of \( y = x \). How is the graph of \( y = x + 5 \) different?

   a. It is steeper.
   b. It is less steep.
   c. It shifts the line up on the y-axis.
   d. It shifts the line down on the y-axis.
   e. It changes it from an uphill line to a downhill line.
   f. I do not know.

8. Which of the following is true?

   a. The slope of a vertical line is always positive.
   b. The slope of a vertical line is always zero.
   c. The slope of a vertical line is always undefined.
   d. The slope of a vertical line is always negative.
   f. I do not know.
9. What is the y-intercept of the line $5x + 3y = 12$?
   a. $(0,5)$
   b. $(0,12)$
   c. $(0,3)$
   d. $(0,4)$
   f. I do not know.

10. Which of the following ordered pairs is the x-intercept on the line $5x - 2y = 20$?
   a. $(-10,0)$
   b. $(0,-10)$
   c. $(4,0)$
   d. $(0,4)$
   f. I do not know.

11. Determine the slope of the linear function represented by the following table-of-values.
   a. 0
   b. 1
   c. 2
   d. 3
   e. $\frac{1}{3}$
   f. I do not know.

12. Which of the following is the solution to the system of equations:
    \[
    \begin{align*}
    4x + y &= -14 \\
    y &= 3x 
    \end{align*}
    \]
   a. $(2,6)$
   b. $(-2,-6)$
   c. $(4,-14)$
   d. $(4,3)$
   f. I do not know.
13. The $x$-values of the linear function, $f(x) = 5x - 4$, are also known as the function’s:

a. range  
b. independent values  
c. slope  
d. dependent values  
f. I do not know.

14. Which of the following lines is parallel to $6x + 2y = 8$?

a. $y = 3x - 1$  
b. $y = -\frac{1}{3}x + 4$  
c. $y = -3x + 9$  
d. $y = \frac{1}{3}x - 8$  
f. I do not know.

15. Which of the following best describes the following graph?

a. A continuous function  
b. A discrete function  
f. I do not know.
16. Which of the following does not represent the slope ($m$) of a linear function?

a. rate of change
b. \( \frac{\text{rise}}{\text{run}} \)
c. \( \frac{y_2 - y_1}{x_2 - x_1} \)
d. \( \frac{\text{change in } x}{\text{change in } y} \)
e. \( \frac{\Delta y}{\Delta x} \)
f. I do not know.

17. Find the slope of the line passing through the points $(-2,-3)$ and $(4,5)$. (Use the graph, if necessary.)

a. \( -\frac{3}{4} \)
b. \( \frac{3}{4} \)
c. \( -\frac{4}{3} \)
d. \( \frac{4}{3} \)
f. I do not know.
18. The menu below shows that the cafeteria offers two meals every Monday, the Stinger Special and the Hungry Hornet. Each meal includes a number of burritos, \(x\), and a number of tacos, \(y\). Using the information, write a system of linear functions to find the cost of one burrito and one taco.

\[
\begin{align*}
\text{Stinger Special} & \quad \text{\$3.50} \\
& \quad \text{(one burrito, two tacos)} \\
\text{Hungry Hornet} & \quad \text{\$6.00} \\
& \quad \text{(two burritos, three tacos)}
\end{align*}
\]

\[
\begin{align*}
a. \quad x + y &= 3.50 \\
x + y &= 6.00 \\
b. \quad x + 2y &= 3.50 \\
2x + 3y &= 6.00 \\
c. \quad x - 2y &= 3.50 \\
2x - 3y &= 6.00 \\
d. \quad 2x + y &= 3.50 \\
3x + 2y &= 6.00 \\
f. \quad \text{I do not know.}
\end{align*}
\]

19. Which of the following linear functions is represented by the given graph?

\[
\begin{align*}
a. \quad f(x) &= \frac{3}{2}x - 1 \\
b. \quad f(x) &= \frac{2}{3}x - 1 \\
c. \quad f(x) &= -\frac{3}{2}x - 1 \\
d. \quad f(x) &= -\frac{2}{3}x - 1 \\
f. \quad \text{I do not know.}
\end{align*}
\]
20. Which of the following linear functions does the given table represent?

<table>
<thead>
<tr>
<th>x</th>
<th>-4</th>
<th>-2</th>
<th>0</th>
<th>1</th>
<th>3</th>
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<tbody>
<tr>
<td>y</td>
<td>11</td>
<td>7</td>
<td>3</td>
<td>1</td>
<td>-3</td>
</tr>
</tbody>
</table>

a. $y = -x + 5$

b. $y = 2x - 9$

c. $y = x + 3$

d. $y = -2x + 3$

f. I do not know.
LINEAR FUNCTIONS TEST WITH RESULTS DATA

Directions: Your responses to this test will be used in a scientific study. Please answer seriously, and do not just guess. If you do not know the answer to a question, choose option f.

1. Which of the following best describes the rate of change within a linear function?
   a. y-intercept
   b. domain
   c. x-intercept
   d. slope
   e. range
   f. I do not know.

<table>
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<th></th>
<th>Control Pretest</th>
<th>Control Posttest</th>
<th>Treatment Pretest</th>
<th>Treatment Posttest</th>
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<td>A</td>
<td>7%</td>
<td>5%</td>
<td>4%</td>
<td>10%</td>
</tr>
<tr>
<td>B</td>
<td>6%</td>
<td>5%</td>
<td>5%</td>
<td>4%</td>
</tr>
<tr>
<td>C</td>
<td>4%</td>
<td>3%</td>
<td>4%</td>
<td>0%</td>
</tr>
<tr>
<td>D</td>
<td>32%</td>
<td>66%</td>
<td>35%</td>
<td>83%</td>
</tr>
<tr>
<td>E</td>
<td>7%</td>
<td>12%</td>
<td>14%</td>
<td>2%</td>
</tr>
<tr>
<td>F</td>
<td>44%</td>
<td>10%</td>
<td>38%</td>
<td>2%</td>
</tr>
</tbody>
</table>

2. Which of the following best describes the following model?
   “When Troy adds 5 marbles to the beaker containing water, the water level is now at 120 mL. When Cathy adds 9 more marbles to the beaker, the level reaches 210 mL.”
   a. A continuous function
   b. A discrete function
   f. I do not know.

<table>
<thead>
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<td>16%</td>
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<td>18%</td>
<td>50%</td>
</tr>
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<td>F</td>
<td>22%</td>
<td>60%</td>
<td>27%</td>
<td>50%</td>
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</table>
3. In a linear function the initial condition is often referred as which of the following?

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<tr>
<td>a. slope</td>
<td>A 13%</td>
<td>11%</td>
<td>12%</td>
<td>10%</td>
</tr>
<tr>
<td>b. range</td>
<td>B 6%</td>
<td>15%</td>
<td>4%</td>
<td>12%</td>
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<td>c. y-intercept</td>
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<td>16%</td>
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<td>d. domain</td>
<td>D 16%</td>
<td>15%</td>
<td>14%</td>
<td>25%</td>
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<tr>
<td>e. x-intercept</td>
<td>E 5%</td>
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<td>1%</td>
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<tr>
<td>f. I do not know.</td>
<td>F 51%</td>
<td>38%</td>
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<td>19%</td>
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4. After graphing a system of linear functions, the solution is seen as which of the following?

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<td>a. the slope of the lines</td>
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<tr>
<td>b. the intersection of the lines</td>
<td>B 11%</td>
<td>45%</td>
<td>14%</td>
<td>54%</td>
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<tr>
<td>c. the x- &amp; y-intercepts of the lines</td>
<td>C 41%</td>
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<td>29%</td>
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<tr>
<td>f. I do not know.</td>
<td>F 28%</td>
<td>7%</td>
<td>39%</td>
<td>0%</td>
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5. Given the following three functions, which numbers represent their slopes.

\[ y = 2 + 3x; \quad f(x) = 4x + 5; \quad F(n) = 6(n - 1) + 7 \]

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<tr>
<td>b. 3, 5, 7</td>
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<td>11%</td>
<td>4%</td>
<td>2%</td>
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<tr>
<td>c. ( y, f, F )</td>
<td>C 11%</td>
<td>4%</td>
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<td>2%</td>
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<td>d. 3, 4, 6</td>
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<td>f. I do not know.</td>
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6. Which aspect of a linear function determines the steepness of the graph of its line?

a. y-intercept  
b. domain  
c. x-intercept  
d. slope  
e. range  
f. I do not know.

<table>
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<th>Control Posttest</th>
<th>Treatment Pretest</th>
<th>Treatment Posttest</th>
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<td>3%</td>
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<td>2%</td>
</tr>
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<td>C</td>
<td>5%</td>
<td>9%</td>
<td>4%</td>
<td>4%</td>
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<tr>
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<td><strong>43%</strong></td>
<td><strong>73%</strong></td>
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<td>6%</td>
<td>2%</td>
<td>5%</td>
<td>8%</td>
</tr>
<tr>
<td>F</td>
<td>21%</td>
<td>6%</td>
<td>25%</td>
<td>2%</td>
</tr>
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</table>

7. Consider the graph of \( y = x \). How is the graph of \( y = x + 5 \) different?

a. It is steeper.  
b. It is less steep.  
c. It shifts the line up on the y-axis.  
d. It shifts the line down on the y-axis.  
e. It changes it from an uphill line to a downhill line.  
f. I do not know.

<table>
<thead>
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8. Which of the following is true?

a. The slope of a vertical line is always positive.  
b. The slope of a vertical line is always zero.  
c. The slope of a vertical line is always undefined.  
d. The slope of a vertical line is always negative.  
f. I do not know.

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9. What is the y-intercept of the line $5x + 3y = 12$?

a. $(0,5)$
b. $(0,12)$
c. $(0,3)$
d. $(0,4)$
f. I do not know.

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10. Which of the following ordered pairs is the x-intercept on the line $5x - 2y = 20$?

a. $(-10,0)$
b. $(0,-10)$
c. $(4,0)$
d. $(0,4)$
f. I do not know.

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<td>F</td>
<td>45%</td>
<td>24%</td>
<td>61%</td>
<td>8%</td>
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</tbody>
</table>

11. Determine the slope of the linear function represented by the following table-of-values.

<table>
<thead>
<tr>
<th>$n$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(n)$</td>
<td>4</td>
<td>7</td>
<td>10</td>
<td>13</td>
<td>16</td>
</tr>
</tbody>
</table>

a. 0
b. 1
c. 2
d. 3
e. $\frac{1}{3}$
f. I do not know.

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<td>C</td>
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<td><strong>D</strong></td>
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<td><strong>32%</strong></td>
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<td>E</td>
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<td>F</td>
<td>53%</td>
<td>29%</td>
<td>56%</td>
<td>12%</td>
</tr>
</tbody>
</table>
12. Which of the following is the solution to the system of equations:

\[ 4x + y = -14 \]
\[ y = 3x \]

a. \((2, 6)\)
b. \((-2, -6)\)
c. \((4, -14)\)
d. \((4, 3)\)
f. I do not know.

