DIFFERENCES OF ALGEBRAIC PERFORMANCES BY 7TH GRADERS
WITH AND WITHOUT DEDUCTIVE REASONING TRAINING

A Thesis
Presented
to the Faculty of
California State University, Chico

In Partial Fulfillment
of the Requirements for the Degree
Master of Arts
in
Social Science

by

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Spring 2010
DIFFERENCES OF ALGEBRAIC PERFORMANCES BY 7TH GRADERS
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Luna Katayama
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DEDICATION

This thesis is dedicated to my parents, Masahiro and Katsuko Katayama, and my sister, Shihomi Mori, who are in Japan.

Without their patience, understanding, caring, and financial support, the completion of this work would not have been possible.
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ABSTRACT

DIFFERENCES OF ALGEBRAIC PERFORMANCES BY 7TH GRADERS WITH AND WITHOUT DEDUCTIVE REASONING TRAINING

by

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Japanese mathematics education has provided students with deductive problem-solving practices in mathematics. However, this type of education is not currently offered in American mathematics classrooms. Learning from Japan’s success in mathematics education, deductive reasoning training may be necessary in American mathematics classrooms to improve mathematics skills of American students. This study postulated that 141 Chico Junior High School (CJHS) seventh graders could improve their algebra scores by using deductive reasoning practices and learning mathematics deductively. Also, the study examined whether or not learning mathematics with deductive reasoning training was more effective in improving the students’ mathematics performances than without it. A survey was given to the participants to explore the possibility that their attitudes toward learning mathematics were different from Japanese students
and to determine if any factors such as their socio-cultural backgrounds may have generally affected their mathematics performances. The test results indicated that deductive reasoning practices helped the students, especially the boys, improve their performances in algebra. However, this did not mean that learning mathematics with deductive reasoning practices was a better learning method than without it. The survey results suggested that the Asian/Pacific islander students seemed to have more positive attitudes toward learning math than the other ethnic students, and the native English speakers understood mathematics lectures better than the English language learners did. Also, the Japanese students seemed to have more positive attitudes toward learning mathematics than the CJHS seventh grade students did.
CHAPTER I

INTRODUCTION TO THE STUDY

Students’ poor academic performance is of great concern in the United States. The renewal of the Elementary and Secondary Education Act in 2001, called No Child Left Behind Act of 2001, instituted state achievement testing in English language arts and mathematics as a way to improve students’ education. States have also implemented high school exit exams to improve education.

The quality of education in the United States has been questioned because of the comparisons of American students’ academic achievement with students in other countries. For example, Progress in International Reading Literacy Study (PIRLS) has tested fourth graders’ reading literacy skills in 35 countries every five years since 2001. Trends in the International Mathematics and Science Study (TIMSS) has compared the mathematics and science achievement of fourth and eighth graders from across 40 countries every four years since 1995. The Programme for International Student Assessment (PISA) has provided standardized reading, mathematics, and science literacy assessments for testing 15-year-old students in 58 countries every three years since 2000.

Out of those international tests, the relatively recent one, PISA 2006, showed that the American students scored remarkably below the international average in mathematics. The United States was ranked 25th with 474 points for mathematics while the international average was 498 points (Baldi, Jin, Skemer, Green, & Herget, 2007,
Particularly, mathematics is an important subject to be learned in order for students to be academically successful or be an effective workforce. However, their mathematics achievement lags behind other developed countries, the major economic competitors of the United States. Among G-8 countries who participated in TIMSS 2007 mathematics test, 10% of American fourth graders reached the advanced international benchmark while 23% of Japanese and 16% of British and Russian fourth graders did, and only 6% of eighth graders in the United States could solve the advanced mathematics problems while 26% of eighth graders in Japan did (U.S. Department of Education, 2009, p. 23).

American student mathematics performance has tended to be specifically compared with Japan’s students’. Japan is one of the major economic competitors of the United States and is “an obvious choice for comparison because it has always scored near the top in international comparisons in mathematics achievement” (Stigler & Hiebert, 1999, p. 17). Also, TIMSS 2007 indicated that Japanese students significantly outperformed other participating G-8 countries’ students in mathematics with higher percentages (U.S. Department of Education, 2009). Being a leading political and economic force in the world, the United States may need to rethink its mathematics education, borrowing what has made Japanese mathematics education more successful.

Mathematics teaching and learning styles are different between the United States and Japan (U.S. Department of Education, 1999). American instructors teach mathematical terms and have students practice skills by connecting them to real life situations because American instructors do not want students to get confused and become frustrated by new mathematics procedures (Stigler & Hiebert, 1999). For example,
teachers set a mathematics problem to measure the circumference of a basketball. Also, American teachers ask students to practice new mathematics procedures many times and later give them more difficult exercises while supposedly minimizing errors, confusion, and frustration that students may have (Stigler & Hiebert, 1999). These practices have become routine in most of mathematics classrooms in the United States.

While students in the United States spend almost all their time practicing routine concepts and procedures which are demonstrated by instructors (Stigler & Hiebert, 1998), students in Japan are taught to solve mathematics problems without directions. They are required to solve many word problems by trying out various approaches. Meanwhile, students are struggling and frustrated by not understanding what they are supposed to do. Struggling through a trial and error process is believed to play an important role in improving students’ mathematics performances by Japanese instructors.

Japanese teachers believe students learn best by first struggling to solve mathematics problems, then participating in discussions about how to solve them, and then hearing about the pros and cons of different methods and the relationships between them. Frustration and confusion are taken to be a natural part of the process, because each person must struggle with a situation or problem first in order to make sense of the information he or she hears later. (Stigler & Hiebert, 1999, p. 91)

After overcoming the hardship, students can connect various solutions and formulas in their understanding by later discussion with an instructor.

(Japanese) teachers act as if mathematics is a set of relationships between concepts, facts, and procedures. These relationships are revealed by developing methods of solution to problems, studying the methods, working toward increasingly efficient methods, and talking explicitly about the relationships of interest. (Stigler & Hiebert, 1999, p. 89)

Japanese instructors believe that understanding the relationship between the process of solving a problem and its result is important in the learning process, and struggling to
solve mathematics problems helps students remember the association between mathematics concepts and methods of solution.

Struggle and frustration are the feelings that American teachers do not want their students to experience in mathematics classrooms, as mentioned earlier. However, struggling or frustrating process in learning can be found not only in mathematics but also in many other situations. For example, when people run a marathon, they go through hard stages, but once they have reached their goal, they are satisfied. When people practice a musical instrument, they may get frustrated when their fingers do not move smoothly, but after they practice the same musical phrases repeatedly, they can eventually play that instrument well and be happy with their accomplishment. Nobody enjoys solving frustrating and confusing mathematics problems, but struggling in finding solutions to the problems is a natural process of learning. After students overcome the hardship to find a way to solve a mathematics problem, they would be satisfied or happy with their achievement, and Japanese teachers believe that overcoming the hardship is more likely to improve the students’ mathematics performances.

In Japanese classrooms, the style of learning mathematics is to teach deductive reasoning skills. Deduction is the movement from the suggestion or hypothesis toward facts and toward developing, applying, and testing (Dewey, 1910). In other words, deductive thinking takes place through the following steps: 1) observing general information; 2) based on the observation, suggesting a conclusion; 3) testing if the suggested conclusion is true; and, 4) finding whether or not the suggested conclusion is true. In Japanese mathematics classrooms, based on the researcher’s experience as a student in Japan, Japanese students are first given new mathematics concepts and
procedures as general information. Then, students are asked to try out new concepts and procedures with other mathematics problems without instructor’s directions. After that, students suggest their own methods of finding a solution. During this time, students individually develop a way to solve a mathematics problem based on concepts or procedures previously given. Then, students share their methods of solution with their classmates or with the instructor to make sure that the students’ solutions to mathematics problems are applicable. Therefore, a Japanese teaching technique in mathematics classrooms is deductive reasoning training. This deductive reasoning training seems to be missing in mathematics classrooms in American middle schools. Implementing deductive reasoning practices in American middle school mathematics classrooms may help American middle schoolers improve their mathematics performances.

Statement of the Problem

Traditional mathematics instruction in American mathematics classrooms consists of little deductive reasoning unlike Japanese mathematics classrooms. By implementing deductive reasoning training to mathematics classrooms in American middle schools, American middle school students may improve their mathematics performances. This study sought to test the following hypotheses:

1. Deductive reasoning training will improve algebra performance of seventh graders.
2. Learn algebra deductively is more effective to improve seventh graders than the American learning style.
3. Gender difference would not affect Hypothesis 1 and Hypothesis 2.
4. Seventh graders’ socio-cultural background is related to their ways of and attitudes toward learning mathematics.

5. Japanese students have more positive attitudes toward learning mathematics than American seventh graders.

6. Types of attitudes toward learning mathematics that American seventh graders have can be defined.

Purpose of the Study

The purpose of the study was to study American seventh graders’ mathematics performance improvement by learning algebra deductively and to study about elements such as American seventh graders’ socio-cultural backgrounds that influence their mathematics performance. In the experiment, one lesson, two quizzes, one assignment of five algebraic problems, and one survey were provided for two groups of seventh graders at Chico Junior High School (CJHS). One group took a lesson to learn basic algebra and practice its deductive reasoning problems in an American traditional way of learning. The other group took a lesson to learn and practice deductive reasoning algebraic problems in a Japanese way of learning. Comparison of the two groups’ assignment and pre- and post-test scores in each group were analyzed. Both the groups were required to write out how to solve problems on the quizzes and assignment. These tests and assignment were to observe participants’ thinking process.

A survey about participants’ socio-cultural background questions was given to the participants. This survey was to see the relationship between the students’ background and their ways of and attitudes toward learning math. Also, some of the
survey questions were compared with the ones which were conducted by the National Institute for Educational Policy Research of Japan (NIER) (2008) to discuss similarities and differences between the CJHS students and Japanese students.

Algebra was used to score students because it must be the starting point to reform mathematics education. A member of the math panel and outgoing president of the National Council of Teachers of Mathematics, Francis “Skip” Fennell (as cited in Vogel, 2008), mentioned, “For far too many children, algebra becomes their first mathematics stumbling block” (p. 34). The former U.S. Education Secretary, Margaret Spellings (as cited in Vogel, 2008), also said, “If students do well in algebra, then they are more likely to succeed in college and be ready for better career opportunities in the global economy of the 21st century” (p. 34). Algebra is an important subject to reduce the number of high school dropouts and to improve students’ mathematics skills.

Seventh graders were tested because Japanese students start to learn algebra in the seventh grade. American students start to learn algebra in the eighth grade, but seventh graders are actually capable of learning basic algebra and solving algebraic problems as evidenced by Japanese seventh graders’ high achievement in algebra. Also, because American seventh graders have not yet had algebra instruction, the advantage is that there will be no biased results when instructions and testing are administered to them.

Significance of the Study

It is important to examine whether deductive reasoning practices and deductive learning can boost middle school students’ performance in mathematics
because this may be the key difference between American and Japanese mathematics education and may prove to be a valuable teaching tool for American mathematics teachers.

Theoretical Bases

*Experiencing* mathematics problem-solving is necessary even though students may have a hard time working out solutions. The Japanese teaching technique includes this type of *experiencing*, which is important in learning. Dewey (1916) explained how *experiencing* took place in learning as follows:

To learn from experience is to make a backward and forward connection between what we do to things and what we enjoy or suffer from things in consequence. Under such conditions, doing becomes a *trying* (which is experience on the active side); an experiment with the world to find out what it is like; the *undergoing* (which is experience on the passive side) becomes instruction – discovery of the connection of things. (p. 164)

Japanese students try new mathematics concepts by practicing mathematics problem-solving repeatedly and by developing their methods of solution. Although they get frustrated by this hard work, they undergo the mathematics procedures, and thus find the connections between concepts and facts. This development of Japanese students’ own methods of solution is the experience in learning mathematics.

When people learn from their experience, thinking is involved. Thinking is the initial stage of the developing experience and is indirectly guided to link an experience and its consequence (Dewey, 1916). How this works is that thinking ascertains facts that help individuals indicate other facts which are not directly ascertained, and this is called *inference* (Dewey, 1910). Usually, *inferring* occurs through observation. This observation is followed by a suggestion which is made as to its consequence, and testing the
suggestion confirms the consequence. For example, suppose that there are white birds. When people see them, they may guess that those white birds are ducks, geese, cranes, etc. Then, people may find out what kind of birds they are by asking other people, looking up reference books, checking up information online, and so on. After this, people confirm whether their guess was right or not. This whole process is deduction. This deduction is significant in the thought process and in learning.

Importantly, deductive reasoning skills are necessary for problem-solving. Dewey (1910) explained why this was so, giving an example as follows:

If the physician does not know the general laws of the physiology of the human body, he has little way of telling what is either peculiarly significant or peculiarly exceptional in any particular case that he is called upon to treat. If he knows the laws of circulation, digestion, and respiration, he can deduce the conditions that should normally be found in a given case. . . . Deductive systems are necessary in order to put the question in a fruitful form. (p. 93)

Not only physicians but also many others can implement deductive reasoning skills to solve problems in their daily life. For instance, when someone has abdominal pain, he/she guesses what the cause of the pain is, such as foods, stress, menstruation, and so on, before going to see a doctor. If he/she does not want to see a doctor, he/she may try to figure out which medicine is necessary to take. If the first choice does not work, the cause of the pain that he/she guessed is proven to be wrong. He/she may try another drug to reduce pain. This whole process is deductive reasoning, and deductive reasoning skills should be used for mathematics problem-solving as well as problem-solving in daily life.

When people learn, pieces of information are collected, are sorted by categories, and are used when needed after “experiment with the world to find out what it is like” (Dewey, 1916, p. 164). Deduction is always involved when creating relationships
of information. “Only deduction brings out and emphasizes consecutive relationships, and only when relationships are held in view does learning become more than a miscellaneous scrap-bag” (Dewey, 1910, p. 97). This explanation matches Japanese teachers’ beliefs about mathematics learning as stated earlier. As Japanese teachers believe, relating mathematics concepts to methods of solution, which are brought by deductive processes, can help students solve other mathematics problems. Fehr (1953) pointed out that patterns of solutions to mathematics problems should be organized into “a logical frame of reference” (p. 35) to be able to solve the problems. By using logical frames of reference, patterns of solutions can be easily remembered and recalled when students solve more difficult mathematics problems. Thus, deduction plays an important role in mathematics learning. “Deductive reasoning is a key in working on mathematics because rigorous logical proof, which is a unique fundamental characteristic of mathematics, is constructed using deductive reasoning” (Ayalon & Even, 2008, p. 236). This learning process using deductive reasoning skills is the way of learning mathematics in Japan. Japanese students think of an easier way to solve a problem, and then, they test it out with other similar problems to confirm if the solution method is good or bad. In other words, Japanese students have been trained with this deductive way of learning.

**Case Study and the Researcher’s Personal Experience as a Student in Japan**

Deductive reasoning is used to solve mathematics problems and to improve mathematics skills of students in Japanese middle schools. However, few deductive reasoning practices have currently been applied to American middle school mathematics classrooms, so American students have been expected to somehow gain deductive
reasoning skills without training. Deductive skills are assumed to be learned indirectly by American students in traditional content areas such as science and mathematics (Hurst, Tan, Meek, Sellers, & McArthur, 2003; Leighton, 2006). In fact, deductive reasoning practices were found in mathematics classrooms of Japanese eighth graders in the TIMSS video study, but not found in those of American eighth graders (U.S. Department of Education, 1999). The video study (U.S. Department of Education, 1999) showed that the Japanese students practiced mathematic deductive reasoning problems in 61% of the lessons (which were videotaped) while the U.S. students did not practice any at all (p.65). From this video study, it is obvious that Japanese mathematics education has focused on deductive reasoning skills. It is doubtful that American students can gain strong deductive reasoning skills in mathematics without explicitly practicing them in class.

In typical mathematics lessons in the United States, as mentioned earlier, students are asked to mostly practice mathematics procedures in class. The TIMSS video study indicated that the average percentage of seatwork time spent in practicing routine procedures was about 98% (Stigler & Hiebert, 1998). This seatwork includes reviewing mathematics concepts learned in previous lesson and homework answers. Kawanaka and Stigler (1999) showed one of the nine U.S. lessons which they researched as an example of the traditional American lesson:

In this lesson, the teacher and students first spent 30 min checking homework answers. The teacher then introduced the Pythagorean theorem by stating and explaining the formula. The teacher drew a freehand triangle on the chalk-board, assigned numbers to two sides, and guided students to figure out the length of the side that was left out. (p. 270)

Given mathematics procedures, Describe/Explain questions which followed the answer to Yes/No or Name/State questions are usually asked in class (Kawanaka & Stigler, 1999).
This is “for assessment purposes more often than to elicit students’ ideas and thoughts” (Kawanaka & Stigler, 1999, pp. 272-273).

In addition, because American instructors focus on evaluating students’ answers in class, they rarely ask students to reflect on each other’s responses (Kawanaka and Stigler, 1999). In fact, American lessons lack time spent in developing or analyzing mathematics problem-solving by inventing other solutions. About 10% of the time was spent to develop mathematics concepts and procedures and below 3% of the time was spent to invent other solutions (Stigler & Hiebert, 1998). This is because American teachers take most of the lesson time to demonstrate how to solve a sample problem in order for students to be able to solve similar mathematics problems by themselves.

On the contrary, Japanese teachers take more lesson time for students to invent, analyze, and prove mathematics problem-solving. About 40% of seatwork time was used to practice procedure, but about 18% of seatwork time was spent to apply concepts and about 43% was to invent or think about methods of solution (Stigler & Hiebert, 1998). An example of a typical Japanese mathematics lesson is presented as follows:

The lesson focuses on one or sometimes two key problems. After reviewing the major point of the previous lesson and introducing the topic for today’s lesson, the teacher presents the first problem. The problem is usually one that students do not know how to solve immediately but for which they have learned some crucial concepts or procedures in their previous lessons. Students are asked to work on the problem for a specified number of minutes and then to share their solutions. The teacher reviews and highlights one or two aspects of the students’ methods of solution or presents another solution method. (Stiger & Hiebert, 1997, “Teaching in Germany,” para. 16)

As indicated earlier, Japanese students are expected to improve deductive reasoning skills through lessons and assignments. Deduction is involved in not only solving mathematics
problems but also figuring out how to solve mathematics problems. Through classwork and homework, Japanese students find and develop their own solutions and solve other problems again and again, applying given mathematics problem-solving rules (Stigler & Hiebert, 1999). These developing, inventing, and analyzing methods of solution by students themselves are deductive reasoning practices.

More specifically, in a Japanese traditional lesson, instructors present the rules by proving them. Then, Japanese students practice solving mathematics problems with the concepts. Particularly, they are often required to write out how they solve problems in the answer sheets. So, teachers can check on the deductive steps that they made and correct them (Stigler & Hiebert, 1999). An example of a lesson would be the following:

Solve the following equations:

3a + 2b = 17
2a + 3b = 18.

Japanese teachers would provide two rules for students to solve this problem in class. One rule is using the least common multiple (LCM). Typically, instructors show the process of solving it by using rules. In this example, the LCM of 2 and 3 is 6. To make 3a in the first statement be 6a, the first equation needs to be multiplied by 2. It is 6a + 4b = 34. To do the same thing to the second statement, it needs to be multiplied by 3. It is 6a + 9b = 54. Then, each component in the first equation is subtracted by each in the second equation and the answers can be found:

\[
\begin{align*}
6a + 4b &= 34 \\
-(6a + 9b) &= 54 \\
0 - 5b &= -20 \rightarrow 5b = 20. \text{ Answer } b = 4. \rightarrow 3a + 2(4) &= 17 \\
&\rightarrow \text{ Answer } a = 3.
\end{align*}
\]
After the teachers’ demonstration, some similar problems would be given to students. Students try out this rule by solving the problems to see if that rule works.

The other rule to solve this problem would be finding the value of a or b from one of the equations. Teachers would show calculations to find the value of a or b in the lesson. From the first equation, \(3a = 17 - 2b\). Therefore, \(a = \frac{17 - 2b}{3}\). This equation is applied to the second equation. \(2(\frac{17-2b}{3}) + 3b = 18\). There is only b in this equation, so the value of b can be found. If \(b = 4\), then \(3a + 2(4) = 17\). The answer is \(a = 3\). Students would be given similar problems and practice on them, applying the rule.

After practicing problem-solving with the two rules, a Japanese teacher asks students to work word problems to which they need to apply the rules. Using either the first rule or the second rule to solve the problem is up to the students. While working to solve the word problems, the students develop or invent their own methods to find equations and solve the equations. After that, the students confirm if they have found a correct equation and solve the equation correctly which was verified later in discussion with the instructor. This process is deduction.

Conclusion

Deductive reasoning skills are critical to improve mathematics performance in mathematics problem-solving and proofing (Ayalon & Even, 2008; College Entrance Examination Board [CEEB], 1959; Inglis & Simpson, 2004; Morris & Sloutsky, 1995; Staver, 1986; Stylianides, 2005) so that the mathematics practices with deductive reasoning have mainly helped students in Japan achieve their advanced mathematics performances. As Japanese students did, the deductive reasoning training may help American middle school students improve their mathematics performances.
Limitations of the Study

The duration of the experiment is so short that adequate responses may not be collected, and testing students only one time may be unreliable. However, it is difficult to conduct longitudinal study or several studies on the same or different groups because the regular curriculum in middle schools should not be repeatedly interrupted by this research. Also, test scores can be influenced by other factors, such as students’ motivation, mood, and health conditions. These factors can not be controlled in the research.

Definitions of Terms

Academic Achievement/Performance

Academic Achievement/Performance is the measure of students’ proficiency in academics.

Deduction

Deduction is the movement from the suggestion or hypothesis toward facts and toward developing, applying, and testing (Dewey, 1910). In deduction, conclusions are based on information provided in the problem (Bruning, Schraw, Norby, & Ronning, 1995).

Deductive Reasoning Training in Learning Mathematics

It is to learn new mathematics concepts through the following deductive steps: 1) finding students own methods of solution, 2) testing if those methods of solution would work on the students own, and 3) concluding their methods of solution would or would not work in later discussion given by instructors.
Education

The general function of education assumes that of direction, control, or guidance (Dewey, 1916). The natural or native impulses of the young do not agree with the life-customs of the group into which they are born. Consequently, they have to be directed or guided. This control is not the same thing as physical compulsion (Dewey, 1916).

Experience

Experience is trying on the active side and undergoing on the passive side (Dewey, 1916).

Inductive Reasoning

Induction is the opposite of deduction. It is the movement from facts/data toward suggestions and hypotheses. In induction, it can be realized that conclusions go beyond the data as in the making of a theory (Bruning et al., 1995).

Learning

“Human learning is defined as a change in behavior acquired through an experience. The learning is usually directed toward specific goals through organized patterns of experience” (Fehr, 1953, p.2).

Mathematics Education

Mathematics Education is how to direct, control, or guide mathematic problem-solving to students.
CHAPTER II

LITERATURE REVIEW

Mathematics requires *thinking* and *reasoning*. Basic calculations, such as addition and subtraction, involve thinking of basic logical relationships between numbers. Advanced calculations, for example, algebra, necessitate thinking and reasoning of complex logical associations between numbers and inferences from logic rules. Holyoak and Morris (2005) defined *reasoning* as one type of *thinking*, and it is interconnected with deduction and induction as follows:

The study of thinking includes several interrelated subfields that reflect slightly different perspective on thinking. *Reasoning*, which has a long tradition that springs from philosophy and logic, places emphasis on the process of drawing inferences (conclusions) from some initial information (premises). In standard logic, an inference is *deductive* if the truth of the premises guarantees the truth of the conclusion by virtue of the argument form. If the truth of the premises renders the truth of the conclusion more credible but does not bestow certainty, the inference is called *inductive* . . . (p. 2)

Thinking includes reasoning, induction, and deduction. Reasoning is derived from either induction or deduction to conclude a fact through inferences originated in premises.

Induction and deduction are primarily different in what to observe and how to conclude. The aim of induction is to make a conclusion from a particular observation.

“Inductive reasoning is to generalize rules or hypotheses from examples and background knowledge” (Rivera & Becker, 2007, p. 144). The following logic would be induction:

“All the swans I have ever seen are white. Therefore, all swans are white” (Evans, 1982,
In this example, swans’ color is generalized from a particular observation. This process of generalization is induction.

On the contrary, to inductive reasoning, as briefly mentioned earlier, deductive reasoning is a process from a general observation to a particular conclusion. The following logic would be deduction: “All swans are white. This bird is a swan. Therefore, this bird is white” (Evans, 1982, p. 1). That all swans are white is a general observation. That this bird is white is a particular conclusion. This deductive logic, called Syllogism, was introduced by Aristotle, an ancient Greek philosopher in the 300s B.C. Aristotle (trans. 1989) explained Syllogism, “For let A belong to every B and B to some C. Then, if to be predicated of every (B) is what was said in the beginning, it is necessary for A to belong to some C” (p. 5). A, B, and C can be substituted by subjects and objects. In the previous deduction example, A is white color. B is swan. C is bird. So, white color belongs to every swan and swan belongs to every bird. Therefore, white color belongs to some bird.

Syllogism consists of two deductive reasoning forms, such as proposition and condition. There are four syllogism relationships in propositional reasoning: “(1) All A are B, (2) No A are B, (3) Some A are B, and (4) Some A are not B” (Evans, 1982, p.74). Usually, one or more combinations of the four relationships are observed in deductive argument in three propositional statements. One conclusion, the last statement in the three propositions, “is a logical consequence of the two former which is called the premises” (Boole, 1847, p. 31). For example, “No singers are tone deaf. Some men are singers. Therefore, some men are not tone deaf” (Evans, 1982, p. 76). The two premises imply
that some men are not tone deaf singers. The last statement is a conclusion. The inference comes in and concludes that some men are not tone deaf.

The other form of deductive argument, conditional reasoning, also has four rules: “(1) If p, then q, (2) If p, then not q, (3) If not p, then q, and (4) if not p, then not q” (Evans, 1982, p.123). The following would be an example of the conditional rules: “If Calvin deposits 50 cents, he will get a coke. Calvin deposits 50 cents. Calvin will get a coke” (Rips, 1994, p .3). The first two statements are given rules. The last statement is a conclusion. This conclusion is made through an inference that Calvin will get a Coke because he deposited 50 cents. From a general observation, the inference reaches to a specific conclusion.

In mathematics, both induction and deduction are necessary for problem-solving. “Inductive reasoning involves examining particular cases, identifying patterns and relationships among those cases, and extending the patterns and relationships” (Kriegler, 2004, Mathematical Thinking Tools, para. 4). An example of inductively solving mathematics problems would be: Find the pattern of the sequence, 0, 2, 4, 6, 8, 10, ...

The answer, 2n, can be delivered, observing the sequenced numbers. There are no numbers that continue after 10, but it is more likely to find 12, based on inductive reasoning.