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<td>75%</td>
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<td>C</td>
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<td>48%</td>
<td>21%</td>
<td>60%</td>
<td>4%</td>
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13. The \(x\)-values of the linear function, \(f(x) = 5x - 4\), are also known as the function’s:

a. range
b. independent values
c. slope
d. dependent values
e. I do not know.

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<td>F</td>
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14. Which of the following lines is parallel to $6x + 2y = 8$?

   a. $y = 3x - 1$
   b. $y = -\frac{1}{3}x + 4$
   c. $y = -3x + 9$
   d. $y = \frac{1}{3}x - 8$
   f. I do not know.

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</table>
   A  | 12%            | 18%              | 6%                | 21%               |
   B  | 14%            | 31%              | 12%               | 13%               |
   C  | 10%            | 22%              | 4%                | 56%               |
   D  | 3%             | 11%              | 6%                | 6%                |
   F  | 60%            | 18%              | 71%               | 2%                |

15. Which of the following best describes the following graph?

   a. A continuous function
   b. A discrete function
   f. I do not know.

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</table>
   A  | 62%            | 56%              | 64%               | 65%               |
   B  | 22%            | 40%              | 18%               | 29%               |
   F  | 14%            | 4%               | 17%               | 6%                |
16. Which of the following does not represent the slope \((m)\) of a linear function?

\(\begin{align*}
\text{a. rate of change} \\
\text{b. rise} \\
\text{c.} \quad \frac{y_2 - y_1}{x_2 - x_1} \\
\text{d.} \quad \frac{\text{change in } x}{\text{change in } y} \\
\text{e.} \quad \frac{\Delta y}{\Delta x} \\
\text{f. I do not know.}
\end{align*}\)

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17. Find the slope of the line passing through the points \((-2,-3)\) and \((4,5)\). (Use the graph, if necessary.)

\(\begin{align*}
\text{a.} \quad -\frac{3}{4} \\
\text{b.} \quad \frac{3}{4} \\
\text{c.} \quad -\frac{4}{3} \\
\text{d.} \quad \frac{4}{3} \\
\text{f. I do not know.}
\end{align*}\)

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18. The menu below shows that the cafeteria offers two meals every Monday, the Stinger Special and the Hungry Hornet. Each meal includes a number of burritos, \( x \), and a number of tacos, \( y \). Using the information, write a system of linear functions to find the cost of one burrito and one taco.

\[
\begin{align*}
a. & \quad x + y = 3.50 \\
& \quad x + y = 6.00 \\
\end{align*}
\]

\[
\begin{align*}
b. & \quad x + 2y = 3.50 \\
& \quad 2x + 3y = 6.00 \\
\end{align*}
\]

\[
\begin{align*}
c. & \quad x - 2y = 3.50 \\
& \quad 2x - 3y = 6.00 \\
\end{align*}
\]

\[
\begin{align*}
d. & \quad 2x + y = 3.50 \\
& \quad 3x + 2y = 6.00 \\
\end{align*}
\]

e. I do not know.
19. Which of the following linear functions is represented by the given graph?

- a. \( f(x) = \frac{3}{2}x - 1 \)
- b. \( f(x) = \frac{2}{3}x - 1 \)
- c. \( f(x) = -\frac{3}{2}x - 1 \)
- d. \( f(x) = -\frac{2}{3}x - 1 \)
- f. I do not know.

20. Which of the following linear functions does the given table represent?

<table>
<thead>
<tr>
<th>x</th>
<th>-4</th>
<th>-2</th>
<th>0</th>
<th>1</th>
<th>3</th>
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<tbody>
<tr>
<td>y</td>
<td>11</td>
<td>7</td>
<td>3</td>
<td>1</td>
<td>-3</td>
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<td>18%</td>
<td>69%</td>
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- a. \( y = -x + 5 \)
- b. \( y = 2x - 9 \)
- c. \( y = x + 3 \)
- d. \( y = -2x + 3 \)
- f. I do not know.
STUDENT SURVEY: Please reply to each statement below to describe your feelings about the activity titled __________________________.

Each choice stands for:
SA—Strongly agree, A—Agree, D—Disagree, SD—Strongly Disagree, U—Undecided

1. This activity was enjoyable.
   SA                        A                          D                          SD                         U

2. This activity was easy.
   SA                        A                          D                          SD                         U

3. This activity was valuable.
   SA                        A                          D                          SD                         U

4. This activity was fair.
   SA                        A                          D                          SD                         U

5. This activity kept my attention.
   SA                        A                          D                          SD                         U

6. The directions were clear and easy to follow.
   SA                        A                          D                          SD                         U

7. I would recommend using this activity to other teachers of Algebra 1 students.
   SA                        A                          D                          SD                         U

8. Please make any comments concerning the above statements. Thank you for your time.
APPENDIX E
TEACHER SURVEY: Please reply to each statement below to describe your feelings about activity titled ________________________________.

Each choice stands for:
SA—Strongly agree, A—Agree, D—Disagree, SD—Strongly Disagree, U—Undecided

1. I feel this activity is connected to the California Content Standards listed in the introduction to the activity.
   SA                       A                          D                          SD                         U

2. I feel this activity has reasonable objectives for the students.
   SA                       A                          D                          SD                         U

3. I feel this activity is more engaging than other lessons I have used to teach the same topic/standard.
   SA                       A                          D                          SD                         U

4. I would recommend using this activity to other teachers of algebra students.
   SA                       A                          D                          SD                         U

5. I feel students have more knowledge of the concepts covered because of this activity.
   SA                       A                          D                          SD                         U

6. The directions were clear and easy to follow.
   SA                       A                          D                          SD                         U

7. The activity addresses various learning styles.
   SA                       A                          D                          SD                         U

8. I feel this activity engages and motivates students.
   SA                       A                          D                          SD                         U

9. Please make any comments concerning the above statements. (Use the back of this survey, if necessary.) Thank you for your time.
APPENDIX F
**STUDENT SURVEY COMMENTS**

| Predictable Patterns                          | It was ok overall.  
|                                             | Fun, Fun           
|                                             | Start off with linear equations first and then harder ones.  
|                                             | I liked it, it helped me understand tables.  
|                                             | There wasn't any concerns, it was fun.  
|                                             | It was a little confusing at first but Mr. R showed us how and made it easy.  
|                                             | I couldn't figure out the last two.  
|                                             | I think it could be harder.  
|                                             | Instructions were kind of useless.  
|                                             | Explain how to do it better. (Give an example)  
|                                             | This activity was enjoyable and it was something new for us to do something fun and easy.  
|                                             | I thought it needed an example.  
|                                             | The activity confused me a bit.  
|                                             | The last two pages were hard.  
|                                             | Work in partners more?  
|                                             | I felt that this activity was easy but fun to learn about and capable of I knew this chapter pretty well before we did it so it was very easy..  
|                                             | The wording was confusing at the beginning but then I caught on.  
|                                             | We should do more partner activities.  
|                                             | Once you got it, it was easy.  
| The Tower of Tumblers                       | The directions were clear and easy to follow.  
|                                             | I didn't enjoy it.  
|                                             | To me, math is never enjoyable.  
|                                             | Make the lab easier.  
|                                             | I didn't like it.  
|                                             | It would have been easy but for some reason I didn't quite understand it.  
| All Tied Up in Knots                        | It was fun and easy to follow.  
|                                             | This was too easy.  
|                                             | It was difficult to understand but different.  
|                                             | I think it was definitely easy and clear to understand when verbally explained. Before doing the assignment, maybe we should go over the problems as a class.  
|                                             | Some of the written questions confused me but it was clarified as we went through it.  
|                                             | I didn't really know what was going on.  

<table>
<thead>
<tr>
<th>Intercepts in Real Life</th>
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</thead>
<tbody>
<tr>
<td>• This was a lot more fun than working out of the book. It was easy for me.</td>
</tr>
<tr>
<td>• It was easy and kept my attention.</td>
</tr>
<tr>
<td>• For once it was an interesting lab to do and it took some thought.</td>
</tr>
<tr>
<td>• This was great! Next time don't use sweaty socks though. (They smelled!)</td>
</tr>
<tr>
<td>• This gave me a better understanding of the subject.</td>
</tr>
<tr>
<td>• While doing this I hadn't realized how fast and easy it was. Especially being paired with someone who wasn't your best friend which focused me more on the subject. It got easier as we moved through the stations.</td>
</tr>
<tr>
<td>• I say that other Algebra 1 teachers should use this.</td>
</tr>
<tr>
<td>• I think its easier when we work in groups.</td>
</tr>
<tr>
<td>• It was done in so little time and rushed me. I did not finish.</td>
</tr>
<tr>
<td>• It was fun and time killing.</td>
</tr>
<tr>
<td>• It was boring, I wouldn't do it again.</td>
</tr>
<tr>
<td>• I needed more time at each station. I didn't finish any of them in class.</td>
</tr>
<tr>
<td>• It became redundant after about three labs, I simply wanted to complete the assignment and did not have to think about the work I was doing because it was redundant.</td>
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<table>
<thead>
<tr>
<th>Can Barbie Survive the Bungee</th>
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</thead>
<tbody>
<tr>
<td>• Fun experiment!</td>
</tr>
<tr>
<td>• This is a fun way to learn linear equations.</td>
</tr>
<tr>
<td>• It was fun, but understanding how to estimate the amount of rubber bands needed was confusing.</td>
</tr>
<tr>
<td>• We didn't have enough time and the problems got confusing but the end was fun.</td>
</tr>
<tr>
<td>• Didn't get equations.</td>
</tr>
<tr>
<td>• It was super fun, exciting, and destructive.</td>
</tr>
<tr>
<td>• I loved this activity especially when we bungeed the Barbies from the stairs.</td>
</tr>
<tr>
<td>• Loved it!</td>
</tr>
<tr>
<td>• The directions were clear but the work was hard.</td>
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Goal: Students will understand the concepts of rate of change initial condition from the study of patterns.

Objectives: Students will be able to:
1. Graph a set of discrete data on a coordinate plane.
2. Derive a linear or non-linear equation from a set of data.
3. Determine if a function from a set of data is linear or non-linear.

California Content Standards in Algebra 1: 6.0 Students graph a linear equation.

California Content Standards in Grade 7: 3.0 Students graph and interpret linear and some nonlinear Functions.

Multiple Intelligence(s): Bodily/Kinesthetic, Visual/Spatial, Interpersonal and Mathematical/Logical

Lesson Length: One 50-minute class period.