Mathematical “deductive reasoning involves drawing conclusions by examining a problem’s structure” (Kriegler, 2004, Mathematical Thinking Tools, para. 4). An example of algebraic logics in propositional reasoning would be: “All Ys are Xs
and all Zs are Ys (two premises). Therefore, all Zs are Xs (a conclusion).” They can be expressed in equations as follows:

\[ Y = X \text{ and } Z = Y. \]

Therefore, \( Z = X. \)

To withdraw the conclusion, \( Z = X, \) its problem structure is examined:

\[ Y = X \implies Y - X = 0 \]
\[ Z = Y \implies Y = Z \implies (Z) - X = 0 \implies Z = X. \]

Also, deductive conditional rules are often used to solve mathematical problems. One of the examples is the following: If \( 2n + 10 = 18, \) then what is the value of \( n \) and \( 4n? \) The answer, \( n = 4 \) can be found from the provided rule, \( 2n + 10 = 18. \) The answer \( n = 4 \) will be a new rule to solve the next problem, \( 4n. \) Since the new rule, \( n = 4 \) is drawn, \( 4n = 16 \) can be concluded. These steps to get the two answers are deduction. Another example for this would be:

Prove if \( a \) and \( b \) are both inverses for \( c, \) then \( ac = ca = 1 \) and \( bc = cb = 1. \) The answer is:

\[ a = (1)a = (bc)a = b(ca) = b(1) = b. \]

A structure of this problem is proved step by step. This step by step proof is deduction.

History of Mathematics Education

Regarding the use of either inductive or deductive reasoning in mathematics, mathematics education reforms have been discussed and implemented in the United States. During the mid-20\(^{th}\) century, mathematical researchers presented that learning mathematics using deductive thinking was significant (Allendoerfer, 1957; CEEB, 1959;
Fehr, 1953; National Council of Teachers of Mathematics [NCTM, 1989] after John Dewey (1910) introduced it as one of the learning phases. Especially, Fehr (1953) emphasized that deduction should be used in learning mathematics “by going over and then generalizing the particular solution, by taking many similar examples and abstracting the common elements of solution, by making a logical chain of known theorems to the new result, and by a mixture of these methods” (p. 35). These methods are a part of what is involved in the deductive reasoning process.

Because of these researchers’ suggestions, a program to include deductive reasoning in all mathematics courses in American high schools was established. For instance, College Entrance Examination Board (CEEB, 1959) published a “Program for College Preparatory Mathematics” listing four objectives. One of the objectives was to gain “an understanding of the deductive method as a method of thought. This includes the ideas of axioms, rules of inference, and methods of proof” (CEEB, 1959, The Report, para. 7). However, this plan was not adopted in high school education at that time. “Unfortunately, …the introductory algebra tended to treat the content in a reductionistic fashion, employing numerous repetitious exercises and giving little attention to reasoning or complex problem-solving” (Kilpatrick & Izsák, 2008, p.9). This failure to apply deductive reasoning in all mathematics courses led to a rethinking of mathematics education reform in the late 20th century. Eventually, instead of deductive reasoning practices, the importance of the inductive approach to mathematics was introduced (Christiansen, 1969; Gawronski, 1971).

Although both inductive and deductive reasoning are important in mathematical problem-solving, training in deductive reasoning has tended to be ignored.
It may be because educators have focused on instructional procedures generalizing concepts of, for example, algebra. Most students have had and still have difficulties in the transition from the arithmetical concept to abstract thinking by using symbolic equations in algebra when they start to learn it (Ketterlin-Geller, Jungjohann, Chard, & Baker, 2007; Leinhardt, Zaslavsky, & Stein, 1990; Steen, 2007). Particularly, the foundation of teaching basic algebra, for which the goal was to understand symbols and how to solve simple equations, was formed in the 1970s (Kilpatrick & Izsák, 2008). This abstract thinking in algebra has been emphasized in the mathematics curriculum (Bendnarz, Kieran, & Lee, 1996; NCTM, 1989). As a result of this, curricula in mathematics education had been developed to help students smoothly transfer from numbers to symbolic equations in the 1990s and the 2000s (Andrews & Sayer, 2006; Lee & Wheeler, 1989; NCTM, 1989, 2000). For example, the NCTM (2000) suggested instructional programs for algebra that students in preschool to K-2 should be able to recognize, describe, and extend patterns and sequences of simple numeric patterns, and students in grades 3 to 12 should be able to generalize numeric patterns with tables, graphs, words, and symbolic rules. Generalizing numeric patterns requires inductive reasoning skills as explained earlier. Through mathematics courses, American students have apparently been encouraged to practice inductive reasoning while ignoring deductive reasoning.

Deductive Reasoning and Cognitive Psychology

Deductive reasoning thought processes have been discussed in Psychology throughout the past decades. Through the discussions, which explained the mechanisms of deductive reasoning thought processes, two theories have been acknowledged the
most: Mental Logic Theory and Mental Models Theory. According to Mental Logic Theory, deductive reasoning is processed, based on “knowledge retrieved from long-term memory, beliefs, opinions, guesses, and various kinds of implicatures” (Braine, 1994, p. 243) and linguistic rules such as “and for conjunction, or for disjunction, not for negation, and if for conditionality” (O’Brien, 1993, p. 31). For instance, the following prepositional statement, *if a bird is pecking a tree, then that bird must be a woodpecker*, contains a rule of conditionality and a rule of inference that the bird is most likely a woodpecker, based on knowledge that woodpeckers peck on trees.

Johnson-Laird (1986) introduced Mental Models Theory, stating that deduction was done by constructing mental representations of semantic contents rather than by depending on linguistic logic. For example, in a prepositional argument, *if A, then B*, people generally represent the association between A and B. More specifically, mental representation can be found in the following statements. “The triangle is to the right of the circle. The circle is to the right of the diamond. Therefore, the triangle is to the right of the diamond” (Johnson-Laird & Yang, 2008, p. 344). In this example, the meaning of the triangle, the circle, and the diamond is represented in the mind, and their locations are spatially located in the mind. These three-dimensional relationships are created by mental representation (Johnson-Laird & Yang, 2008).

Which theory postulates a deductive reasoning process in a human’s mind better than the other is still being debated because both have weaknesses in their approaches. For instance, for Mental Logic Theory, whether children who have less knowledge than adults can make deductions was questionable. According to Mental Models Theory, it is difficult for people to represent many objects or premises in the
mind at the same time (O’Brien, Braine, & Yang, 1994). Both theories are still the
subjects of an ongoing discussion meant to identify and solve their weak points.
However, in regard to children’s ability to deduce, many empirical studies have proven
that they were capable of deductive reasoning as described below.

Cognitive Development and Deductive Reasoning

Children’s ability to engage in deductive reasoning in different age groups has
been studied by many researchers. The studies found that the ability of logical thinking
individually varied and was different between children at different ages (DeLisi, 1979;
Kalechofsky, 1971; Mattheis, Spooner, & Coble, 1985; Piaget, 1957, 1972). In addition,
as children grew up, their ability to make deductions had improved (Hawkins, Pea, Glick,
& Scribner, 1984; Mattheis et al., 1985; Markovits, Schleifer, & Fortier, 1989; Müller,
Sokol, & Overton, 1999; Oakes, 1984; Overton, Ward, Black, Noveck & O’Brien, 1987;
Piaget, 1972; Pillow, Hill, Boyce, & Stein, 2000; Reid, 2002; Saarni, 1973; Shapiro &
O’Brien, 1970; Stylianides, 2005; Stylianides & Stylianides, 2008; Ward & Overton,
1990). Four-year-olds do not deductively infer information and rarely referred to the
premise (Pillow et al., 2000; Reid, 2002), but when clues are given to them, they are able
to solve deductive reasoning problems (Stylianides & Stylianides, 2008). Subsequently,
children, ages 6 and 7, are sometimes able to explain partial inferences based on premise
information and able to make deductions, but they do not know that deducing is more
certain than guessing (Pillow et al., 2000; Reid, 2002; Markovits et al., 1989). At the age
of eight and nine, children are able to be more consistent in inferring and deducing
(Overton et al., 1987; Pillow et al., 2000; Shapiro & O’Brien, 1970). Finally, children age
10 are steadily able to obtain inferential premises from deductive reasoning (Stylianides & Stylianides, 2008; Watters & English, 1995).

According to Piaget (1957), the ability to test hypotheses systematically is present in children, ages 11 to 15. Eleven-year-olds are able to distinguish logical or illogical syllogisms (Markovits et al., 1989). Children, grade seven or above, emerge to be able to distinguish between possibilities whether hypotheses are true or not and what actual reality is, and they are also able to coordinate the possibility levels hierarchically (Müller et al., 1999). Fourteen to 15-year-olds are capable of constructing hypotheses and deducing the consequences from the hypotheses (Overton et al., 1987; Piaget, 1972).

For most studies, it has been suggested that children, ages 10 or older, are capable of solving deductive reasoning problems. Therefore, from the cognitive development perspective, middle school students certainly have the ability to reason deductively and are able to be trained with deductive reasoning practices.

Algebra

Algebra is important for children to learn because it is needed for vocational and academic preparation and used in daily life. In the business world, workers are expected to translate work-related problems into general mathematical models from calculating discounts for merchandise to operating technology-based equipment and machinery (Ketterlin-Geller et al., 2007, p. 67). Algebra courses are required in colleges to achieve academic performance in mathematics, sciences, and engineering. Also, it is useful in everyday life to understand the quantity by expressing symbolical equations.
For example, the usage of gallons of gasoline per mile can be calculated by applying algebraic formulas (Ketterlin-Geller et al., 2007, p. 67).

Learning algebra is often involved in deductive reasoning. For instance, various problem-solving strategies would be used for algebra word, equation, and proof problems (Kriegler, 2004; Mayer, 1982; Pedemonte, 2008). Importantly, studies showed that students at ages 10 to 16 who were trained with deductive reasoning outperformed on algebraic equations, conditional responses, and proofs than those students who were not trained with deductive reasoning and were or were not trained with inductive reasoning (Barrish, 1970; Morris & Sloutsky, 1995). Therefore, children at ages 10 or older should be trained to learn algebra with deductive reasoning.

In addition, researchers and curriculum developers suggested that children should learn algebra at an early age (Cavanagh, 2008; Ketterlin-Geller et al., 2007; Vogel, 2008). American students start to learn Algebra I in the eighth grade and do not usually learn basic algebra before Algebra I. In Japan, students learn basic algebra in elementary schools (Watanabe, 2008) and study Algebra I at seventh grade in the way of deductive reasoning. Since students at ages 10 or older are deductively able to solve mathematical problems as proven in the developmental deductive reasoning studies, American students should learn basic algebra before secondary school in order to be successful in learning Algebra I.

Conclusion

The usage of deductive reasoning in mathematics is very significant. About 50 years ago, deduction was the key to teaching mathematics. However, because students
had and still have a hard time transferring from the arithmetic concept to the symbolic logic, teaching abstract thinking has been a focal point of teaching mathematics, especially algebra. A generalized approach to teaching algebraic thinking is currently being emphasized in the mathematics curriculum – the implementation of inductive lessons. Inductive reasoning is also used in solving mathematical problems, but deduction may be more effective in improving students’ performance. Whether the mechanism of deductive reasoning depends on linguistic rules or mental representations is still being debated, but it is certain that children, ages 10 or older, are able to reason deductively. Learning from Japanese students’ success in mathematics, learning mathematics deductively should be made the focus of middle school classrooms, and learning basic algebra with deductive reasoning should be taught to students at an early age in school.
CHAPTER III

METHODOLOGY

In the present, traditional mathematics instruction in American classrooms applies little deductive reasoning. To examine if deductive reasoning training helps American seventh graders improve their mathematics performances, this study compared algebra performance of seventh grade students who received traditional instruction with those who received instruction through deductive reasoning.

Five regular seventh grade mathematics classes at a public middle school in Chico, California, Chico Junior High School (CJHS), participated in the study. All classes were taught by the same instructor, Mrs. Vallarie Jensen, who loves and sincerely cares about the students. The subjects were divided into control groups and experimental groups in every class period, which created five control groups and five experimental groups. The control groups were given traditional American mathematics teaching instruction while the experimental groups were provided deductive reasoning training in their classrooms.

Before the experiment, the students’ classrooms were observed on January 21 and 22, 2009. The purpose of this observation was to see what the students were learning at that time so that the research test questions could be adjusted to the students’ skills, was to make sure that the teaching styles in the classes were similar to the
traditional American styles (previously mentioned in Chapter I and II), and was to check the students’ performances in the classes.

Data used for statistical tests were collected from a pre-test and a post-test which the students responded on. A survey was created by the researcher, and its responses of the students were collected to explore if the students’ backgrounds, such as gender, education levels of their parents, ethnicity, and first language, influenced their attitudes toward learning mathematics. In order to compare differences in attitudes toward learning mathematics between American seventh graders and Japanese students, the CJHS seventh graders’ responses to six questions of the survey, which was created by the researcher, were compared to Japanese students’ responses to six questions of a survey which was conducted by National Institute for Educational Policy Research (NIER) of Japan (2008c, 2008d).

Pre-Experimental Observation

Mrs. Jensen was teaching the students fractions on January 21 and exponents, multiplying powers, and dividing powers on January 22. Because many students at CJHS did not understand basic math, such as fractions, divisions, multiplications, etc., Mrs. Jensen had been reviewing the basic math rather than teaching more advanced topics. Mrs. Jensen’s teaching style was similar to the traditional American mathematics instruction described in Chapter I. Checking homework answers was observed at the beginning of the classes. She read the answers aloud and the students corrected or checked their answers on the worksheets assigned the day(s) before. This
was followed by having students work on new worksheet(s)/homework or teaching a new math procedure.

The routine math procedures were given to the students as worksheets, and they individually worked on the worksheets. The math drills/worksheets were created by Mrs. Jensen in order to have the students practice solving equations (see Appendix A). If the students could not finish all drill problems in class, the rest became their homework. According to Ms. Jensen, the students were assigned those worksheets almost every day.

When a new mathematics concept was introduced, Mrs. Jensen recommended that the students should write out what it was, how it was used, and what could be known by applying the procedure to math problems on their own sheets called *frames* (see Appendix B). *Frames* are original teaching instrument of Mrs. Jensen’s, which was used in all the CJHS mathematics classrooms. The students were allowed to look at those *frames* which they filled in while they solved the similar mathematics problems in class, at home, and even during exams.

After introducing a new mathematics concept, Mrs. Jensen worked sample mathematics problems with the students while guiding them with yes/no or right/wrong questions. Also, the call-and-response style was frequently observed in her classrooms. For example, one of the sample questions was that there were two fractions and the students were asked to find out if they were equivalent.

\[
\frac{47}{52} \ ? \frac{33}{38}
\]

Once the students learned cross multiplication, they were asked to work on the other fraction equivalency questions. Then, Mrs. Jensen said, “We do this by ……?” The
students responded, “Cross-multiply!” Also, it was seen that the students were raising their hands to answer the teacher’s questions. If students’ responses were correct, they were rewarded with candy.

Regarding the students’ performances in the classes, they were, overall, very motivated to verbally respond to the questions during the lectures and to work on the drills on their own when they were told to do so. The students’ responses were given at an impressively fast-pace, but almost all students were able to keep up with that pace. Different math skill levels were seen in each class period and there were noticeable differences among the individual students of each period in their math abilities.

Additional Differences in American and Japanese Seventh Grade Classes

During the observation and the experiment at CJHS, in addition to the differences in seventh grade classrooms between the United States and Japan noted in the research, (previously explained in Chapter I and II), several more significant differences were noticed, such as class length, amount of weekly assigned homework, daily class schedule, daily time spent at school, number of distractions during class, and tardiness of students.

The CJHS students seem to spend more time in class than Japanese middle school students do. The CJHS mathematics class is held for 51 minutes every Monday through Friday except for Wednesday when the class is only 45 minutes long. There are a few exceptions to this. The CJHS first period usually takes time to listen to the speaker-generated announcements and the pledge of allegiance totally about 5 minutes at the beginning of class. The CJHS fourth period (supposedly from 11:44 A.M. to 12:29 P.M.)
seventh graders are allowed to leave class about 5 to 10 minutes early to go to lunch in order for them to avoid crowds at the food court.

Unlike the CJHS class length, in Japanese middle schools, mathematics class is usually 50 minutes long and is held three or four days a week. As a result, the CJHS students spent a total of 249 minutes in class for a week (about 224 minutes for the first period and about 199 to 224 minutes the fourth period), and this is greater than what the Japanese students spent in class for a week, which is a total of between 150 minutes and 200 minutes.

Additionally, the CJHS students may have more chances to come to class late than Japanese students do. A few students who showed up late were seen during the observation and the experiment, a total of seven days. The CJHS students seemed to rush because they have only a five-minute break to move from one classroom to another. Some students may not be able to make it to class on time because of the short break. Also, because the students move from one classroom to another within such a short break, it is almost impossible for students to review new math procedures during break.

On the contrary, in Japanese schools, students stay in one classroom every school day. There are ten-minute breaks, and these breaks are not used for students to move from one classroom to another, but for teachers to do so. Lunch is generally delivered to classrooms. Accordingly, Japanese students tend to be in class on time. Also, some Japanese students use breaks to review new math procedures which they just learned, discussing them with their classmates. Therefore, Japanese students may take more time to review what they learned than the CJHS students.
While the CJHS students spent more time in class and were less likely to review what they had learned after class than Japanese students, the CJHS students are more likely to have more time to do homework at home than Japanese students do. The CJHS students have six periods a day till 2:46 P.M. After the last period, many students, except for those who sign up and stay for after-school programs, usually go home. Mrs. Jensen stays to teach an extra mathematics class lasting 51 minutes to students who have signed up for the class.

Compared to this, most Japanese junior high school students tend to regularly stay at school until about 5:00 or 6:00 P.M. In Japanese public middle schools, there are generally six or seven periods a day. Usually, the first period starts at 8:00 A.M. and the last period ends at 3:00 or 4:00 P.M. After all the periods, Japanese students must clean up their school facilities, including their classrooms, until about 5:00 P.M. and are required to attend their club activities until about 6:00 P.M. if they choose to be in the activities.

The other difference between the CJHS and Japanese middle schools is the amount of homework assigned for a week. The CJHS students in the math classes tend to have assignments almost every math class. When the observation was done, the CJHS students got worksheets #14 and #15 assigned on January 21, 2009 and worksheets #16 and #17 assigned on January 22, 2009. By January 23, 2009, the students were supposed to complete a total of 17 worksheets.

Unlike the CJHS students, students at Japanese junior high schools rarely have homework assigned from every math class. Probably, it may be given to Japanese students every other week or less often.
The CJHS students often got distracted during the lecture or while working on the math practices. For example, some messenger students came to the class and handed in slips of papers to Mrs. Jensen and she stopped her lecture. After that, a student or students in the math class were called to the principal’s office or were notified of their library book due date. The students who were called to the principle’s office left the classroom in the middle of their work. The students who were told the deadline of their library book stopped their work.

Not only messenger students but also other students from another classroom or teachers may come to the class. For example, there was the attendance competition against the eighth graders where they were assigned to wear the headbands on the first day of the experiment and flowers on the second day of the experiment at CJHS. In order to count how many students in the class were wearing headbands or flowers, several counting students came to the math class and stopped Mrs. Jensen from lecturing and the students from solving math equations.

There were also other factors possibly causing the students’ distraction. Rewarding candy sometimes happened while the students were solving the math equations. When the students chose a piece of candy from a basket, the students stopped their work on the math problems.

In Japan, junior high school students are less likely to have the distractions which CJHS students have during class. There are no messenger students in Japanese schools. Basically, in Japan, it is considered to be rude if students move from one classroom to another for any reason during class. For an emergency call, the announcement would be made through speakers or an emergent message would be
brought by teachers to a particular student at the beginning of the class, but not during the class. There are generally no attendance competitions at junior high schools, but the teachers in each class may check on students’ attendance, instead of students from other classrooms counting the number of students in the classroom.

Also, Japanese students are not allowed to chew gum or eat candy and snacks during school. Candy would never be provided to students during class. Even if a teacher gives candy to students, it would not be provided until the end of class or after all students’ work is done.

The Research Design

Two types of research designs were utilized. A quantitative experimental research with a single-subject design was conducted to examine whether deductive reasoning practiced in seventh grade classrooms could improve students’ performance in algebra. Also, a cross-sectional survey was designed to investigate what factors may affect their math performance.

For the quantitative experimental research, a control group was taught through traditional American styles while an experimental group was provided Japanese learning styles with deductive reasoning practices. The pre-test and the post-test were given to both groups before and after the lessons. The differences of the pre-test and post-test scores were compared between a control and an experimental group and between boys and girls in each group.
The multiple-choice survey was conducted with 18 questions. This was designed to find relationships among the students’ attitudes toward mathematics and their gender differences, ethnic backgrounds, and parents’ education levels.

The CJHS survey used the six questions on NIER (2008c, 2008d), which were translated into English by the researcher. The questions are Q6, Q7, Q10, Q12, Q14, and Q13 on the CJHS survey. The responses of those questions on the CJHS survey were compared with the responses of Japanese sixth graders and ninth graders on the NIER (2008c, 2008d) survey. The comparisons were to analyze the cultural differences between the United States and Japan in terms of students’ ways of and attitudes toward learning math and to discuss if the deductive reasoning practices and the deductive way of learning math were applicable to American students. However, the CJHS survey scale included neutral response choices, maybe and sometimes which were not available to the Japanese survey. The NIER (2008c, 2008d) survey included a response choice, other, instead of the neutral response choices.

Japanese seventh graders did not attend the NIER’s (2008c, 2008d) survey research, so sixth graders would be the closest match to compare the survey results. However, algebra is a critical subject to learn during Japanese middle school, which consists of three-year programs, so the survey responses of the ninth graders as algebra learners as well as the Japanese sixth graders would be essential to be compared with the CJHS seventh graders’ survey responses.
Participants

The mathematics instructor and 141 seventh graders (63 boys and 78 girls) at CJHS in Chico, California volunteered to participate. Because the students in each class period showed different levels of math skills, to mix students with various math skill levels in the two research groups, the participants were divided in half in each period. One half was the control group and the other half was the experimental group.

The participants were randomly selected for these groups in each period. Numbers written on the cards were sequentially placed on the left top corner of each student’s desk. The students who sat on the even-numbered desks were chosen as the control group. The students who sat on the odd-numbered desks were selected as the experimental group.

The control group consisted of a total of 65 students (30 boys, 35 girls): 13 (5 boys and 8 girls) from the first period, 15 (6 boys and 9 girls) from the second period, 9 (6 boys and 3 girls) from the third period, 12 (4 boys and 8 girls) from the fourth period, and 16 (9 boys and 7 girls) from the fifth period.

The experimental group had a total of 76 students (33 boys, 43 girls): 15 (5 boys and 10 girls) from the first period, 15 (7 boys and 8 girls) from the second period, 13 (4 boys and 9 girls) from the third period, 16 (11 boys and 5 girls) from the fourth period, and 17 (6 boys and 11 girls) from the fifth period.

Eight students from the control group and four students from the experimental group were absent on the lecture days.

The survey was taken by 137 students (64 boys and 73 girls). The demographic of the student body was 46% white, 32.1% Mexican/Hispanic/Latino,
14.6% Asian/Pacific Islander, 3.6% Black/African American, and 3.6% Alaska Native/American Indian. In addition, the native English speakers were 65% and the English learners were 35%.

All participants were provided candy for their participation, encouragement, and reward for their work.

Instrument

The instruments including the two tests, an assignment, sample algebraic word problems used in class, and survey were created by the researcher. The algebraic word problems on the tests, the assignment, and the sample word problems used in the experiment were created referring to several word problems from a CJHS seventh graders’ textbook, *Mathematics: Concepts and Skills*, the National Institute for Educational Policy Research of Japan (2008a, 2008b), and Japanese websites (e.g., “Chugaku sugaku,” n.d.; “Chuaku eigo sugaku,” n.d.), and an article, *Development of Algebraic Reasoning in Children and Adolescents: A Cross-Cultural and Cross-Curricular Perspective* (Morris & Sloutsky, 1997, p.5).

The tests and the assignment contained five algebraic word problems each, and there were three to five sentences in each problem. To examine the students’ solution methods and thought processes, a performance-based assessment was taken and each algebraic word problem contained plenty of space for the participants to write out their math-solving processes. The same format was used for an assignment so that the students could practice word problems at home or in class.
The pre-test and post-test included exactly the same questions except for the numbers on the math questions, a few letters which were used as unknown variables, and the order of the word problems (see Appendix C). Also, the sample algebraic word problems (see Appendix D) and homework (see Appendix E) which were totally different from the two tests were provided for the students. However, these problems were associated with deductive reasoning practices just like the two tests. The students’ desk numbers were written on the two tests and the assignment.

The word problems were chosen as an instrumental material to test the students because semantic mathematics problems are fundamentally utilized in Japanese mathematics classrooms. In fact, word problems including real life situations allow learners to easily apply math concepts. Also, linguistic comprehension and deductive reasoning are interconnected as mentioned earlier. Therefore, using word problems is significant to help the students experience Japanese learning styles in math class.

The survey included five background questions and thirteen attitude-toward-math-related questions (see Appendix F). The students were not required to write their desk numbers on the survey.

Treatment

On the lecture day, the deductive reasoning practices were taught to the experimental group. The Japanese learning styles used for this experiment were to have the students deductively work on the math problems on their own, with the instructor, or with classmates and to have the students share their various creative solution methods for a math problem and to take time to review the materials with their classmates.
The students were repeatedly encouraged by the instructor to find their own solution methods and were required to write out the various ways of solving the algebraic problems on the white board or to explain the solutions without writing them out on the white board in class. This was to teach the students to creatively find the answers. The students were asked to be in a group to solve algebraic word problems, which were prepared as homework, and to help each other in a group, guided to share their own solution methods with their group members. The assignment answers without their solution methods were given to the students to have them experience deducing the ways to solve the algebraic problems from looking at the answers. Most of the students completed their assignment within about 20 minutes in class.

On the other hand, the students in the control group took the American traditional lessons which gave them in class explanations, procedure practices, and guided practices. The students individually worked on the sample algebraic word problems in class. They were not asked to work on the assignment problems in class and were not grouped to solve the sample word problems which Mrs. Jensen presented in class. Instead, the assignment without its answers was given right after the pre-test on the first experiment day. Technically, the control group students had almost one day to work on the assignment before having to turn it in, which is more time than the experimental group students spent on completing the assignment in class. Candy was provided to all the students every experiment day.
The Experimental Procedure

This study took place for five days: May 21-27 and June 1 of 2009. The lectures and the students’ performances were videotaped for these five days. The math instructor was given the brief lecture instructions in advance and the additional or changed instructions nearly 10 minutes before the beginning of every lecture day.

On the first experiment day, all the students took the pre-test. Before the test, in order for the students to be able to translate the word problems to math languages such as add and subtract or symbols such as, +, ÷, ×, etc. Mrs. Jensen gave all the students a lecture about words which could mean addition, subtraction, multiplication, and division. For example, *sum*, *plus*, *all together*, *more than*, *increased by*, and so on can be found instead of the word, *add*, or the symbol, +. The students were asked to write out the word differences on their *frames* so they would be able to interpret what the word problems were asking them to do by looking at their frame. Before the students left the class, the assignment without its answers was handed to the control group students.

On the second day, the experimental group remained in the class and was given a Japanese-style lesson with deductive reasoning practices while the control group students were asked to turn in their assignments at the beginning of the class, and then were asked to leave class to join the PE class students.

For the experimental group, a lecture was provided, presenting a few sample word problems. Then, Mrs. Jensen wrote how to solve those problems on the white board, explaining the solutions to the students. Meanwhile, the students were frequently asked if they had different solution methods for each problem. If the students solved the problems differently, they were encouraged to share their own way of solving the
problems, coming up to the front and writing their own solution methods on the whiteboard.

After the lecture, the students were placed in groups of four or five. An assignment with its answers was provided, but its solution methods were not. All of the problems on the assignment were solved by the groups in class. This was to create time for the students to discuss the materials and to review the lecture with their classmates like some Japanese students do during their break time. Being in a group, the students were helping each other solve the problems and were sharing their own solution methods with the group members. After all the work was done, one student from each group presented how the group solved the algebraic word problems on the whiteboard and explained their solution methods. Several students who did not show their work on their assignment sheet during class were required to complete their work at home and bring it back to the next class.