Materials:
- Either individual copies of the warm-up or an overhead transparency copy.
- Choose 6 – 10 of the linear function pattern (pages 5 – 14) as well as 2 – 4 of the non-linear function patterns (pages 15 – 18). Make copies for each individual student or have the students work in pairs.
- Individual copies of the post activity Quiz.
- Optional: Each graph could be done with color pencils to visually enhance the activity.

Assessment: Classwork, Homework and Quiz.

References: The Pattern and Function Connection, Fulton and Lombard.

Extensions: Students could create their own Predictable Patterns using different shapes, then exchange them with a partner to solve and complete in class.

Attachments: Warm-up and KEY (pages 3 & 4), the pattern worksheets (pages 5 – 18), homework (pages 19 – 22), the answer KEY (page 23), the quiz and answer KEY (pages 24 – 26)
Directions:
1. Start the student individually with the warm-up (page 3). (5 – 10 min)
2. Have student volunteers complete the warm-up problems on the board and check for understanding. (5 min)
3. Explain the goals of the day, and then have students either work individually or in pairs.
4. Give them approximately 25 – 30 minutes to compete.
5. Have student compare answers and check for understanding with their peers. (5 min)

Closure:
1. Discuss the new concepts of rate of change, initial condition and their graphical relationships. Also discrete functions and linear versus non-linear functions.
2. Discuss the challenges and successes of this activity
3. Assign the homework due tomorrow.
4. Inform them of the possibility of a Quiz in the future.

Ideas adapted from The Pattern and Function Connection by Teacher to Teacher Press
Using the following domain \{-2, -1, 0, 1, 2, 3, \} to create a table of values, then write the range for each of the functions.

1. \( g(x) = 4x - 1 \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( G(x) )</td>
<td>-9</td>
<td>-5</td>
<td>-1</td>
<td>3</td>
<td>7</td>
<td>11</td>
</tr>
</tbody>
</table>

Range = \{ –9, –5, –1, 3, 7, 11 \}

2. \( y = -x + 5 \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>7</td>
<td>6</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>2</td>
</tr>
</tbody>
</table>

Range = \{ 2, 3, 4, 5, 6, 7 \}

3. \( v(t) = 3t^2 - 4 \)

<table>
<thead>
<tr>
<th>( t )</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( v(t) )</td>
<td>8</td>
<td>-1</td>
<td>-4</td>
<td>-1</td>
<td>8</td>
<td>23</td>
</tr>
</tbody>
</table>

Range = \{ –4, –1, 8, 23 \}

4. \( A(r) = - \frac{1}{2} r + 3 \)

<table>
<thead>
<tr>
<th>( r )</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A(r) )</td>
<td>4</td>
<td>3.5</td>
<td>3</td>
<td>2.5</td>
<td>2</td>
<td>1.5</td>
</tr>
</tbody>
</table>

Range = \{\frac{3}{2}, 2, \frac{5}{2}, 3, \frac{7}{2}, 4 \}
Directions: Let $n$ represent the picture number of the function. Let $A(n)$ represent the total number of shapes in that picture. Fill in the table. Find the expression that represents the pattern in terms of $n$. Graph and label the discrete function.

 función A

<table>
<thead>
<tr>
<th>$n$</th>
<th>$A(n)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>15</td>
</tr>
<tr>
<td>4</td>
<td>20</td>
</tr>
<tr>
<td>5</td>
<td>25</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>10</td>
<td>50</td>
</tr>
</tbody>
</table>

Questions:

1. What is the rate of change from picture to picture?

2. Use the pattern to find the initial value (when $n = 0$). Include it on your table and graph.
Function B

Directions: Let $n$ represent the picture number of the function. Let $B(n)$ represent the total number of shapes in that picture. Fill in the table. Find the expression that represents the pattern in terms of $n$. Graph and label the discrete function.

<table>
<thead>
<tr>
<th>$n$</th>
<th>$B(n)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
</tr>
</tbody>
</table>

Questions:

1. What is the rate of change from picture to picture?

2. Use the pattern to find the initial value (when $n = 0$). Include it on your table and graph.
Directions: Let $n$ represent the picture number of the function. Let $C(n)$ represent the total number of shapes in that picture. Fill in the table. Find the expression that represents the pattern in terms of $n$. Graph and label the discrete function.

<table>
<thead>
<tr>
<th>$n$</th>
<th>$C(n)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
</tr>
</tbody>
</table>

Questions:

1. What is the rate of change from picture to picture?

2. Use the pattern to find the initial value (when $n = 0$). Include it on your table and graph.
Directions: Let $n$ represent the picture number of the function. Let $D(n)$ represent the total number of shapes in that picture. Fill in the table. Find the expression that represents the pattern in terms of $n$. Graph and label the discrete function.

<table>
<thead>
<tr>
<th>$n$</th>
<th>$D(n)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
</tr>
</tbody>
</table>

Questions:

1. What is the rate of change from picture to picture?

2. Use the pattern to find the initial value (when $n = 0$). Include it on your table and graph.
Function $G$

Directions: Let $n$ represent the picture number of the function. Let $G(n)$ represent the total number of shapes in that picture. Fill in the table. Find the expression that represents the pattern in terms of $n$. Graph and label the discrete function.

<table>
<thead>
<tr>
<th>n</th>
<th>$G(n)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
</tr>
<tr>
<td>3</td>
<td>30</td>
</tr>
</tbody>
</table>

Questions:

1. What is the rate of change from picture to picture?

2. Use the pattern to find the initial value (when $n = 0$). Include it on your table and graph.
Directions: Let $n$ represent the picture number of the function. Let $H(n)$ represent the total number of shapes in that picture. Fill in the table. Find the expression that represents the pattern in terms of $n$. Graph and label the discrete function.

<table>
<thead>
<tr>
<th>$n$</th>
<th>$H(n)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
</tr>
<tr>
<td>5</td>
<td>10</td>
</tr>
</tbody>
</table>

Questions:
1. What is the rate of change from picture to picture?
2. Use the pattern to find the initial value (when $n = 0$). Include it on your table and graph.
Function J

Directions: Let $n$ represent the picture number of the function. Let $J(n)$ represent the total number of shapes in that picture. Fill in the table. Find the expression that represents the pattern in terms of $n$. Graph and label the discrete function.

<table>
<thead>
<tr>
<th>$n$</th>
<th>$J(n)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
</tr>
</tbody>
</table>

Questions:

1. What is the rate of change from picture to picture?

2. Use the pattern to find the initial value (when $n = 0$). Include it on your table and graph.
Function K

Directions: Let $n$ represent the picture number of the function. Let $K(n)$ represent the total number of shapes in that picture. Fill in the table. Find the expression that represents the pattern in terms of $n$. Graph and label the discrete function.

Questions:

1. What is the rate of change from picture to picture?

2. Use the pattern to find the initial value (when $n = 0$). Include it on your table and graph.
Directions: Let \( n \) represent the picture number of the function. Let \( M(n) \) represent the total number of shapes in that picture. Fill in the table. Find the expression that represents the pattern in terms of \( n \). Graph and label the discrete function.

Questions:

1. What is the rate of change from picture to picture?

2. Use the pattern to find the initial value (when \( n = 0 \)). Include it on your table and graph.
Directions: Let $n$ represent the picture number of the function. Let $P(n)$ represent the total number of shapes in that picture. Fill in the table. Find the expression that represents the pattern in terms of $n$. Graph and label the discrete function.

<table>
<thead>
<tr>
<th>$n$</th>
<th>$P(n)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
</tr>
<tr>
<td>5</td>
<td>13</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>10</td>
<td>25</td>
</tr>
</tbody>
</table>

Questions:
1. What is the rate of change from picture to picture?
2. Use the pattern to find the initial value (when $n = 0$). Include it on your table and graph.
Function Q

Directions: Let $n$ represent the picture number of the function. Let $Q(n)$ represent the total number of shapes in that picture. Fill in the table. Find the expression that represents the pattern in terms of $n$. Graph and label the discrete function.

Questions:
1. What is the rate of change from picture to picture?
2. Use the pattern to find the initial value (when $n = 0$). Include it on your table and graph.
Directions: Let $n$ represent the picture number of the function. Let $R(n)$ represent the total number of shapes in that picture. Fill in the table. Find the expression that represents the pattern in terms of $n$. Graph and label the discrete function.

Questions:

1. What is the rate of change from picture to picture?

2. Use the pattern to find the initial value (when $n = 0$). Include it on your table and graph.
Directions: Let $n$ represent the picture number of the function. Let $T(n)$ represent the total number of shapes in that picture. Fill in the table. Find the expression that represents the pattern in terms of $n$. Graph and label the discrete function.

<table>
<thead>
<tr>
<th>$n$</th>
<th>$T(n)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>7</td>
</tr>
<tr>
<td>5</td>
<td>10</td>
</tr>
</tbody>
</table>

Questions:

1. What is the rate of change from picture to picture?

2. Use the pattern to find the initial value (when $n = 0$). Include it on your table and graph.
Directions: Let $n$ represent the picture number of the function. Let $U(n)$ represent the total number of shapes in that picture. Fill in the table. Find the expression that represents the pattern in terms of $n$. Graph and label the discrete function.

Questions:

1. What is the rate of change from picture to picture?

2. Use the pattern to find the initial value (when $n = 0$). Include it on your table and graph.
Directions: Let $n$ represent the picture number of the function. Let $E(n)$ represent the total number of shapes in that picture. Fill in the table. Find the expression that represents the pattern in terms of $n$. Graph and label the discrete function.

<table>
<thead>
<tr>
<th>$n$</th>
<th>$E(n)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>$\vdots$</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
</tr>
</tbody>
</table>

Questions:
1. What is the rate of change from picture to picture?
2. Use the pattern to find the initial value (when $n = 0$). Include it on your table and graph.
Directions: Let \( n \) represent the picture number of the function. Let \( F(n) \) represent the total number of shapes in that picture. Fill in the table. Find the expression that represents the pattern in terms of \( n \). Graph and label the discrete function.

**Function F**

<table>
<thead>
<tr>
<th>( n )</th>
<th>( F(n) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>\ldots</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
</tr>
</tbody>
</table>

**Questions:**

1. What is the rate of change from picture to picture?

2. Use the pattern to find the initial value (when \( n = 0 \)). Include it on your table and graph.
Directions: Let \( n \) represent the picture number of the function. Let \( L(n) \) represent the total number of shapes in that picture. Fill in the table. Find the expression that represents the pattern in terms of \( n \). Graph and label the discrete function.

Function \( L \)

<table>
<thead>
<tr>
<th>( n )</th>
<th>( L(n) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>( \vdots )</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
</tr>
</tbody>
</table>

Questions:

1. What is the rate of change from picture to picture?

2. Use the pattern to find the initial value (when \( n = 0 \)). Include it on your table and graph.
Function W

Directions: Let \( n \) represent the picture number of the function. Let \( W(n) \) represent the total number of shapes in that picture. Using the given expression that represents the pattern in terms of \( n \), draw a possible picture for \( n = 1, 2 \& 3 \). Then fill in the table. Also, graph and label the discrete function.