On the third day, the experimental group left the classroom and went to the library. The control group remained in the classroom and experienced the American traditional learning styles. At the beginning of the class, their assignment sheets which were graded by the researcher were passed back to them while all the students who turned in their assignment were given a piece of candy. The students who scored higher than average were rewarded one more piece of candy in order to motivate all the students to work harder in class. Since the control group had had a day off the day before, the assignment was reviewed to freshen their memory. Five students were asked to show their work on the whiteboard and describe their solution methods. While listening to the students who were presenting their solution methods, the math teacher corrected their
errors as needed. This review was to begin by checking homework answers as usual. After reviewing all the assignment problems, the new math problems (see Appendix D) were practiced during class. They had no additional assignment. At the end of class, the post-test was provided. It was only five to 10 minutes before the end of class, so most students could not complete the post-test.

On the fourth day, the two groups were swapped again. The experimental group remained in the classroom and the control group went to the library. Because there were three holidays and a weekend before that day, it was necessary for the students to review the information. As for the control group, the assignment, which was graded by the researcher, was passed back to the students and the students who turned in the assignment were given a piece of candy. The students who scored higher than average were also rewarded with one more piece of candy. Several students were asked to show their solution methods on the white board and to explain them. While the presentations were going on, the students were frequently asked to share different ways of solving the assignment word problems. After all the problems of the assignment were reviewed, a few new math word problems were practiced in class. After that, the post-test was assessed. It was given only 10 or 15 minutes, so most students could not solve all the post-test problems within that time.

On the last day, all students remained in the classroom, and they took a survey and solved the rest of the post-test problems.
Data Collection Procedures

The pre-test was administered to the control and the experimental group under the same circumstances. It was the very first day of the experiment. All the students remained in the classroom and took the test after the brief lecture to explain how to interpret word problems. During the pre-test, some students who discussed with their classmates how to solve the problems were observed, but they were not told to refrain from talking with their classmates. There were a few students who asked the instructor questions about the word problem questions and the instructor answered their questions. The researcher did not interfere with the students.

The post-test was assessed under a similar circumstance for both the control and the experimental groups. For the control group, the post-test was given to the students right after the lecture, but the time for the post-test varied in each period as follows: the first, second, and third periods had about 15 minutes for the test, the fourth period had about 20 minutes, and the fifth period had about 5 minutes.

For the experimental group, the post-test was administered right after the lecture and the time given for the post-test was as follows: the first period did not have time to do the post-test at all, the second period had about 15 minutes for the post-test, the third period had about 20 minutes, the fourth period had 10 minutes, and the fifth period had 15 minutes.

On the last experiment day, the post-test sheets were returned to both the control and experimental group students and all the students worked on the rest of the post-test questions together. There seemed to be few students who talked with their classmates to find out how to solve the post-test problems, but many students did not
even seem to try to solve the problems. The researcher still did not interfere with the students.

All the tests and the assignment were scored by the researcher. The tests were graded twice. The tests were first graded without the grading procedures and the students who made an effort to answer earned extra points. However, the average of the results was pretty high and did not indicate that there were different performances between the control and the experimental group. After all, the tests were graded again in order to fit the purposes of this research, using the grading procedures. The tests were graded based on whether the following five categories were found or not: deductive reasoning, correct answer, correct use of variables, correct equation, and equation solved correctly (see Appendix G). Each of the five problems on the test was evaluated according to each category, and each category was worth four points. Each problem on the test was worth 20 points. The total score for the test was 100 points.

If a student concluded a specific answer by trying out different numbers, this was considered as an example of inductive reasoning as explained in Chapter II, and four points were deducted out of 20 points in each problem. The reason why this category was included in the grading procedures is that this test was to see if the students gained deductive reasoning skills. In fact, it was recognized that many students inductively solved specifically one of the five word problems which had to be solved deductively (see Question (5) on the post-test on Appendix H). Many students correctly figured out the correct equations and tried inserting a number to induce their answers, or most of the students simply tried inserting several numbers in the equation set up in their minds and induced their answers from these trials (see Question (5) on the post-test on Appendix I).
There were few students who deduced their answers on question (5) on the post-test (see Appendix J).

Data Analysis Procedures

The total scores of each student were collected for each period. Using SPSS, the analysis was made by comparing the pre-test score mean with the post-test score mean between or among the following groups: the control and the experimental groups and gender. A paired two-tailed $t$ test was used to examine if the experimental group significantly improved their test scores from the pre-test to the post-test. An independent $t$ test was used to see whether there was a significant difference on the post-test scores between the control group and the experimental group. Another independent $t$ test was used to determine if gender differences significantly affected the test scores. A two-way ANOVA was used to analyze whether the group difference and/or gender difference had a significant main or interaction effect between the pre-test and the post-test. The survey responses were converted into quantitative analysis.

The students who were absent during the experiment were eliminated from the entire data because lack of their performances would be invalid. Among the students who did participate all the seven experiment days, there were many students who scored lower on the post-test than on the pre-test. Because the post-test was held during the last week of the semester, the students seemed to be less motivated to solve the word problems on that day than on the first day of the experiment. For example, three students in the control group got 18, 24, and 54 on the pre-test, but scored zero on the post-test. Five students in the experimental group scored 4, 8, 10, 28, and 32 on the pre-test, but got zero on the
post-test. Because the same math problems for both of the tests were given to the students, their performance on the post-test was expected to be relatively similar to or better than the ones on the pre-test. Accordingly, the students who lacked effort on the post-test compared to the pre-test should be excluded because their results are meaningless. The result of a paired \( t \) test result with data excluding these less motivated students was compared with the result of a paired \( t \) test with the data including all the students.

For elimination of less motivated students, a criterion was arranged. Students who scored 20 points or more less on the post-test than on the pre-test were not included in data analysis. A 20 point minimum decrease was set as an acceptable/non-acceptable threshold because one problem was worth 20 points. If students worked on one problem less on the post-test than on the pre-test, it can be said that they did not even try to solve or did not want to work on the problem for the post-test although they did solve or did work on the problem for the pre-test. Based on this criterion, a total of 21 students from the control group and a total of 16 students from the experimental group were eliminated from the analysis in addition to excluding the absent students.

The survey responses to Q6, Q7, Q10, Q12, Q14, and Q13 were comparable to the Japanese survey responses which were given to sixth graders and ninth graders by NIER (2008c, 2008d), so the comparison analysis between the American seventh graders and the Japanese sixth and ninth graders was made.
CHAPTER IV

RESULTS AND DISCUSSION

Six hypotheses were analyzed: 1) Deductive reasoning training would significantly improve algebra performances of seventh graders; 2) Learning mathematics deductively, which is the Japanese way of learning mathematics, would significantly work better than the American teaching technique; 3) Gender differences would not influence Hypothesis 1 and Hypothesis 2; 4) The students’ backgrounds including gender, their parents’ education levels, ethnicity, and native language would significantly associate with their ways of and attitudes toward learning mathematics; 5) Japanese students would have more positive attitudes toward learning mathematics than American students; and, 6) Types of attitudes toward learning mathematics that American seventh graders have can be defined. The findings for the hypotheses are as follows.

Presentation of the Findings

The data analyzed for this research excluded students who were absent for part of the intervention or testing and those who showed low motivation as explained in Chapter III, Methodology. The reason why this data was used is that a result of the control group including all the students did not make sense to analyze. The control group including all the students statistically significantly decreased their test scores from the pre-test to the post-test even though the two tests, except for the numbers, the variables
used, and the order of the problems, were identical. Table 1 shows means and standard deviations for the pre-test and the post-test scores for all the students and for the groups with excluded students’ scores removed.

Table 1

*Means and Standard Deviation*

<table>
<thead>
<tr>
<th>Test</th>
<th>Mean</th>
<th>Standard deviation</th>
<th>Sample size</th>
</tr>
</thead>
</table>
| Including All the Students
| The Control Group
| Pre       | 40.42  | 16.28              | 65          |
| Post       | 34.82  | 20.98              | 65          |
| The Experimental Group
| Pre       | 38.22  | 18.76              | 74          |
| Post       | 38.04  | 24.23              | 74          |
| Excluding the Absent and the Low Motivation Students
| The Control Group
| Pre       | 39.32  | 17.23              | 44          |
| Post       | 41.86  | 17.53              | 44          |
| The Experimental Group
| Pre       | 39.90  | 18.50              | 58          |
| Post       | 45.57  | 21.51              | 58          |
Comparison of scores between the control group and the experimental group with absent and low motivation students’ scores eliminated showed that there was not a statistically significant difference when tested with an independent $t$ test for the pre-test, $t(100) = -.16$, $p = .87$, thereby indicating that the control and experimental groups were similar at the beginning of the experiment.

Findings Related to Hypothesis 1

Does deductive reasoning training improve algebra performance of the seventh graders? To answer this question, the pre-test and the post-test scores of the experimental group and of the control group were compared. A paired $t$ test comparing the pre-test and the post-test scores of the experimental group revealed, $t(58) = -2.92$, $p = .01$ (two-tailed), and of the control group revealed, $t(44) = 17.23$, $p = .27$ (two-tailed).

The seventh graders who learned algebra through deductive reasoning significantly improved their scores on the post-test, however those who did not learn algebra deductively did not significantly improve their scores (see Table 1).

The paired $t$ test also showed that the pre-test and the post-test were reliable tests ($p = .00$). Therefore, the students who scored high on the pre-test were very likely to score high on the post-test and the students who scored lower on the pre-test were very likely to score lower on the post-test. Regarding its correlation, the experimental group ($r = .62$) had a higher positive correlation than the control group ($r = .74$).

Findings Related to Hypothesis 2

Is the deductive reasoning training more effective than the traditional American teaching technique? This was answered by a comparison of the post-test scores between the control group and the experimental groups. The independent $t$ test indicated
that there were no statistical significant differences between the two groups on the post-test, $t(100) = -.93, p = .35$ (two-tailed). Therefore, it did not prove that the deductive reasoning training worked better to improve algebra performances of the seventh graders than the American traditional learning style.

Findings Related to Hypothesis 3

Does gender difference affect Hypothesis 1? This question was asked to determine if the gender difference can be found in improving algebra performances by the seventh graders when learning algebra deductively. The performance differences between genders were compared by an independent $t$ test to answer this question.

According to an independent $t$ test, there was no significant difference between the boys and the girls in the control group on the pre-test, $t(43) = -1.97, p = .06$ (two-tailed), but there was on the post-test, $t(43) = -3.14, p = .00$ (two-tailed) (see Table 2). Unlike the control group, the experimental group did not show significant performance differences between the genders on both the pre-test, $t(57) = -.16, p = .88$ (two-tailed), and the post-test, $t(58) = .37, p = .71$ (two-tailed) (see Table 2).

Also, to make sure the independent $t$ test for the gender comparison was appropriately analyzed for a combination of the gender and the groups such as the control and the experimental groups, the two-way ANOVA was tested. According to the two-way ANOVA, there was no significant interaction effect between the groups and the genders on the pre-test (see Table 3) although its chart looked like there were different mean scores (see Figure 1). The performance differences on the pre-test between the genders were the same for the control and the experimental groups, and the performance differences on the pre-test between the two groups were the same for each gender. Also,
Table 2

*Independent T Test Statistics for the Gender Differences*

<table>
<thead>
<tr>
<th>Test</th>
<th>Gender</th>
<th>Mean</th>
<th>Standard deviation</th>
<th>Sample size</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>The Control Group</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pre</td>
<td>Boys</td>
<td>33.81</td>
<td>16.00</td>
<td>21</td>
</tr>
<tr>
<td></td>
<td>Girls</td>
<td>43.58</td>
<td>17.13</td>
<td>24</td>
</tr>
<tr>
<td>Post</td>
<td>Boys</td>
<td>32.95</td>
<td>15.87</td>
<td>21</td>
</tr>
<tr>
<td></td>
<td>Girls</td>
<td>48.33</td>
<td>16.84</td>
<td>24</td>
</tr>
<tr>
<td></td>
<td>The Experimental Group</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pre</td>
<td>Boys</td>
<td>39.04</td>
<td>17.90</td>
<td>27</td>
</tr>
<tr>
<td></td>
<td>Girls</td>
<td>39.81</td>
<td>19.53</td>
<td>32</td>
</tr>
<tr>
<td>Post</td>
<td>Boys</td>
<td>45.93</td>
<td>18.73</td>
<td>28</td>
</tr>
<tr>
<td></td>
<td>Girls</td>
<td>43.84</td>
<td>24.02</td>
<td>32</td>
</tr>
</tbody>
</table>

there was no significant main effect on the gender difference and no significant main effect between the two groups on the post-test (see Table 3). These results supported the result of the independent t test for the gender difference that the gender difference did not affect algebra performances on the pre-test of the control and the experimental groups.

However, on the post-test, there was a significant interaction effect between the groups and the genders (see Table 4 and Figure 2). The performance differences on the post-test between the genders were not the same for the control group and the experimental group. Likewise, the performance differences on the post-test between the
Table 3

Analysis of Variance for the Pre-Test

<table>
<thead>
<tr>
<th>Source</th>
<th>df</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Between subjects</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Group (Gr)</td>
<td>1</td>
<td>.04</td>
<td>.84</td>
</tr>
<tr>
<td>Gender (Ge)</td>
<td>1</td>
<td>2.21</td>
<td>.14</td>
</tr>
<tr>
<td>Gr X Ge</td>
<td>1</td>
<td>1.60</td>
<td>.21</td>
</tr>
<tr>
<td><strong>S within-group</strong></td>
<td>100</td>
<td></td>
<td></td>
</tr>
<tr>
<td>error</td>
<td></td>
<td>(320.37)</td>
<td></td>
</tr>
</tbody>
</table>

Note. Values enclosed in parentheses represent mean square errors. S = subjects. *p<.05. **p<.01.

two groups were not the same for each gender. There was no significant main effect on the group differences and no significant main effect on the gender differences (see Table 4). The result of the interaction effect supported the independent t test for the gender difference that there were significant performance differences on the post-test based on the gender difference.

Does the gender difference affect Hypothesis 2? This question was to examine if regardless of the gender difference, deductive reasoning training was more effective to improve algebra performances of the seventh graders than the American learning style. The boys in the control group were compared with those in the experimental group, and so were the girls in the two groups. The independent t test for a comparison of the boys between the control group and the experimental group illustrated that their performances
on the pre-test were not significantly different, $t(46) = -1.05, p = .30$ (two-tailed), but their performances on the post-test were significantly different, $t(47) = -2.56, p = .01$ (two-tailed) (see Table 2). There were no significant differences between the girls in the two groups on both the pre-test, $t(54) = .75, p = .46$ (two-tailed) and the post-test, $t(54) = .82, p = .42$ (two-tailed) (see Table 2).

Findings Related to Hypothesis 4

Do the seventh graders’ backgrounds such as gender, their parents’ education levels, ethnicity, and first language significantly associate with their attitudes toward learning mathematics? For this question, the survey responses were analyzed. The chi-
Table 4

*Analysis of Variance for the Post-Test*

<table>
<thead>
<tr>
<th>Source</th>
<th>df</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between subjects</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Group (Gr)</td>
<td>1</td>
<td>1.20</td>
<td>.28</td>
</tr>
<tr>
<td>Gender (Ge)</td>
<td>1</td>
<td>2.94</td>
<td>.09</td>
</tr>
<tr>
<td>Gr X Ge</td>
<td>1</td>
<td>5.67*</td>
<td>.03</td>
</tr>
<tr>
<td>S within-group</td>
<td>101</td>
<td>(385.29)</td>
<td></td>
</tr>
</tbody>
</table>

Note. Values enclosed in parentheses represent mean square errors. S = subjects. *p<.05. **p<.01.

square results for the associations between the survey questions and the students’ parents’ education levels are displayed in Table 5 and in Table 6. There was a significant relationship between Q12 and both the fathers’ and the mothers’ education levels.

From the contingency table for the chi-square test, there were discrepancies between the observed count and the expected count in the survey answer categories in Q12, always, almost always, and sometimes. The higher the scholastic degrees the fathers held, the more the number of students relatively reported always. The lower the scholastic degrees the fathers held, the more the number of students moderately answered sometimes. Thus, more of the fathers with the higher degree than the fathers with the lower degree influenced their children to work mathematics problems/homework until they were solved. This trend was same with the mothers’ education levels.
Table 7 illustrates that associations between the survey questions and the genders. The gender differences significantly affected the students’ responses to Q6. The survey answers to Q6 showed that the observed count exceeded the expected count in *strongly agree* for the boys and in *maybe* for the girls. Therefore, the boys liked mathematics more than the girls.

According to the chi-square test results for the ethnicity association with the survey questions (see Table 8), there were significant relationships between Q7, Q16, and Q18 and the ethnicity differences.

*Figure 2.* The two-way ANOVA with two factors (group X gender) on post-test.
Table 5

*Associations Between the Survey Questions and the CJHS Students’ Fathers’ Education Levels*

<table>
<thead>
<tr>
<th>Questions</th>
<th>( \chi^2 )</th>
<th>df</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>6) I like mathematics.</td>
<td>19.36</td>
<td>20</td>
<td>.50</td>
</tr>
<tr>
<td>7) Learning mathematics is important.</td>
<td>20.30</td>
<td>15</td>
<td>.16</td>
</tr>
<tr>
<td>8) I understand mathematics frames that I wrote in class.</td>
<td>15.23</td>
<td>15</td>
<td>.44</td>
</tr>
<tr>
<td>9) Mathematics is one of my favorite subjects.</td>
<td>26.41</td>
<td>20</td>
<td>.15</td>
</tr>
<tr>
<td>10) What kind of mathematics do you use outside of class?</td>
<td>9.50</td>
<td>20</td>
<td>.98</td>
</tr>
<tr>
<td>11) I try to think how to use mathematics in everyday life.</td>
<td>25.09</td>
<td>25</td>
<td>.46</td>
</tr>
<tr>
<td>12) I try to work mathematics problems/homework until they are solved.</td>
<td>29.09**</td>
<td>20</td>
<td>.00</td>
</tr>
<tr>
<td>13) I make a note for mathematics problem-solving on my frame sheets or</td>
<td>18.44</td>
<td>25</td>
<td>.82</td>
</tr>
<tr>
<td>bindernotes so that I can look back and see how to solve similar</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>mathematics problems in the future.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>14) I try to figure out an easier way to solve mathematics problems.</td>
<td>20.79</td>
<td>25</td>
<td>.70</td>
</tr>
<tr>
<td>15) I set a specific time to study mathematics at home.</td>
<td>19.17</td>
<td>25</td>
<td>.79</td>
</tr>
<tr>
<td>16) I read the mathematics textbook before coming to class.</td>
<td>13.78</td>
<td>20</td>
<td>.84</td>
</tr>
<tr>
<td>17) I review frames before beginning mathematics homework.</td>
<td>16.32</td>
<td>25</td>
<td>.91</td>
</tr>
<tr>
<td>18) After getting mathematics quizzes/tests back, I review mistakes that</td>
<td>20.34</td>
<td>25</td>
<td>.73</td>
</tr>
<tr>
<td>I made, for example, misunderstanding mathematics questions,</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>calculation errors, or careless errors.</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Note.* *p*<.05. ***p*<.01.
<table>
<thead>
<tr>
<th>Questions</th>
<th>$\chi^2$</th>
<th>df</th>
<th>$p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>6) I like mathematics.</td>
<td>11.17</td>
<td>20</td>
<td>.94</td>
</tr>
<tr>
<td>7) Learning mathematics is important.</td>
<td>16.70</td>
<td>15</td>
<td>.34</td>
</tr>
<tr>
<td>8) I understand mathematics frames that I wrote in class.</td>
<td>17.33</td>
<td>15</td>
<td>.30</td>
</tr>
<tr>
<td>9) Mathematics is one of my favorite subjects.</td>
<td>12.55</td>
<td>20</td>
<td>.90</td>
</tr>
<tr>
<td>10) What kind of mathematics do you use outside of class?</td>
<td>19.36</td>
<td>20</td>
<td>.55</td>
</tr>
<tr>
<td>11) I try to think how to use mathematics in everyday life.</td>
<td>19.74</td>
<td>25</td>
<td>.76</td>
</tr>
<tr>
<td>12) I try to work mathematics problems/homework until they are solved.</td>
<td>47.10**</td>
<td>20</td>
<td>.00</td>
</tr>
<tr>
<td>13) I make a note for mathematics problem-solving on my frame sheets or bindernotes so that I can look back and see how to solve similar mathematics problems in the future.</td>
<td>12.19</td>
<td>25</td>
<td>.99</td>
</tr>
<tr>
<td>14) I try to figure out an easier way to solve mathematics problems.</td>
<td>20.33</td>
<td>25</td>
<td>.73</td>
</tr>
<tr>
<td>15) I set a specific time to study mathematics at home.</td>
<td>23.12</td>
<td>25</td>
<td>.57</td>
</tr>
<tr>
<td>16) I read the mathematics textbook before coming to class.</td>
<td>13.10</td>
<td>20</td>
<td>.87</td>
</tr>
<tr>
<td>17) I review frames before beginning mathematics homework.</td>
<td>20.72</td>
<td>25</td>
<td>.71</td>
</tr>
<tr>
<td>18) After getting mathematics quizzes/tests back, I review mistakes that I made, for example, misunderstanding Mathematics questions, calculation errors, or careless errors.</td>
<td>17.13</td>
<td>25</td>
<td>.88</td>
</tr>
</tbody>
</table>

*Note. *$p$<.05. **$p$<.01.*
Table 7

*The Associations Between the Survey Questions and the CJHS Students’ Genders*

<table>
<thead>
<tr>
<th>Questions</th>
<th>$\chi^2$</th>
<th>df</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>6) I like mathematics.</td>
<td>9.96*</td>
<td>4</td>
<td>.04</td>
</tr>
<tr>
<td>7) Learning mathematics is important.</td>
<td>.99</td>
<td>3</td>
<td>.81</td>
</tr>
<tr>
<td>8) I understand mathematics frames that I wrote in class.</td>
<td>1.41</td>
<td>3</td>
<td>.70</td>
</tr>
<tr>
<td>9) Mathematics is one of my favorite subjects.</td>
<td>1.57</td>
<td>4</td>
<td>.81</td>
</tr>
<tr>
<td>10) What kind of mathematics do you use outside of class?</td>
<td>.80</td>
<td>4</td>
<td>.94</td>
</tr>
<tr>
<td>11) I try to think how to use mathematics in everyday life.</td>
<td>4.87</td>
<td>5</td>
<td>.43</td>
</tr>
<tr>
<td>12) I try to work mathematics problems/homework until they are solved.</td>
<td>6.80</td>
<td>4</td>
<td>.15</td>
</tr>
<tr>
<td>13) I make a note for mathematics problem-solving on my frame sheets or bindernotes so that I can look back and see how to solve similar mathematics problems in the future.</td>
<td>7.17</td>
<td>5</td>
<td>.21</td>
</tr>
<tr>
<td>14) I try to figure out an easier way to solve mathematics problems.</td>
<td>6.32</td>
<td>5</td>
<td>.28</td>
</tr>
<tr>
<td>15) I set a specific time to study mathematics at home.</td>
<td>8.66</td>
<td>5</td>
<td>.12</td>
</tr>
<tr>
<td>16) I read the mathematics textbook before coming to class.</td>
<td>8.62</td>
<td>4</td>
<td>.07</td>
</tr>
<tr>
<td>17) I review frames before beginning mathematics homework.</td>
<td>6.32</td>
<td>5</td>
<td>.28</td>
</tr>
<tr>
<td>18) After getting mathematics quizzes/tests back, I review mistakes that I made, for example, misunderstanding mathematics questions, calculation errors, or careless errors.</td>
<td>3.06</td>
<td>5</td>
<td>.69</td>
</tr>
</tbody>
</table>

*Note. *$p<.05$. **$p<.01$.*
Table 8

The Associations Between the Survey Questions and the CJHS Students’ Ethnicities

<table>
<thead>
<tr>
<th>Questions</th>
<th>$\chi^2$</th>
<th>df</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>6) I like mathematics.</td>
<td>15.05</td>
<td>16</td>
<td>.52</td>
</tr>
<tr>
<td>7) Learning mathematics is important.</td>
<td>28.14**</td>
<td>12</td>
<td>.01</td>
</tr>
<tr>
<td>8) I understand mathematics frames that I wrote in class.</td>
<td>10.16</td>
<td>12</td>
<td>.60</td>
</tr>
<tr>
<td>9) Mathematics is one of my favorite subjects.</td>
<td>21.99</td>
<td>16</td>
<td>.14</td>
</tr>
<tr>
<td>10) What kind of mathematics do you use outside of class?</td>
<td>25.89</td>
<td>16</td>
<td>.06</td>
</tr>
<tr>
<td>11) I try to think how to use mathematics in everyday life.</td>
<td>25.96</td>
<td>20</td>
<td>.17</td>
</tr>
<tr>
<td>12) I try to work mathematics problems/homework until they are solved.</td>
<td>22.00</td>
<td>16</td>
<td>.14</td>
</tr>
<tr>
<td>13) I make a note for mathematics problem-solving on my frame sheets or bindernotes so that I can look back and see how to solve similar mathematics problems in the future.</td>
<td>22.14</td>
<td>20</td>
<td>.33</td>
</tr>
<tr>
<td>14) I try to figure out an easier way to solve mathematics problems.</td>
<td>26.31</td>
<td>20</td>
<td>.16</td>
</tr>
<tr>
<td>15) I set a specific time to study mathematics at home.</td>
<td>24.45</td>
<td>20</td>
<td>.22</td>
</tr>
<tr>
<td>16) I read the mathematics textbook before coming to class.</td>
<td>29.99*</td>
<td>16</td>
<td>.02</td>
</tr>
<tr>
<td>17) I review frames before beginning mathematics homework.</td>
<td>26.45</td>
<td>20</td>
<td>.15</td>
</tr>
<tr>
<td>18) After getting mathematics quizzes/tests back, I review mistakes that I made, for example, misunderstanding mathematics questions, calculation errors, or careless errors.</td>
<td>31.11*</td>
<td>20</td>
<td>.05</td>
</tr>
</tbody>
</table>

Note. *$p<.05$. **$p<.01$. 

For Q7, the Black/African American students held a residual between the observed and the expected count with the survey answer category, *maybe*, and the Mexican/Hispanic/Latino students did so with *agree*. So, comparatively, it can be said that the Mexican/Hispanic/Latino students thought that learning mathematics was important more than the Black/African American students did.

Regarding Q16, the Asian/Pacific Islander students with *almost always*, *sometimes*, and *almost never*, and the Black/African American, the Mexican/Hispanic/Latino, and the White students with *never* were the categories where the observed count was greater than the expected count. So, this can be analyzed as the Asian/Pacific Islander students read the mathematics textbook before coming to class more often than the Black/African American, the Mexican/Hispanic/Latino, and the White students did.

The survey answer categories to Q18 which showed discrepancies between the observed count and the expected count were *always* for the Asian/Pacific Islander students and the Black/African American students, *almost always* for the White students, and *sometimes* for the Mexican/Hispanic/Latino students. The Asian/Pacific Islander students and the Black/African American students reviewed mistakes that they made after getting mathematics quizzes/tests back more often than the White students did, and the White students did so more than the Mexican/Hispanic/Latino students did.