<table>
<thead>
<tr>
<th>( n )</th>
<th>( W(n) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>...</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>2n – 1</td>
</tr>
</tbody>
</table>

Questions:

1. What is the rate of change from picture to picture?

2. Use the pattern to find the initial value (when \( n = 0 \)). Include it on your table and graph.
<table>
<thead>
<tr>
<th>FUNCTION</th>
<th>EQUATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>A(n) = 3n + 1</td>
</tr>
<tr>
<td>B</td>
<td>B(n) = -2n + 14</td>
</tr>
<tr>
<td>C</td>
<td>C(n) = 2n + 2</td>
</tr>
<tr>
<td>D</td>
<td>D(n) = 4n - 3</td>
</tr>
<tr>
<td>E</td>
<td>E(n) = -4n + 18</td>
</tr>
<tr>
<td>F</td>
<td>F(n) = 6n + 2</td>
</tr>
<tr>
<td>G</td>
<td>G(n) = n</td>
</tr>
<tr>
<td>H</td>
<td>H(n) = 3n + 5</td>
</tr>
<tr>
<td>J</td>
<td>J(n) = 8n - 2</td>
</tr>
<tr>
<td>K</td>
<td>K(n) = 4n</td>
</tr>
<tr>
<td>L</td>
<td>L(n) = 3n + 2</td>
</tr>
<tr>
<td>M</td>
<td>M(n) = -5n + 21</td>
</tr>
<tr>
<td>P</td>
<td>P(n) = 3n + 6</td>
</tr>
<tr>
<td>Q</td>
<td>Q(n) = n^2</td>
</tr>
<tr>
<td>R</td>
<td>R(n) = ( ! (n)(n + 1) )</td>
</tr>
<tr>
<td>T</td>
<td>T(n) = 2^n</td>
</tr>
<tr>
<td>U</td>
<td>U(n) = (n + 2)^2</td>
</tr>
</tbody>
</table>
Predictable Patterns QUIZ  

**Function A**

1. 

2. 

3. 

1) How many tiles are in the 4\textsuperscript{th} step of function $A$?  
2) How many tiles are in step 10 of function $A$?  
3) What is the rate of change of function $A$?  
4) What is the initial value of function $A$?  
5) Graph function $A$ below.  
6) Function rule, $A$ in terms of $n$: $A(n) =$ _________.  
7) Make a t-table for function $B$ below.  
8) What is the rate of change of function $B$?  
9) What is the initial value of function $B$?  
10) Function rule, $B$ in terms of $n$: $B(n) =$ _________.  
11) What is $B(15)$ or what is $B(n)$ when $n = 15$?  
12) Extra Credit: On the back of this paper create a table, a graph, and a possible representation of the 1\textsuperscript{st} three steps of the function: $C(n) = 3n - 2$  

**Function B**

1. 

2. 

3. 

<table>
<thead>
<tr>
<th>$n$</th>
<th>$A(n)$</th>
<th>$B(n)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Answer Column:**

1.  
2.  
3.  
4.  
5. see below  
6.  
7. see below  
8.  
9.  
10.  
11.  

**Function A**

**Function B**

**Graph:**

![Graph of Function A](image1)  

![Graph of Function B](image2)
Predictable Patterns QUIZ (KEY)  Name________________

Function A  Function B

1) How many tiles are in the 4th step of function A?
2) How many tiles are in step 10 of function A?
3) What is the rate of change of function A?
4) What is the initial value of function A?
5) Graph function A below.
6) Function rule, A in terms of n: A(n) = __________.
7) Make a t-table for function B below.
8) What is the rate of change of function B?
9) What is the initial value of function B?
10) Function rule, B in terms of n: B(n) = __________.
11) What is B(15) or what is B(n) when n = 15?
12) Extra Credit: On the back of this paper create a table, a graph, and a possible representation of the 1st three steps of the function: C(n) = 3n – 2

<table>
<thead>
<tr>
<th>A(n)</th>
<th>n</th>
<th>B(n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
<td>37</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
<td>40</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
<td>43</td>
</tr>
</tbody>
</table>

Answer Column:

1. ___39____
2. ____111____
3. ___12____
4. ___–9____
5. see below
6. A(n) = 12x – 9
7. see below
8. ___–3____
9. ____18____
10. B(n) = –3x + 18
11. B(15) = –27
12) A possible answer for $C(n) = 3n - 2$

<table>
<thead>
<tr>
<th>n</th>
<th>C(n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-2</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
</tr>
<tr>
<td>5</td>
<td>13</td>
</tr>
<tr>
<td>10</td>
<td>27</td>
</tr>
</tbody>
</table>

Diagram showing the relationship between $n$ and $C(n)$.
Teacher Notes – "The Towers of Tumblers"

**Goal:**
Students collect, organize, and graph data. As they investigate the function \( y = mx + b \), they further develop their understanding of independent variable, dependant variable, rate of change (slope), initial value (y-intercept), concrete, discrete, domain and range. They translate from and among concrete models, tables, graphs, verbal descriptions, and algebraic rules. The student will also solve a system of linear equations and interpret its solution.

**Objectives:**
Students will be able to:
1. Measure lengths in centimeters.
2. Graph sets of discrete data on a coordinate plane.
3. Derive linear equations from each set of collected data using slope (rate of change) and y-intercept (initial condition).
4. Make predictions of the application using the linear functions.
5. Solve a system of linear equations and interpret its solution to the application.

**California Content Standards in Algebra 1:**
5.0 Students solve multistep problems, including word problems, involving linear equations.
6.0 Students graph a linear equation and compute the \( x \)- and \( y \)-intercepts.
17.0 Students determine the domain of independent variables and the range of dependent variables defined by a graph, a set of ordered pairs, or a symbolic expression.

**California Content Standards in Grade 7:**
3.0 Students graph and interpret linear functions.

**Multiple Intelligence(s):**
Bodily/Kinesthetic, Visual/Spatial, Interpersonal and Mathematical/Logical

**Lesson Length:**
One 50-minute class period.

**Materials:**
- Individual copies or overhead copy of the warm-up
- Overhead copy of warm-up KEY
- Individual copies of activity worksheet
- Graph paper
- Rulers
- Two sizes of cups (5 of each for each group)
  - Suggested sizes and makes:
    - 8oz Styrofoam
    - 20+ oz tall paper cup (i.e. Starbucks Venti paper cups)
- Colored pencils to differentiate the graphs and data collection for each size cup, (optional)
- Calculators (optional)

Assessment: Check for understanding throughout the activity and at the closure. The extension activity could be used as an assessment.

Extensions: Students use a linear function to determine the number of stacking cups to fill the classroom doorway. (See "Extension Activity" worksheet)

Attachments: Warm-up and KEY (pages 3 & 4), "The Towers of Tumblers" activity worksheet (page 5), "The Towers of Tumblers" follow-up questions overhead copy (page 6), extension activity teacher page (page 7)

Directions:
1. Start the students individually with the warm-up (page 3). (5 min)
2. Have student volunteers complete the warm-up problems on the board and check for understanding. (5 min)
3. Explain the goals of the day and the procedures of working with groups.
4. Hand out the activity worksheets, graph paper, 5 of each cup to each group, rulers and colored pencils.
5. Students either work in pairs or groups of three or four.
6. Give them approximately 35 – 40 minutes to compete.
7. Have student compare answers and check for understanding with their peers. (5 min)

Closure:
1. Discuss each follow-up question in part 4.
2. Take time to connect the concepts rate of change, y-intercept, domain, range, discrete versus continuous, linear functions and solutions to systems of linear functions with this activity.
3. Discuss the challenges and successes of this activity.
4. Optional: Assign the extension activity if time permits or use on a different day.
"The Towers of Tumblers" – Warm-up

1. Given the following table of values, determine the linear equation that defines its relationship.

<table>
<thead>
<tr>
<th>t</th>
<th>v(t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>-21</td>
</tr>
<tr>
<td>-1</td>
<td>-11</td>
</tr>
<tr>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>5</td>
<td>19</td>
</tr>
</tbody>
</table>

2. Solve the system of linear equations by graphing:

\[ y = \frac{5}{2}x - 1 \]
\[ y = -\frac{1}{4}x + 10 \]

3. Solve the system of linear equations by the substitution method:

\[ y = 2x - 1 \]
\[ 2x - 3y = 5 \]

4. Solve the system of linear equations by the elimination method:

\[ 3x - y = 6 \]
\[ 2x + 3y = 4 \]
“The Towers of Tumblers” – Warm-up - KEY

1. Given the following table of values, determine the linear equation that defines its relationship.

<table>
<thead>
<tr>
<th>t</th>
<th>v(t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>-21</td>
</tr>
<tr>
<td>-1</td>
<td>-11</td>
</tr>
<tr>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>5</td>
<td>19</td>
</tr>
</tbody>
</table>

\[ \text{slope} = m = \frac{19 - 9}{5 - 3} = \frac{10}{2} = 5 \]

\[ y - y_i = m(x - x_i) \rightarrow v - v_i = m(t - t_i) \]

\[ v - 9 = 5(t - 3) \]

\[ v(t) = 5t - 6 \]

Solve the system of linear equations by graphing:

\[ A \quad y = \frac{5}{2}x - 1 \]

\[ B \quad y = -\frac{1}{4}x + 10 \]

**solution:** (4,9)

2. Solve the system of linear equations by the substitution method:

\[ y = 2x - 1 \]

\[ 2x - 3y = 5 \]

\[ 2x - 2(2x - 1) = 5 \]

\[ 2x - 6x + 3 = 5 \]

\[ -4x + 3 = 5 \]

\[ -4x = 2 \]

\[ x = -\frac{1}{2} \]

\[ y = 2x - 1 \]

\[ y = 2\left(-\frac{1}{2}\right) - 1 \]

\[ y = -1 - 1 \]

\[ y = -2 \]

**Solution** \[ \left( -\frac{1}{2}, -2 \right) \]

3. Solve the system of linear equations by the elimination method:

\[ 3x - y = 6 \rightarrow 3(3x - y = 6) \rightarrow 9x - 3y = 18 \]

\[ 2x + 3y = 4 \rightarrow \rightarrow 2x + 3y = 4 \]

\[ 11x + 0 = 22 \]

\[ 11x = 22 \]

\[ x = 2 \]

\[ 3y = 0 \]

\[ y = 0 \]

**Solution** \( (2,0) \)
**Part 1: The problem**

You will be given a few of small tumblers (cups) and a few large tumblers (cups). Your job is to determine if it is possible to have a stack of short cups with the same number of cups as a stack of large cups while the two stacks are the same height. Consider only a traditional stack of cups where one cup is slipped into another cup.