Table 9 shows that the survey results of associations between the survey questions and what the students’ first language was. Whether the students’ first language is English or not significantly influenced the students’ answers to Q8, Q10, Q12, and Q14.
Table 9

The Associations between the Survey Questions and the CJHS Students’ First Language

<table>
<thead>
<tr>
<th>Questions</th>
<th>$\chi^2$</th>
<th>df</th>
<th>$p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>6) I like mathematics.</td>
<td>2.61</td>
<td>4</td>
<td>.63</td>
</tr>
<tr>
<td>7) Learning mathematics is important.</td>
<td>6.06</td>
<td>3</td>
<td>.11</td>
</tr>
<tr>
<td>8) I understand mathematics frames that I wrote in class.</td>
<td>8.53*</td>
<td>3</td>
<td>.04</td>
</tr>
<tr>
<td>9) Mathematics is one of my favorite subjects.</td>
<td>7.15</td>
<td>4</td>
<td>.13</td>
</tr>
<tr>
<td>10) What kind of mathematics do you use outside of class?</td>
<td>10.06*</td>
<td>4</td>
<td>.04</td>
</tr>
<tr>
<td>11) I try to think how to use mathematics in everyday life.</td>
<td>5.81</td>
<td>5</td>
<td>.33</td>
</tr>
<tr>
<td>12) I try to work mathematics problems/homework until they are solved.</td>
<td>13.71**</td>
<td>4</td>
<td>.01</td>
</tr>
<tr>
<td>13) I make a note for mathematics problem-solving on my frame sheets or bindernotes so that I can look back and see how to solve similar mathematics problems in the future.</td>
<td>3.00</td>
<td>5</td>
<td>.70</td>
</tr>
<tr>
<td>14) I try to figure out an easier way to solve mathematics problems.</td>
<td>14.24*</td>
<td>5</td>
<td>.01</td>
</tr>
<tr>
<td>15) I set a specific time to study mathematics at home.</td>
<td>7.93</td>
<td>5</td>
<td>.16</td>
</tr>
<tr>
<td>16) I read the mathematics textbook before coming to class.</td>
<td>4.62</td>
<td>4</td>
<td>.33</td>
</tr>
<tr>
<td>17) I review frames before beginning mathematics homework.</td>
<td>3.63</td>
<td>5</td>
<td>.61</td>
</tr>
<tr>
<td>18) After getting mathematics quizzes/tests back, I review mistakes that I made, for example, misunderstanding mathematics questions, calculation errors, or careless errors.</td>
<td>6.02</td>
<td>5</td>
<td>.30</td>
</tr>
</tbody>
</table>

Note. *$p$<.05. **$p$<.01.
In the survey answers to Q8, the categories where the observed frequencies exceeded the expected frequencies were *strongly agree* for the native English speakers and *maybe* for the English learners. The native English speakers understood mathematics frames that they wrote in class better than the English learners did.

For Q10, discrepancies between the observed frequencies and the expected frequencies were seen in the native English speakers using all the five kinds of mathematics and the English learners using only one kind of mathematics. So, the native English speakers used more addition, subtraction, multiplication, division, and measuring outside of class than the English learners did.

The contingency table for the chi-square test regarding Q12 and the students’ first language illustrated that the observed count exceeded the expected count in *always* and *almost always* for the native English speakers and in *sometimes* for the English learners. The native English speakers tried to work mathematics problems/homework until they were solved more often than the English learners did.

For Q14, the native English speakers reported *always* and *almost always* where the observed count was greater than the expected count while the English learners answered *sometimes*, *almost never*, and *never* where the observed count was greater than the expected count. Therefore, the native English speakers tried to figure out an easier way to solve mathematics problems more frequently than the English learners did.

**Findings Related to Hypothesis 5**

Do Japanese students learn mathematics with more positive attitudes than American seventh graders? To answer this question, comparisons between the CJHS seventh grade students’ survey response ratios of Q6, Q7, Q11, Q12, Q13, and Q14 and
the Japanese sixth and ninth graders’ were analyzed. The responses to Q6 showed that the ratio of the students who liked mathematics was similar among the three groups (49% of the CJHS students, 66% of the Japanese sixth graders, and 53% of the Japanese ninth graders) (see Table 10, Figure 3, 4, and 5).

Table 10

*Attitude Response Table for Q6*

<table>
<thead>
<tr>
<th>Group</th>
<th>Strongly Agree (SA)</th>
<th>Agree (A)</th>
<th>A Total of SA and A</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q6) I like math</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CJHS 7th Graders</td>
<td>11.7%</td>
<td>37.2%</td>
<td>48.9%</td>
</tr>
<tr>
<td>Japanese 6th Graders</td>
<td>36.3%</td>
<td>29.2%</td>
<td>65.5%</td>
</tr>
<tr>
<td>Japanese 9th Graders</td>
<td>26.1%</td>
<td>27.1%</td>
<td>53.2%</td>
</tr>
</tbody>
</table>

The ratios of the responses to Q7 indicated that most of the students in the three groups commonly thought that learning mathematics was important (96% of the CJHS students, 92% of the Japanese sixth graders, and 78% of the Japanese ninth graders) (see Table 11, Figure 6, 7, and 8).

According to the responses to Q11, CJHS students (26%) tried to think how to use mathematics in everyday life less than the Japanese students did (65% of the Japanese sixth graders and 35% of the Japanese ninth graders) (see Table 12, Figure 9, 10, and 11).

For Q12, a relatively similar portion of the students in the three groups answered that they tried to work mathematics problems/homework until they were solved
Figure 3. The ratio of the CJHS students’ responses to Q6.

Figure 4. The ratio of the Japanese sixth grade students’ responses to Q6.¹

¹ From the survey results which was conducted by NIER (2008c, 2008d).
Figure 5. The ratio of the Japanese ninth grade students’ responses to Q6.²

Table 11

Attitude Response Table for Q7

<table>
<thead>
<tr>
<th>Group</th>
<th>Strongly Agree (SA)</th>
<th>Agree (A)</th>
<th>A Total of SA and A</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q7) Learning math is important</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CJHS 7th Graders</td>
<td>54.0%</td>
<td>42.3%</td>
<td>96.4%</td>
</tr>
<tr>
<td>Japanese 6th Graders</td>
<td>69.1%</td>
<td>22.6%</td>
<td>91.7%</td>
</tr>
<tr>
<td>Japanese 9th Graders</td>
<td>42.1%</td>
<td>36.0%</td>
<td>78.1%</td>
</tr>
</tbody>
</table>

² From the survey results which was conducted by NIER (2008c, 2008d).
Figure 6. The ratio of the CJHS students’ responses to Q7.

Figure 7. The ratio of the Japanese sixth grade students’ responses to Q7.\(^3\)

\(^3\) From the survey results which was conducted by NIER (2008c, 2008d).
Figure 8. The ratio of the Japanese ninth grade students’ responses to Q7.4.

Table 12

*Attitude Response Table for Q11*

<table>
<thead>
<tr>
<th>Group</th>
<th>Strongly Agree (SA)</th>
<th>Agree (A)</th>
<th>A Total of SA and A</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q11) I try to think how to use math in everyday life</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CJHS 7th Graders</td>
<td>8.1%</td>
<td>17.6%</td>
<td>25.7%</td>
</tr>
<tr>
<td>Japanese 6th Graders</td>
<td>31.0%</td>
<td>34.2%</td>
<td>65.2%</td>
</tr>
<tr>
<td>Japanese 9th Graders</td>
<td>10.9%</td>
<td>23.6%</td>
<td>34.5%</td>
</tr>
</tbody>
</table>

4 From the survey results which was conducted by NIER (2008c, 2008d).
Figure 9. The Ratio of the CJHS Students’ Responses to Q11.

Figure 10. The ratio of the Japanese sixth grade students’ responses to Q11.\textsuperscript{5}

\textsuperscript{5} From the survey results which was conducted by NIER (2008c, 2008d).
(60% of the CJHS students, 76% of the Japanese sixth graders, and 64% of the Japanese ninth graders) (see Table 13, Figure 12, 13, and 14).

Table 13

*Attitude Response Table for Q12*

<table>
<thead>
<tr>
<th>Group</th>
<th>Strongly Agree (SA)</th>
<th>Agree (A)</th>
<th>A Total of SA and A</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q12) I try to work math problems/homework until they are solved.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CJHS 7th Graders</td>
<td>16.8%</td>
<td>43.1%</td>
<td>59.9%</td>
</tr>
<tr>
<td>Japanese 6th Graders</td>
<td>40.6%</td>
<td>35.5%</td>
<td>76.1%</td>
</tr>
<tr>
<td>Japanese 9th Graders</td>
<td>27.1%</td>
<td>36.6%</td>
<td>63.7%</td>
</tr>
</tbody>
</table>

*From the survey results which was conducted by NIER (2008c, 2008d).*
Figure 12. The ratio of the CJHS students’ responses to Q12.

Figure 13. The ratio of the Japanese sixth grade students’ responses to Q12.\(^7\)

\(^7\) From the survey results which was conducted by NIER (2008c, 2008d).
Figure 14. The ratio of the Japanese ninth grade students’ responses to Q12.\(^8\)

The survey result for Q13 showed that compared to the CJHS students (42%), twice as many of the Japanese sixth graders (81%) made a note for mathematics problem-solving on their *frame* sheets/binder notes so they could look back and see how to solve similar mathematics problems in the future. More of the Japanese ninth graders (77%) did so than the CJHS students also did so (see Table 14, Figure 15, 16, and 17).

The ratios of the responses to Q14 showed that a fairly high ratio of the students in the three groups reported that they tried to find an easier way to solve mathematics problems (68% of the CJHS students, 77% of the Japanese sixth graders, and 63% of the Japanese ninth graders) (see Table 15, Figure 18, 19, and 20).

\(^8\) From the survey results which was conducted by NIER (2008c, 2008d).
Table 14  
*Attitude Response Table for Q13*

<table>
<thead>
<tr>
<th>Group</th>
<th>Strongly Agree (SA)</th>
<th>Agree (A)</th>
<th>A Total of SA and A</th>
</tr>
</thead>
<tbody>
<tr>
<td>CJHS 7\textsuperscript{th} Graders</td>
<td>14.4%</td>
<td>27.3%</td>
<td>41.7%</td>
</tr>
<tr>
<td>Japanese 6\textsuperscript{th} Graders</td>
<td>50.5%</td>
<td>30.9%</td>
<td>81.4%</td>
</tr>
<tr>
<td>Japanese 9\textsuperscript{th} Graders</td>
<td>42.6%</td>
<td>34.5%</td>
<td>77.1%</td>
</tr>
</tbody>
</table>

Q13) I make a note for math problem-solving on my frame sheets or bindernotes so that I can look back and see how to solve similar math problems in the future.

*Figure 15.* The ratio of the CJHS students’ responses to Q13.
Figure 16. The ratio of the Japanese sixth grade students’ responses to Q13.9.

Figure 17. The ratio of the Japanese ninth grade students’ responses to Q13.

From the survey results which was conducted by NIER (2008c, 2008d).
Table 15

*Attitude Response Table for Q14*

<table>
<thead>
<tr>
<th>Group</th>
<th>Strongly Agree (SA)</th>
<th>Agree (A)</th>
<th>A Total of SA and A</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q14: I try to figure out an easier way to solve math problems.</td>
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</tr>
<tr>
<td>CJHS 7th Graders</td>
<td>30.3%</td>
<td>37.9%</td>
<td>68.2%</td>
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<tr>
<td>Japanese 6th Graders</td>
<td>43.0%</td>
<td>34.1%</td>
<td>77.1%</td>
</tr>
<tr>
<td>Japanese 9th Graders</td>
<td>29.3%</td>
<td>33.7%</td>
<td>63.0%</td>
</tr>
</tbody>
</table>

*Figure 18.* The ratio of the CJHS students’ responses to Q14.
Figure 19. The ratio of the Japanese sixth grade students’ responses to Q14.\textsuperscript{10}

Figure 20. The ratio of the Japanese ninth grade students’ responses to Q14.

\textsuperscript{10} From the survey results which was conducted by NIER (2008c, 2008d).
Findings Related to Hypothesis 6

What attitudes do American seventh graders have toward learning mathematics? This can be answered by the survey responses of the CJHS students. The response ratios of other survey questions are shown in the following figures. Figure 21 illustrated that many of the CJHS students (74%) answered that they understood mathematics frames they wrote in class.

![Pie chart showing responses to Q8](image)

*Figure 21. The ratio of the CJHS students’ responses to Q8.*

There was the same portion of CJHS students who said that mathematics was one of their favorite subjects (38%) and who said mathematics was not one of their favorites (38%) (see Figure 22).

Figure 23 showed that 33% of CJHS students used all five kinds of mathematics, 27% of them used four of the five kinds, and 20% of them used three of the five kinds, 13% of them used two of the five kinds, and 7% used one of the five kinds.
Figure 22. The ratio of the CJHS students’ responses to Q9.

Figure 23. The ratio of the CJHS students’ responses to Q10.
Half of the CJHS students (50%) did not set a specific time to study mathematics at home (see Figure 24).

![Figure 24. The ratio of the CJHS students’ responses to Q15.](image)

Figure 24. The ratio of the CJHS students’ responses to Q15.

Figure 25 indicated that most of the CJHS students (91%) did not read the mathematics textbook before coming to class.

![Figure 25. The ratio of the CJHS students’ responses to Q16.](image)

Figure 25. The ratio of the CJHS students’ responses to Q16.
Twenty-two percent of CJHS students reviewed frames before beginning mathematics homework, and 27% did not do so (see Figure 26). However, more of the students (51%) answered that maybe they did so.

![Figure 26. The ratio of the CJHS students’ responses to Q17.](image)

More than half of CJHS students (54%) reviewed mistakes they made after getting mathematics quizzes/tests back (see Figure 27).

![Figure 27. The ratio of the CJHS students’ responses to Q18.](image)
Discussion of the Findings

Discussion of the Findings Related to Hypothesis 1

Hypothesis 1 that deductive reasoning would improve algebra performances of seventh graders was supported. The participants in the experimental group significantly increased their mathematics scores after the treatment was given. Therefore, deductive reasoning training successfully helped the students solve their mathematics word problems and gain deductive reasoning skills. In contrast, the students in the control group did not significantly improve their performances on the post-test. Hence, the students who did not learn algebra deductively remained the same between the pre-test and the post-test.