**Part 2: Collecting Data**

Pick one of the cup sizes and set one cup down on the table and measure its height. Slide a second cup in or around the first cup and measure the height of the stack. Repeat this process until there are five cups in the stack. Make your measurements as accurately as is reasonably possible, but remember that when collecting real data small errors will occur. Put your data in an input/output table. Do the same for the other size cups.

**Part 3: Data analysis**

Since small errors occur when collecting data, and there are small variations between cups and how the cups are stacked, the data collected will not fit exactly into a mathematical equation. Keep this in mind as you do the following.

1. Create a graph of the data. Do the two relationships appear linear? Determine a line “of best fit” for each cups’ data. There are a variety of ways of doing this, but the idea is to find a line that comes close to most, if not all, of the points. Draw the graph of these lines on the same graph as the data points.
2. Come up with equations of these lines of best fit. You may do this from information on the graph or information from the table. If you use the table, remember the first difference won’t be constant because the data is not exact and there are variations in the cups. You will have to pick a “best fit” first difference.
3. Make note of the slopes and y-intercepts of your lines of best fit.

A line of best fit and its equation are said to “model” the real life situation.

**Part 4: Follow up questions**

1. What does the slope tell you about the activity? What property of the cup does it represent?
2. Why were the slopes different for the line of best fit for the big cups and the line of best fit for the small cups?
3. What does the y-intercept tell you about the activity? What property of the cup does it represent?
4. Why were the y-intercepts different for the line of best fit for the big cups and the line of best fit for the small cups?
5. Use your equations to determine the number cups needed for the two stacks to have the same number of cups and be of the same height.
6. If the cups you were using had a height of 12.4 cm and a lip of 2.6 cm, what would be the equation for the height of a stack?
7. Is this relationship better described using a discrete or a continuous function?
"The Towers of Tumblers"—Follow-up Questions

1. What does the slope tell you about the activity? What property of the cup does it represent?

2. Why were the slopes different for the line of best fit for the big cups and the line of best fit for the small cups?

3. What does the y-intercept tell you about the activity? What property of the cup does it represent?

4. Why were the y-intercepts different for the line of best fit for the big cups and the line of best fit for the small cups?

5. Use your equations to determine the number cups needed for the two stacks to have the same number of cups and be of the same height.

6. If the cups you were using had a height of 12.4 cm and a lip of 2.6 cm, what would be the equation for the height of a stack?

7. Is this relationship better described using a discrete or a continuous function?
"The Towers of Tumblers" – Extension Activity

Doorway Activity

❖ Materials:
  - 5 cups of the same size (but different than the previous activity)
  - A ruler
  - Graph paper
  - Calculator (optional)

❖ Guiding Question(s):
  - What is the maximum number of tumblers (cups) that we could stack in the height of the doorway?
  - What is the maximum number of tumblers (cups) that we would need to completely fill the doorway with a wall of cups?

❖ Other notes:
  - Buy enough cups to determine the actual number of cups needed.
  - Either supply tape measures for the students to determine the doorway dimensions or supply that information for them.
Teacher Notes – “Intercepts in Real Life”

Goal: Students will create, graph and interpret linear equations in standard form from “real life” situations. The graphing focus is graphing by finding the intercepts.

Objectives: Students will be able to:
1. Write a linear equation in standard form given an application problem.
2. Use the equation to evaluate an outcome given a set of data.
3. Graph the linear equation to find all the possible solutions that work for a set outcome.
4. Interpret the meaning of the intercepts and the other solutions to linear function for the given application.
5. Determine if the relationship is discrete or continuous.

California Content Standards in Algebra 1:
5.0 Students solve multistep problems, including word problems, involving linear equations.
6.0 Students graph a linear equation and compute the x- and y-intercepts.
17.0 Students determine the domain of independent variables and the range of dependent variables defined by a graph, a set of ordered pairs, or a symbolic expression.

California Content Standards in Grade 7:
3.0 Students graph and interpret linear functions.

Multiple Intelligence(s): Bodily/Kinesthetic, Visual/Spatial, Interpersonal and Mathematical/Logical

Lesson Length: One 50-minute class period.

Materials:
- Individual copies or overhead copy of the warm-up
- Individual copies of activity worksheets.
- Printed station signs,
  - Optional: use colored cardstock, laminate, etc.
- Optional: Extra props (socks, scarves, stopwatch, money, etc.) that could be used at each station,
- Optional: Color pencils or markers to differentiate the three types of stations.
Assessment: Check for understanding throughout the activity and at the closure. The extension activity could be used as an assessment.

Extensions: Students could create their own situation that would produce a similar linear function. Then share those with a partner to complete the questions and graph as a quiz or follow-up exercise. (See “Extension Activity” worksheet)

Attachments: Warm-up and KEY (pages 3 & 4), the “Intercepts in Real Life” activity worksheets (pages 5 – 10), the “Intercepts in Real Life” activity worksheets Solution KEYS (pages 11 – 16), station signs (pages 17 – 24), extension activity worksheet (page 25)

Directions:
1. Setup the stations prior to class.
2. Start the student individually with the warm-up (page 3). (5 min)
3. Have student volunteers complete the warm-up problems on the board and check for understanding. (5 min)
4. Explain the goals of the day and the procedures of rotating from station to station.
5. Hand out the activity worksheets.
6. Students either work in pairs or groups of three or four.
7. Give them approximately 35 – 40 minutes to compete.
8. Have student compare answers and check for understanding with their peers. (5 min)

Closure:
1. Discuss the question of discrete versus continuous in part d and the possible answer found in part e.
2. Take time to connect the concepts of x- and y-intercept, domain, range, discrete versus continuous and linear function with this activity.
3. Discuss the challenges and successes of this activity.
4. Optional: Assign the extension activity as homework or extra credit.
"Intercepts in Real Life"– Warm-up

1. The menu below shows that the cafeteria offers a burrito/taco meal on Monday. The meal includes two burritos and four tacos. Let $x$ represent the cost of one burrito and $y$ represent the cost of one taco. Using the information, write a linear function to represent all the possible costs of one burrito and one taco.

   **MONDAY’S MENU**
   
   Hungry Hornet.............$8.00
   (two burritos, four tacos)

2. Find the $x$- and $y$-intercepts of the function $x + 3y = 9$

3. Graph the following linear function by first finding the $x$- and $y$-intercepts. Then plot them and draw the line that passed through those points.

   $2x + 5y = 20$
1. The menu below shows that the cafeteria offers a burrito/taco meal on Monday. The meal includes two burritos and four tacos. Let \( x \) represent the cost of one burrito and \( y \) represent the cost of one taco. Using the information, write a linear function to represent all the possible costs of one burrito and one taco.

\[
2x + 4y = 8.00
\]

**MONDAY'S MENU**

*Hungry Hornet………………$8.00*  
(two burritos, four tacos)

2. Find the \( x \)- and \( y \)-intercepts of the function \( x + 3y = 9 \)

<table>
<thead>
<tr>
<th>( x )-intercept</th>
<th>( y )-intercept</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x + 3(0) = 9 )</td>
<td>( 0 + 3y = 9 )</td>
</tr>
<tr>
<td>( x + 0 = 9 )</td>
<td>( 3y = 9 )</td>
</tr>
<tr>
<td>( x = 9 )</td>
<td>( y = 3 )</td>
</tr>
<tr>
<td>(9,0)</td>
<td>(0,3)</td>
</tr>
</tbody>
</table>

3. Graph the following linear function by first finding the \( x \)- and \( y \)-intercepts. Then plot them and draw the line that passed through those points.

\[
2x + 5y = 20
\]

<table>
<thead>
<tr>
<th>( x )-intercept</th>
<th>( y )-intercept</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 2x + 5(0) = 20 )</td>
<td>( 2(0) + 5y = 20 )</td>
</tr>
<tr>
<td>( 2x + 0 = 20 )</td>
<td>( 0 + 5y = 20 )</td>
</tr>
<tr>
<td>( 2x = 20 )</td>
<td>( 5y = 20 )</td>
</tr>
<tr>
<td>( x = 10 )</td>
<td>( y = 4 )</td>
</tr>
<tr>
<td>(10,0)</td>
<td>(0,4)</td>
</tr>
</tbody>
</table>
“Intercepts in Real Life”

Ticket Problem: Reserved tickets for the basketball game cost $____ each while each general admission ticket costs $____. Let $x$ represent the number of reserved tickets sold and let $y$ represent the number of general admission tickets sold.

a. Write an equation for the total revenue from ticket sales.

b. Use your expression found in part a to find how much money is received for the following ticket sales:
   i. 8 reserved and 10 general
   ii. 10 reserved and 8 general

c. A member of the Manatowa Maniacs must sell $_____ worth of reserved and general admission tickets combined.

   Write an equation stating this.

   ______________________________

d. Find the intercepts for the equation found in part c and use them to plot the graph of the equation. Is this a discrete or continuous function?

   ______________________________

   ______________________________

e. List all the points with positive integer values that the line crosses. What does each set of ordered pairs on the graph of the equation represent in real life?
“Intercepts in Real Life”

Ticket Problem: Reserved tickets for the basketball game cost $____ each while each general admission ticket costs $____. Let x represent the number of reserved tickets sold and let y represent the number of general admission tickets sold.

a. Formulate an expression that represents the total ticket sales.

b. Use your expression found in part a, to find how much money is received for the following ticket sales?
   i. 8 reserved and 10 general
   ii. 10 reserved and 8 general

c. A member of the Manatowa Maniacs must sell $____ worth of reserved and general admission tickets combined.
   Write an equation stating this. ______________________________

d. Find the intercepts for the equation found in part c and use them to plot the graph of the equation. Is this a discrete or continuous function?

```
<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>50</td>
</tr>
<tr>
<td>10</td>
<td>0</td>
</tr>
</tbody>
</table>
```

e. List all the points with positive integer values that the line crosses. What does each set of ordered pairs on the graph of the equation represent in real life?
“Intercepts in Real Life”

Socks and Scarves Problem: Socks cost $____ a pair and each scarf costs $______. Natalie Attired buys some of each. Let \( x \) represent the number of pair of socks and let \( y \) represent the number of scarves she buys.

a. Use your expression found in part a, to find how much would Natalie pay if she bought the following items?
   i. 7 pairs of socks and 4 scarves
   ii. 4 pairs of socks and 7 scarves

c. Natalie’s budget is set at $60, she needs to buy a combination of sock and scarves without going over her budget.
   Write an equation stating this. ________________

d. Find the intercepts for the equation found in part c and use them to plot the graph of the equation. Is this a discrete or continuous function?

e. List all the points with positive integer values that the line crosses. What does each set of ordered pairs on the graph of the equation represent in real life?
Station #____

“Intercepts in Real Life”

Socks and Scarves Problem: Socks cost $____ a pair and each scarf costs $_____. Natalie Attired buys some of each. Let \( x \) represent the number of pair of socks and let \( y \) represent the number of scarves she buys.

a. 

\[
\begin{align*}
4 & \quad \text{Socks} \\
5 & \quad \text{Scarves}
\end{align*}
\]

b. Use your expression found in part a, to find how much would Natalie pay if she bought the following items?

i. 7 pairs of socks and 4 scarves

ii. 4 pairs of socks and 7 scarves

c. Natalie’s budget is set at $60, she needs to buy a combination of sock and scarves without going over her budget. Write an equation stating this.

c. ______________________________

d. Find the intercepts for the equation found in part c and use them to plot the graph of the equation. Is this a discrete or continuous function?

\[
\text{Graph}
\]

e. List all the points with positive integer values that the line crosses. What does each set of ordered pairs on the graph of the equation represent in real life?
"Intercepts in Real Life"

Walking and Running Problem: Dan Damam can walk at a rate of _______ yards per minute and run at a rate of _______ yards per minute. Let \( x \) represent the number of minutes he walks and let \( y \) represent the number of minutes he runs.

a. Dan's Distance = Rate \( \cdot \) Time

<table>
<thead>
<tr>
<th>Mode</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>Walking</td>
<td>( x ) minutes at _____/min represents: _______ yards (expression)</td>
</tr>
<tr>
<td>Running</td>
<td>( y ) minutes at _____/min represents: _______ yards (expression)</td>
</tr>
</tbody>
</table>

Write an expression for Dan's total distance traveled: __________________________

b. Use your expression found in part a, to find how far does Dan Damam go if he:
   i. walks 7 minutes and runs 3 minutes  
   ii. walks 3 minutes and runs 7 minutes

c. Dan puts on his cross training shoes and heads to the store. The store is exactly 1200 yards away, he needs to run and/or walk to get there.

Write an equation stating this. __________________________

d. Find the intercepts for the equation found in part c and use them to plot the graph of the equation. Is this a discrete or continuous function?

\[ x\text{-intercept: ( , )} \]
\[ y\text{-intercept: ( , )} \]

e. List all the points with positive integer values that the line crosses. What does each set of ordered pairs on the graph of the equation represent in real life?
“Intercepts in Real Life”

**Walking and Running Problem:** Dan Daman can walk at a rate of _______ yards per minute and run at a rate of _______ yards per minute. Let $x$ represent the number of minutes he walks and let $y$ represent the number of minutes he runs.

a. 

Dan’s Distance = Rate $\cdot$ Time

| Walking: $x$ minutes at _______/min represents: _______ yards (expression) |
| Running: $y$ minutes at _______/min represents: _______ yards (expression) |

Write an expression for Dan’s total distance traveled: ______________________

b. Use your expression found in part a, to find how far does Dan Daman go if he:
   i. walks 7 minutes and runs 3 minutes
   ii. walks 3 minutes and runs 7 minutes

c. Dan puts on his cross training shoes and heads to the store. The store is exactly 1200 yards away, he needs to run and/or walk to get there.

Write an equation stating this. __________________________

d. Find the intercepts for the equation found in part c and use them to plot the graph of the equation. Is this a discrete or continuous function?

| x-intercept: (_____,____) |
| y-intercept: (______,____) |

e. List all the points with positive integer values that the line crosses. What does each set of ordered pairs on the graph of the equation represent in real life?
**Station #1**

**“Intercepts in Real Life” – SOLUTION KEY**

**Ticket Problem:** Reserved tickets for the basketball game cost $6, each while each general admission ticket costs $4. Let \( x \) represent the number of reserved tickets sold and let \( y \) represent the number of general admission tickets sold.

a. **Ticket Sales**

\[
\begin{align*}
\text{Reserved tickets:} & \quad x \text{ at } \$6 \text{ represents: } 6x \text{ (expression)} \\
\text{General Admission Tickets:} & \quad y \text{ at } \$4 \text{ represents: } 4y \text{ (expression)} \\
\text{Write an expression for the total number of tickets sold: } & \quad 6x + 4y
\end{align*}
\]

b. Use your expression found in part a, to find how much money is received for the following ticket sales?

i. 8 reserved and 10 general
   \[
   6x + 4y = 6(8) + 4(10) = 48 + 40 = 88
   \]

ii. 10 reserved and 8 general
   \[
   6x + 4y = 6(10) + 4(8) = 60 + 32 = 92
   \]

c. A member of the Manitowa Maniacs must sell \( 48 \) worth of reserved and general admission tickets combined.

Write an equation stating this.

\[
6x + 4y = 48
\]

d. Find the intercepts for the equation found in part c and use them to plot the graph of the equation. Is this a discrete or continuous function?

- **x-intercept**: \( 6x = 48 \) \( x = 8 \)
- **y-intercept**: \( 4y = 48 \) \( y = 12 \)

\[
\begin{array}{c|c|c|c}
\hline
x & y & \\
6x + 4y = 48 & 6x + 4(0) = 48 & 6(0) + 4y = 48 \\
6x = 48 & 4y = 48 \\
x = 8 & y = 12 \\
\hline
\end{array}
\]

x-intercept: \( (8, 0) \)

y-intercept: \( (0, 12) \)

---

e. List all the points with positive integer values that the line crosses. What does each set of ordered pairs on the graph of the equation represent in real life?

\( (0, 12), (2, 9), (4, 6), (6, 3), (8, 0) \) Each of these ordered pairs represent the possible combinations of reserved and general tickets adding up to \$48. 

---

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Station # 4

“Intercepts in Real Life” – SOLUTION KEY

Ticket Problem: Reserved tickets for the basketball game cost $5 each while each general admission ticket costs $2.50. Let \( x \) represent the number of reserved tickets sold and let \( y \) represent the number of general admission tickets sold.

a. **Ticket Sales**

<table>
<thead>
<tr>
<th>Reserved tickets:</th>
<th>( x ) at $5 represents: ( 5x ) (expression)</th>
</tr>
</thead>
<tbody>
<tr>
<td>General Admission Tickets:</td>
<td>( y ) at $2.50 represents: ( 2.50y ) (expression)</td>
</tr>
</tbody>
</table>

Write an expression for the total number of tickets sold: \( 5x + 2.5y \)

b. Use your expression found in part a, to find how much money is received for the following ticket sales?

i. 8 reserved and 10 general
   \( 5x + 2.5y \)
   \( 5(8) + 2.5(10) \)
   \( 40 + 25 \)
   \$65

ii. 10 reserved and 8 general
   \( 5x + 2.5y \)
   \( 5(10) + 2.5(8) \)
   \( 50 + 20 \)
   \$70

c. A member of the Manatowa Maniacs must sell \( \underline{35} \) worth of reserved and general admission tickets combined.

Write an equation stating this. \( 5x + 2.5y = \underline{35} \)

d. Find the intercepts for the equation found in part c and use them to plot the graph of the equation. Is this a discrete or continuous function?

\( 5x + 2.5y = 35 \)

\( x \)-intercept: \( x \)

\( 5x + 2.5(0) = 35 \)

\( 5x = 35 \)

\( x = 7 \)

\( y \)-intercept: \( y \)

\( 5(0) + 2.5y = 35 \)

\( 2.5y = 35 \)

\( y = 14 \)

\( x \)-intercept: \( (7, 0) \)

\( y \)-intercept: \( (0, 14) \)

e. List all the points with positive integer values that the line crosses. What does each set of ordered pairs on the graph of the equation represent in real life?

\( (0, 14), (1, 12), (2, 10), (3, 8), (4, 6), (5, 4), (6, 2), (7, 0) \). Each of these ordered pairs represent the possible combinations of reserved and general tickets adding up to $35.
Station # 2

“Intercepts in Real Life” – SOLUTION KEY

**Socks and Scarves Problem:** Socks cost $4 per pair and each scarf costs $7.50. Natalie Allred buys some of each. Let \( x \) represent the number of pair of socks and let \( y \) represent the number of scarves she buys.

\[ \text{Write an expression for Natalie's total bill: } 4x + 7.5y \]

\[ \text{Natalie's Bill} \]

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Socks</strong></td>
<td></td>
</tr>
<tr>
<td>( x ) pairs at $4 per pair represents: ( 4x ) (expression)</td>
<td></td>
</tr>
<tr>
<td><strong>Scarves</strong></td>
<td></td>
</tr>
<tr>
<td>( y ) at $7.50 per scarf represents: ( 7.50y ) (expression)</td>
<td></td>
</tr>
</tbody>
</table>

\[ \text{b. Use your expression found in part a, to find how much would Natalie pay if she bought the following items?} \]

\[ \text{i. 7 pairs of socks and 4 scarves} \]

\[ 4x + 7.5y \]

\[ 4(7) + 7.5(4) \]

\[ 28 + 30 \]

\[ 58 \]

\[ \$58 \]

\[ \text{ii. 4 pairs of socks and 7 scarves} \]

\[ 4x + 7.5y \]

\[ 4(4) + 7.5(7) \]

\[ 16 + 52.5 \]

\[ 68.5 \]

\[ \$68.50 \]

\[ \text{c. Natalie’s budget is set at } \$60, \text{ she needs to buy a combination of sock and scarves without going over her budget.} \]

\[ \text{Write an equation stating this.} \]

\[ 4x + 7.5y = 60 \]

\[ \text{d. Find the intercepts for the equation found in part c and use them to plot the graph of the equation. Is this a discrete or continuous function?} \]

\[ \begin{array}{l}
4x + 7.5y = 60 \\
x-intercept \quad y-intercept \\
4x + 7.5(0) = 60 \quad 4(0) + 7.5y = 60 \\
x = 60 \quad 7.5y = 60 \\
x = 15 \quad y = 8 \\
\end{array} \]

\[ x-intercept: (15, 0) \]

\[ y-intercept: (0, 8) \]

\[ \text{e. List all the points with positive integer values that the line crosses. What does each set of ordered pairs on the graph of the equation represent in real life?} \]

\[ (0, 8) \text{ & } (15, 0) \text{ only Each of these ordered pairs represent the only possible combinations of socks and scarves that would total } \$60 \text{ exactly.} \]
Station # 5

"Intercepts in Real Life" – SOLUTION KEY

**Socks and Scarves Problem:** Socks cost $ 5 a pair and each scarf costs $ 6. Natalie Allred buys some of each. Let x represent the number of pair of socks and let y represent the number of scarves she buys.

a. **Natalie's Bill**

<table>
<thead>
<tr>
<th>Socks:</th>
<th>x pairs at $ 5 per pair represents: 5x (expression)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scarves:</td>
<td>y at $ 6 per scarf represents: 6y (expression)</td>
</tr>
</tbody>
</table>

Write an expression for Natalie’s total bill: 5x + 6y

b. Use your expression found in part a, to find how much would Natalie pay if she bought the following items?

i. 7 pairs of socks and 4 scarves  
   5x + 6y = 35 + 24 = $59

ii. 4 pairs of socks and 7 scarves  
   4x + 7.5y = 20 + 42 = $62

c. Natalie’s budget is set at $60, she needs to buy a combination of sock and scarves without going over her budget.

Write an equation stating this. 5x + 6y = 60

d. Find the intercepts for the equation found in part c and use them to plot the graph of the equation. Is this a discrete or continuous function?

- 5x + 6y = 60
- x-intercept: y = 0  
  5x + 6(0) = 60
- y-intercept: x = 0  
  5(0) + 6y = 60
- x-intercept: (12, 0)
- y-intercept: (0, 10)

![Graph](image)

e. List all the points with positive integer values that the line crosses. What does each set of ordered pairs on the graph of the equation represent in real life? (0, 6), (6, 5), (15, 0) only. Each of these ordered pairs represent the only possible combinations of socks and scarves that would total $60 exactly.
“Intercepts in Real Life” – SOLUTION KEY

Walking and Running Problem: Dan Daman can walk at a rate of 50 yards per minute and run at a rate of 200 yards per minute. Let $x$ represent the number of minutes he walks and let $y$ represent the number of minutes he runs.

a. Dan’s Distance = Rate $\times$ Time

Walking: $x$ minutes at 50 yds/min represents: $50x$ yards (expression)

Running: $y$ minutes at 200 yds/min represents: $200y$ yards (expression)

Write an expression for Dan’s total distance traveled: $50x + 200y$

b. Use your expression found in part a, to find how far does Dan Daman go if he:

i. walks 7 minutes and runs 3 minutes

\[ 50(7) + 200(3) = 50\times + 200y \]

ii. walks 3 minutes and runs 7 minutes

\[ 50(3) + 200(7) = 50\times + 200y \]

\[ 350 + 1400 = 1750 \text{ yards} \]

\[ 150 + 1400 = 1550 \text{ yards} \]

c. Dan puts on his cross training shoes and heads to the store. The store is exactly 1200 yards away, he needs to run and/or walk to get there.

Write an equation stating this: $50x + 200y = 1200$

d. Find the intercepts for the equation found in part c and use them to plot the graph of the equation. Is this a discrete or continuous function?

\[ 50x + 200y = 1200 \]

$x$-intercept: $y = 0$

\[ 50x + 200(0) = 1200 \]

\[ 50x = 1200 \]

\[ x = 24 \]

$y$-intercept: $x = 0$

\[ 50(0) + 200y = 1200 \]

\[ 200y = 1200 \]

\[ y = 6 \]

$x$-intercept: $(24, 0)$

$y$-intercept: $(0, 6)$

e. List all the points with positive integer values that the line crosses. What does each set of ordered pairs on the graph of the equation represent in real life?

$(0, 6), (4, 5), (8, 4), (12, 3), (16, 2), (20, 1), (24, 0)$ Each of these ordered pairs represent the only possible combinations of exact minutes walking and running that would total 1200 yards.
Station # 6  

“Intercepts in Real Life” – SOLUTION KEY

**Walking and Running Problem:** Dan Daman can walk at a rate of ___100___ yards per minute and run at a rate of ___300___ yards per minute. Let \( x \) represent the number of minutes he walks and let \( y \) represent the number of minutes he runs.

**a.**

Dan’s Distance = Rate \( \times \) Time

**Walking:** \( x \) minutes at 100 yds/min represents: ___100x___ yards (expression)

**Running:** \( y \) minutes at 300 yds/min represents: ___300y___ yards (expression)

Write an expression for Dan’s total distance traveled: ___100x + 300y___

**b.** Use your expression found in part a, to find how far does Dan Daman go if he:

<table>
<thead>
<tr>
<th>I.</th>
<th>walks 7 minutes and runs 3 minutes</th>
<th>II.</th>
<th>walks 3 minutes and runs 7 minutes</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 100x + 300y )</td>
<td>( 100x + 300y )</td>
<td>( 100(7) + 300(3) )</td>
<td>( 100(3) + 300(7) )</td>
</tr>
<tr>
<td>700 + 900</td>
<td>2100</td>
<td>300 + 2100</td>
<td>2400</td>
</tr>
<tr>
<td>1600 yards</td>
<td>2400 yards</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

c. Dan puts on his cross training shoes and heads to the store. The store is exactly 1200 yards away, he needs to run and/or walk to get there.

Write an equation stating this: ___100x + 300y = 1200___

d. Find the intercepts for the equation found in part c and use them to plot the graph of the equation. Is this a discrete or continuous function?

<table>
<thead>
<tr>
<th>( 100x + 300y = 1200 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x ) – intercept</td>
</tr>
<tr>
<td>100x + 300(0) = 1200</td>
</tr>
<tr>
<td>100x = 1200</td>
</tr>
<tr>
<td>( x = 12 )</td>
</tr>
</tbody>
</table>

| x-intercept: ___(12, 0)___ |
| y-intercept: ___(0, 4)___ |

**e.** List all the points with positive integer values that the line crosses. What does each set of ordered pairs on the graph of the equation represent in real life?

\( (0, 4), (3, 3), (6, 2), (9, 1), \) & \( (12, 0) \). Each of these ordered pairs represent the only possible combinations of exact minutes walking and running that would total 1200 yards.
STATION:  
# 1

Manatowa Maniacs
members must sell $48
worth of tickets
STATION: #3

Dan Daman is running at a rate of 200 yards per minute.

(Microsoft, n.d.)
Manatowa Maniacs members must sell $35 worth of tickets.
Dan Daman is running at a rate of 300 yards per minute.

(Microsoft, n.d.)
Extension Activity – “Intercepts in Real Life”

Problem:

Let $x$ represent the number ____________________________
and let $y$ represent the number ____________________________.

a. 

b. Use your expression found in part a, to ____________________________?

i. ____________________________  ii. ____________________________

c. ____________________________

Write an equation stating this. ____________________________

d. Find the intercepts for the equation found in part c and use them to plot the graph of the equation. Is this a discrete or continuous function?

![Graph of the equation]

e. What does each set of ordered pairs on the graph of the equation represent in real life?
Goal: Students will graph and analyze data to determine a function that represents and models a problem situation. Students will interpret the relationship and answer questions related to the function including the meaning of the parameters in the problem situation.

Objectives: Students will be able to:
1. Measure lengths in centimeters.
2. Graph a set of discrete data on a coordinate plane.
3. Derive a linear equation from a set of collected data using slope (rate of change) and y-intercept (initial condition).
4. Make predictions of the application using the linear functions.

California Content Standards in Algebra 1:
5.0 Students solve multistep problems, including word problems, involving linear equations.
6.0 Students graph a linear equation and compute the x- and y-intercepts.
17.0 Students determine the domain of independent variables and the range of dependent variables defined by a graph, a set of ordered pairs, or a symbolic expression.

California Content Standards in Grade 7:
3.0 Students graph and interpret linear functions.

Multiple Intelligence(s): Bodily/Kinesthetic, Visual/Spatial, Interpersonal and Mathematical/Logical

Lesson Length: One 50-minute class period.

Materials:
- Individual copies or overhead copy of the warm-up
- Individual copies of activity worksheets.
- Two one-meter length ropes with different thickness for each pair or group of students.
- Meter stick for each pair or group of students.
- Color pencils or markers.
Assessment: Check for understanding throughout the activity and at the closure.

Extensions:

1. Students could determine how many bed sheets it would take for a person to escape out of a prison tower, 10 stories up (approximately 100 feet or 30 meters). The bed sheet ladder is created with a knot at every foot or 30 cm of sheet.

2. Students could be given a 3 foot piece of very thick rope (i.e. 3/4 thickness). With the rope and the procedures learned from the activity, the students would need to determine the length of that rope necessary to climb down a prison tower, 10 stories up (approximately 100 – 120 feet). The rope ladder is created with a knot at every foot.

Attachments: Warm-up and KEY (pages 3 & 4), the “All Tied Up in Knots” activity worksheets (pages 5 – 7)

Directions:

1. Start the student individually with the warm-up (page 3). (5 min)
2. Have student volunteers complete the warm-up problems on the board and check for understanding. (5 min)
3. Explain the goals of the day.
4. Ask the question: “Can anyone give me an example in life when you have seen a rope or something similar with knots being used for practical purposes?” Let them come up with a few examples. If not mentioned, interject the following: rope swing, tree house ladder, a bed sheet used as a escape from a burning building. Then inform them that today’s activity is an exploration of those things.
5. Hand out the activity worksheets, ropes and meter sticks.
6. Students either work in pairs or groups of three or four.
7. Give them approximately 35 – 40 minutes to compete.
8. Have student compare answers and check for understanding with their peers. (5 min)

Closure:

1. Discuss the possible answers found in part 3.
2. Take time to connect the concepts of slope, y-intercept, domain, range, independent variable versus dependent variable, discrete versus continuous and linear function with this activity.
3. Discuss the challenges and successes of this activity.
4. **Optional** Assign the extension activity as extra credit.

** Ideas adapted from All Tied up in Knots by www.algebralab.org, Mainland High School**
Determine the linear function from each table of values.

1. \[ \begin{array}{c|c} x & y \\ \hline -1 & 5 \\ 0 & 3 \\ 1 & 1 \\ 2 & -1 \\ 3 & -3 \end{array} \]

2. \[ \begin{array}{c|c} x & f(x) \\ \hline 2 & 5 \\ 3 & 8 \\ 4 & 11 \\ 5 & 14 \\ 6 & 17 \end{array} \]

3. \[ \begin{array}{c|c} t & v(t) \\ \hline -3 & -21 \\ -1 & -11 \\ 1 & -1 \\ 3 & 9 \\ 5 & 19 \end{array} \]

4. \[ \begin{array}{c|c} K & R \\ \hline -4 & 110 \\ -2 & 105 \\ 0 & 100 \\ 2 & 95 \\ 4 & 90 \end{array} \]
Determine the linear function from each table of values.

1. \[y = -2x + 3\]

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>5</td>
</tr>
<tr>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>-1</td>
</tr>
<tr>
<td>3</td>
<td>-3</td>
</tr>
</tbody>
</table>

2. \[f(x) = 3x - 1\]

<table>
<thead>
<tr>
<th>(x)</th>
<th>(f(x))</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>4</td>
<td>11</td>
</tr>
<tr>
<td>5</td>
<td>14</td>
</tr>
<tr>
<td>6</td>
<td>17</td>
</tr>
</tbody>
</table>

3. \[v(t) = 5t - 6\]

<table>
<thead>
<tr>
<th>(t)</th>
<th>(v(t))</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>-21</td>
</tr>
<tr>
<td>-1</td>
<td>-11</td>
</tr>
<tr>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>5</td>
<td>19</td>
</tr>
</tbody>
</table>

4. \[R = -\frac{5}{2}K + 100\]

<table>
<thead>
<tr>
<th>(K)</th>
<th>(R)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>110</td>
</tr>
<tr>
<td>-2</td>
<td>105</td>
</tr>
<tr>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td>2</td>
<td>95</td>
</tr>
<tr>
<td>4</td>
<td>90</td>
</tr>
</tbody>
</table>
ALL TIED UP IN KNOTS

Overview/Objectives:

In this activity you will estimate the length of two ropes with different numbers of knots tied in each. You will develop a linear model for the data and make predictions about lengths of a rope with different numbers of knots.

Materials needed:

- One meter stick
- Two different ropes
- Two different colored pencils or markers
- Individual activity worksheets.

Procedures for Part 1:

1. Before tying any knots in the ropes, measure the length of the ropes in centimeters and record the lengths in the table.
2. Tie one knot in each rope. Tie the knots snuggly, but be sure you can untie them at the end of the activity.
3. Measure the length of each rope (in cm) with one knot and record the length.
4. Tie a second knot in each rope. Measure the length of each rope and record the length. Do not tie one knot on top of another knot.
5. Repeat step #4 for three, four, five, and six knots in each rope.

<table>
<thead>
<tr>
<th>Number of Knots</th>
<th>Length of Rope A (cm)</th>
<th>Length of Rope B (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
ALL TIED UP IN KNOTS

Procedures for Part 2:

Since small errors occur when measuring and collecting data, and there are small variations between the knots, the data collected will not fit exactly into a mathematical equation. Keep this in mind as you do the following.

1. Using a sheet of graph paper, create a graph of each set of data on the same set of axis. Be sure to use different colors representing each rope. Label and number each axis. Does each relationship appear linear?

2. Determine a line “of best fit” for each set of data. There are a variety of ways of doing this, but the idea is to find a line that comes close to most, if not all, of the points. Draw each graph of the line on the same graph as the data points. The line of best fit and its equation are said to “model” the real life situation.

3. Using the graph and the ordered pairs from each set of data, write an equation of each line of best fit.
ALL TIED UP IN KNOTS

Question to Answer for Part 3:

1. What is the independent variable in this situation?

2. What is the dependent variable in this situation?

3. Why were the slopes different for the line of best fit for the two types of rope?

4. What does the slope tell you about the activity? What property of the rope does it represent?

5. Why were the y-intercepts different for the line of best fit for the two types of rope?

6. What does the y-intercept tell you about the activity? What property of the rope does it represent?

7. The x-intercept of a line is the point at which it crosses the x-axis. What is the x-intercept of each line of best fit? What does the x-intercept tell you about the activity? Is this realistic?

8. What is the domain and range of your function?

9. Without tying the knots, determine how long your rope will be with 10 knots tied in it?

10. Without tying the knots, determine how many knots you would have to tie to make the rope 40 cm long.

11. Given a rope with the same thickness as your thicker rope at an initial length of 300 cm. After tying 10 knots in the rope, what is the new length?

12. Determine the length of the thicker rope needed to make a rope ladder with a knot at every 25 cm that someone could use to climb down from the window of a fourth floor apartment (approximately 12 meters high).
Teacher Notes – “Can Barbie Survive the Bungee?”

Goal:
Students collect, organize, and graph data. As they investigate the function \( y = mx + b \), they further develop their understanding of independent variable, dependant variable, rate of change (slope), initial value (y-intercept), concrete, discrete, domain and range. They translate from and among concrete models, tables, graphs, verbal descriptions, and algebraic rules.

Objectives:
Students will be able to:
1. Measure lengths in centimeters.
2. Graph sets of discrete data on a coordinate plane.
3. Derive linear equations from each set of collected data using slope (rate of change) and y-intercept (initial condition).
4. Make predictions of the application using the linear functions.

California Content Standards in Algebra 1:
5.0 Students solve multistep problems, including word problems, involving linear equations.
6.0 Students graph a linear equation and compute the x- and y-intercepts.
17.0 Students determine the domain of independent variables and the range of dependent variables defined by a graph, a set of ordered pairs, or a symbolic expression.

California Content Standards in Grade 7:
3.0 Students graph and interpret linear functions.

Multiple Intelligence(s):
Bodily/Kinesthetic, Visual/Spatial, Interpersonal and Mathematical/Logical

Lesson Length:
Approximately two 50-minute class periods or one 90 minute block.

Assessment:
Check for understanding throughout the activity and at the closure.

Attachments:
- “Can Barbie Survive the Bungee?” activity worksheets (pages 3 – 6)
- “Can Barbie Survive the Bungee?” follow-up questions overhead copy (page 7)
Materials:
- Individual copies of activity worksheets
- Overhead copy of the follow-up questions
- Rulers
- One Meter stick for each group
- One Barbie doll for each group
- Approximately 40 rubber bands for each group
  - Suggested sizes and makes:
    - Size 64 from an office supply store
- Large graph paper or poster (butcher) paper
  - Dimensions: 1 foot by 6 feet
- Calculators
- Tape

Directions:
1. Prior to the day of the activity, find a location on your campus that the students can bungee their Barbie dolls in the Bungee Barbie Contest. For example, the bleachers of a football field works well. You will need to measure the distance from the top to the ground ahead of time. Write this measurement in the blank provided in the conjecture and question 10 of the worksheet.
2. Introduce the class period with a video clip(s) of bungee jumpers. These can be found and previewed through YouTube. (optional)
3. Put the students in groups of 2 or 3.
4. Pass out all the materials and worksheets necessary for the day’s activity.
5. Instruct the students to find a place against the wall to set up their experiment with their group.
6. Have the students follow the instructions on the activity worksheet, then answer the questions that follow.
7. If time allows, review the questions with them using the overhead copy of the follow-up questions.
8. Have the students create their final "bungee cord" using as many rubber bands as needed.
9. Take the class to the determined location in step 1 and conduct the contest. Set up a meter stick just behind the drop zone and have a few set of eyes determine the distance from the ground Barbie’s head end up.

Closure:
1. Take time again to connect the concepts rate of change, y-intercept, domain, range, discrete versus continuous and linear functions with this activity.
2. Discuss the challenges and successes of this activity.
3. Celebrate and award the winners of the contest.

Extension:
Use graphing calculators to find the Linear Regression line.

** Ideas adapted from Barbie Bungee by Illuminations, NCTM **
Can Barbie Survive the Bungee?

Name(s): _____________________________

In this activity, you will simulate a bungee jump using a Barbie® doll and rubber bands.

Before you conduct the experiment, formulate a conjecture:

I believe that _____ is the maximum number of rubber bands that will allow Barbie to successfully and dramatically complete a jump from a height of _____ cm.

The object is to have her come as close to the ground as possible without hitting the ground. Now, conduct the experiment to test your conjecture.

PROCEDURE:
Complete each step below. As you complete each step, put a check mark in the box to the left.

- Tape a large piece of paper to the wall from the floor to a height of about six feet.

- Draw a line near the top to indicate the height from which Barbie will make each jump, her “jump line”.

- Create a double-loop to wrap around Barbie’s feet. A double-loop is made by securing one rubber band to another with a slip knot, as shown (below left).

- Wrap the open end of the double-loop tightly around Barbie’s feet, as shown (below right).
Attach a second rubber band to the first one, again using a slip knot, as shown below.

Attach two more rubber bands to the second one, again using slip knots.

With four rubber bands now attached (the one around her legs does not count), hold the end of the rubber bands at the jump line with one hand, and drop Barbie head first from the line with the other hand. Have a partner make a mark to the lowest point that Barbie reaches on this jump.

Measure the jump distance in centimeters, and record the value in the data table in Question 1. You may wish to repeat this jump several times and take the average, to ensure accuracy. Accuracy is important—Barbie’s life could depend on it!

Repeatedly attach additional rubber bands for each new jump, measure the jump distance, and record the results in the data table.

When you’ve completed the data table, answer Questions 2 – 13.

**QUESTIONS:**

1. Complete the data table below.

<table>
<thead>
<tr>
<th>NUMBER OF RUBBER BANDS ((x))</th>
<th>JUMP DISTANCE IN CENTIMETERS ((y))</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>
2. Make a scatter plot of your data. Label and indicate the scale on each axis.

3. On the graph above, sketch a line of best fit.

4. What is the relationship between the number of rubber bands and jump distance? Is this relationship discrete or continuous?

5. What is the equation for your line of best fit? Define the variables in the equation.

6. Use your equation to determine what distance Barbie would fall using 100 rubber bands. (Take a moment and think -- will 100 be an x-value or a y-value?)
7. Barbie wants to bungee off the Eiffel Tower. It is approximately 300.5 meters tall. How many rubber bands will you need so Barbie just brushes her hair (hopefully not her head) on the ground?

8. What is the slope of your equation, and what does it represent in this context? Looking at the units of the slope is helpful.

9. What is the $y$-intercept of your equation, and what does it represent in this context?

10. Based on your data, what would you predict is the maximum number of rubber bands so that Barbie could still safely jump from ______ cm (the contest jump height)?

11. Are your predictions reliable? Justify your answer. Be sure to consider your methods of collecting, recording, and plotting data.

12. How do your predictions from Question 10 compare to the conjecture you made before doing the experiment? What prior knowledge did you have (or not have) that helped (or hindered) your ability to make a good conjecture?

13. Now it is time to test your calculations in a Barbie Bungee Contest! Tie the number of rubber bands from your prediction in Question 10 and wait for further instructions.
4. What is the relationship between the number of rubber bands and jump distance?

(Microsoft, n.d.)

5. What is the equation for your line of best fit? Define the variables in the equation.

6. Use your equation to determine what distance Barbie would fall using 50 rubber bands. (Take a moment and think -- will 50 be an x-value or a y-value?)

7. Barbie wants to bungee off the Eiffel Tower. It is approximately 300.5 meters tall. How many rubber bands will you need so Barbie just brushes her hair (hopefully not her head) on the ground?

(Microsoft, n.d.)

8. What is the slope of your equation, and what does it represent in this context? Looking at the units of the slope is helpful.

9. What is the y-intercept of your equation, and what does it represent in this context?

10. Based on your data, what would you predict is the maximum number of rubber bands so that Barbie could still safely jump from ______ cm?

11. Are your predictions reliable? Justify your answer. Be sure to consider your methods of collecting, recording, and plotting data.

12. How do your predictions from Question 10 compare to the conjecture you made before doing the experiment? What prior knowledge did you have (or not have) that helped (or hindered) your ability to make a good conjecture?
REFERENCES