Discussion of the Findings Related to Hypothesis 2

Hypothesis 2 that deductive reasoning practices would work better to improve algebra performance of seventh graders than the American traditional teaching instruction was refuted. There were no significant overall mean differences on the post-test between the control and the experimental groups. This meant that deductive reasoning training was not more or less effective to improve algebra performances of the seventh graders than the American traditional teaching technique. Though deductive reasoning practices positively affected the students’ performances according to the finding related to Hypothesis 1, this did not mean that the Japanese learning styles were generally more effective than the American traditional learning styles.
Discussion of the Findings Related to Hypothesis 3

Gender difference affected Hypothesis 1. The boys in the American learning style group scored significantly lower than the girls in the group on the post-test, but the boys and the girls in the Japanese learning style group scored similarly on the post-test. This result is most likely related to individual differences between the students.

Also, gender difference influenced Hypothesis 2. The boys who experienced the American way of learning decreased their scores on the post-test while the boys who experienced the Japanese way of learning increased their test scores on the post-test. The decline of the boys’ test scores in the American learning style group was probably still caused by their loss of motivation. However, this was not the case for the boys in the Japanese learning style group although the test was provided to all the students under the same circumstances. Therefore, the boys who experienced the Japanese learning styles could somehow keep up their motivation to work on the post-test and improve their performances, but the boys who experienced the American learning styles lost their motivation to work on the algebraic word problems. However, unlike the boys, the girls’ mean score differences between the two groups were not significant on both of the tests. So, whether or not the Japanese learning style would generally help students keep up their motivation to work on mathematics problems is still uncertain. Therefore, it can be concluded that the Japanese learning style probably helped the students work on the algebraic word problems and solve the deductive reasoning practices.
Discussion of the Findings Related to Hypothesis 4

Hypothesis 4 that students’ socio-cultural backgrounds would associate with their attitudes toward learning mathematics was confirmed. The students’ ways of and attitudes toward learning mathematics were identified based on their backgrounds such as gender, their parents’ education levels, ethnicity, and native language. The parents’ education levels were related to the students’ motivation to solve mathematics problems/homework. This is probably because the parents who held higher degrees and felt comfortable to solve mathematics problems most likely helped their children do homework at home, and this may have led the students to work mathematics problems/homework until they were solved.

Gender has a bearing on the likes and dislikes of mathematics. The result was that the boys liked mathematics more than the girls did. However, this did not reflect on the test scores for both the pre-test and the post-test. The boys in the control group significantly scored lower than the girls in the control group, and the boys and the girls in the experimental group scored similarly. So, that the boys liked mathematics more than the girls did not mean that boys could perform better on mathematics than girls in general.

The students’ attitudes toward learning mathematics were different, depending on their ethnicity. The Asian/Pacific Islander students had the most positive attitudes toward learning mathematics among the ethnicities. The White students were ranked in the second, and the Mexican/Hispanic/Latino students were ranked in the third. This
finding may indicate that Asian students tend to work on mathematics more positively than others.

Whether or not the students’ native tongue was English influenced their ways of learning mathematics. The native English speakers more effectively utilized their resources such as *frames* and more positively tried to work on mathematics problems than the English learners did. This is possibly because the English learners did not understand lectures given in English and assignments written in English as much as the native English speakers did, so the English learners tended to give up solving mathematics problems.

**Discussion of the Findings Related to Hypothesis 5**

Hypothesis 5 that Japanese students learn mathematics more positively than American seventh graders was true with some types of attitudes. Because of the difference in response choices between the CJHS and NIER (2008c, 2008d) surveys, only *strongly agree* and *agree* responses are reported.

There were not only differences but also commonalities in their attitudes toward learning mathematics between the CJHS students and the Japanese students. Most of the students generally understood the importance of learning mathematics. Over half of the students in each group (CJHS, Japanese sixth, and Japanese ninth graders) were motivated to work mathematics problems until they were solved and tried to figure out easier ways to solve them. Therefore, regardless of cultural differences, most students in the U.S. and Japan commonly have positive attitudes toward learning mathematics.
Oppositely, differences between the CJHS students and Japanese students were recognized as to the following survey questions: 1) if the students liked mathematics, 2) if they tried to think how to use mathematics in everyday life, and 3) if they made a note for mathematics problem-solving on their frame sheets or bindernotes in order for them to look back and see how to solve similar mathematics problems in the future. Regarding the first situation noted above, a contradictory result was shown. The Japanese students liked mathematics more than the CJHS students did.

This past mathematics experience may be also related to another result that few CJHS students tried to apply mathematics in everyday situations unlike many Japanese students who did so. Because Japanese students practice solving semantic mathematics problems in class as mentioned in Chapters I, the students may easily connect mathematics formulas to daily life situations which may require mathematics. However, American students less likely practice solving word problems in mathematics classrooms, compared to Japanese students. Therefore, it may be more difficult for American middle school students to use their knowledge of mathematics in everyday life than Japanese students.

The remarkable difference between the CJHS students and the Japanese students was seen in regard to the question about making a note for mathematics problem-solving. The majority of the Japanese students made a note for mathematics problem-solving on their bindernotes unlike less than half of the CJHS students who did so. Without making notes of important mathematics formulas or how to solve mathematics problems, it may be difficult for students to recall what they learned before. Also, reviewing these notes is critical to improve mathematics performances. Because of the
huge difference in this survey question between the CJHS students and the Japanese students and the importance of reviewing notes, this cultural influence on students’ ways of learning may need special attention.

**Discussion of the Findings Related to Hypothesis 6**

To Hypothesis 6, types of attitudes toward learning mathematics were defined as follows. Based on the other survey questions, most of the CJHS students did not regularly study mathematics at home, did not prepare for mathematics lectures before coming to class, and did not review their frames before doing homework. However, most of them understood mathematics frames that they wrote in class. All the students were almost equally dispersed in the five survey answers for the question, if mathematics was one of their favorite subjects. More than half of them used either all the five or the four kinds of mathematics outside of class and reviewed their mistakes on quizzes/tests after getting them back.

Like the CJHS students who were asked to make frames in class, if American students were usually asked to make notes for mathematics problem solving, they would do so and would also understand their notes. However, students may not regularly study mathematics at home and may not review their notes before doing homework. Students would review their errors that they made on their quizzes/tests after getting the quizzes/tests back, but would not review their notes before the quizzes/tests. Building study mathematics habits, making students’ own notes to solve mathematics problems, and reviewing these notes before taking quizzes/tests would be recommended to improve American students’ mathematics performances.
CHAPTER V

SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

Deductive reasoning skills are necessary to solve mathematics problems. About 50 years ago, the importance of deductive reasoning training in mathematics courses in American middle schools was addressed. Nevertheless, this idea was not accepted and is rarely included in the current mathematics programs for middle schools in the United States. When students first learn algebra, many of them struggle with transitioning from arithmetical concepts to algebraic symbolic equations. To help students understand algebra better, teaching them generalized numeric patterns of symbolic equations with routine mathematics problem-solving practices has been the focus in mathematics education. This requires inductive reasoning skills, and thus training deductive reasoning skills has been ignored. Unlike the United States, with deductive reasoning practices, Japan has succeeded in its mathematics education. Learning from Japan’s success in mathematics, implementing deductive reasoning training in mathematics classrooms may be a key to improving American students’ mathematics performances.

The study was conducted to determine if the following six hypotheses: 1) American seventh graders could improve their algebra performances by learning algebra deductively as seen in Japanese seventh graders’ mathematics classrooms; 2) Learning
algebra deductively would work better to improve algebra performances of American seventh grade students than the American traditional learning style; 3) The gender difference would not affect Hypothesis 1 and Hypothesis 2; 4) The students’ socio-cultural backgrounds including gender, their parents’ education levels, ethnicity, and primary language would influence their ways of and attitudes toward learning mathematics; 5) Japanese students have more positive attitudes toward learning mathematics than American seventh graders; and 6) Types of attitudes that American seventh graders have toward learning can be defined.

A pre-test/post-test experiment was designed. One hundred forty-one CJHS seventh graders and their teacher, Mrs. Jensen, volunteered to participate in the study. Mrs. Jensen gave lectures about algebraic word problems to one group with deductive reasoning training and to the other group without it. Several algebraic word problems and one assignment containing five algebraic word problems similar to the pre-test and the post-test were provided to the students during the lectures. A survey was given to the students at the end of the experiment.

For the hypothesis 1, 2, and 3, the comparisons of the test scores were made between the pre-test and the post-test, between the control group and the experimental group, and between the genders. Paired t tests were used to determine if the students could improve their test scores after experiencing deductive reasoning training. Independent t tests were used to examine if the students who experienced deductive reasoning practices performed better than those who did not. Two-way ANOVA was used to see if there was an interaction and/or main effect between the control/experimental group and the genders.
For the hypothesis 4, the survey was analyzed in order to find if there were any factors such as the education levels of the students’ parents, the students’ ethnicities, and their primary language that affected mathematics performances of the students. Also, for the hypothesis 5, the students’ responses to the five questions of the survey were compared to Japanese students’ responses to a survey which was conducted by NIER (2008c, 2008d) in Japan in order to examine whether or not Japanese students had more positive attitudes toward learning mathematics than American students.

The result found in this experiment supported the hypothesis that deductive reasoning training helps the students improve their algebraic performances. This showed that deductive reasoning training was useful and applicable to American students. Also, the seventh graders were relatively able to understand and solve the algebraic word problems although they had never learned algebra before, and thus, it was expected that they would translate word problems to algebraic symbolic equations with difficulty. In fact, the students were able to translate the contents of the problems in a way that they could comprehend fully and recall what they understood on the post-test within the test time given. From the findings, it can be said that American seventh graders could manage learning new algebraic concepts with deductive reasoning practices.

Based on the result of the refuted hypothesis which is that the seventh graders who experience deductive reasoning practices would perform better than those who do not, it can be concluded that the American way of learning mathematics was as effective as the Japanese way of learning mathematics to teach seventh graders algebraic word problems. However, there is a possibility that the Japanese learning style would work better than the American learning style because the boys who experienced the Japanese
learning style increased their mean scores from the pre-test to the post-test while their scores in the classes with the American learning style decreased. One reason why the boys who experienced the American learning style underperformed was loss of motivation to take the post-test during the last week of the semester. However, loss of motivation was not found in many of the boys who experienced the Japanese learning style. Therefore, the Japanese learning style somehow kept the boys motivated to work hard on the algebraic word problems more so than the American learning style.

Unlike the boys, the performances of the girls remained the same between the pre-test and the post-test. It can be generally concluded that girls may not improve their mathematics performances by obtaining training in deductive reasoning and learning mathematics deductively. An apparent reason for the difference between the boys and the girls is not found, but deductive reasoning practices and learning algebra deductively may have been a better fit for the boys than for the girls. This can be explained by the survey result that the boys liked mathematics more than the girls as well. The boys have more positive attitudes toward learning mathematics than the girls. Thus, the boys who learned mathematics deductively may have improved their performances more than the girls. However, there was a female student in the Japanese learning style group of the fifth period who showed her work excellently in class. She helped her classmates improve their mathematics performances, and she was the only student who received full credit on the post-test (see Appendix J). Therefore, it should not be disregarded that there would be some girls who could improve their mathematics performances by learning mathematics deductively.
Besides the gender differences for the mathematics performances, there are other elements such as the students’ ethnicity and their native language that influence students in learning mathematics. For example, the minority groups, except for Asians/Pacific Islanders, seemed to have less positive attitudes toward learning mathematics than the majority group. This tendency was found in the minority groups who speak English as a second language compared to the majority group of the native English speakers. Unfortunately, because the students’ desk numbers were not recorded for the survey, the test scores could not be compared with the students’ backgrounds. Consequently, it was not found whether or not learning mathematics deductively influenced American students depending on their ethnicity and their primary language.

Also, compared to the Japanese sixth graders and ninth graders, American seventh graders have less positive attitudes toward learning mathematics. In other words, Japanese students seemed to enjoy and like mathematics more than American students. As mentioned earlier, Japanese students may struggle or be frustrated by not finding appropriate methods of solving problems when they are asked to solve word problems without instructors’ directions. However, once the students overcome the hardships by getting correct answers, they would be satisfied and be happy with their accomplishments. Students would then have more fun with mathematics. Also, experiencing mathematics problem-solving and independently finding solution methods may help them have more fun in mathematics classrooms because they could experiment with mathematics problems in any way they want to. This fun feeling would be similar to the one when exploring adventures. Through experiences and fun activities, as mentioned by John Dewey (1910, 1916), students can probably remember what they learned by
learning mathematics deductively better than by only in-class explanations and guided practices. Therefore, deductive reasoning training in mathematics classrooms may make students feel more satisfied with their achievements in mathematics than the American traditional learning style, and thus, the students may like mathematics more.

Future Research and Recommendations

Longitudinal research would be recommended because it may give better data to analyze test score differences between the control group and the experimental group. Because students usually need a lot of time to work on mathematics problem-solving and to find their own solution methods, mastering new mathematics concepts deductively would not likely be accomplished within a two or three day lecture session. The longer students get trained in deductive reasoning in mathematics, the more they can get used to deductive reasoning approaches and easily find their own ways of solving mathematics problems. Meanwhile, better data can be collected for the study. Recalling that the boys between the two groups showed significant performance differences, there is a potential that there may be significant differences in mathematics performances with and without deductive reasoning training. To determine if this potential is true, an extended length of time for the study would be highly recommended.

It is suggested that students should take a pre-test and a post-test under similar circumstances in order to be accurately compared. Because the post-test was provided to the participants at the last week of the semester, many students lost their motivation to work hard on the post-test, and their test scores were lower than the pre-test. The data may have been different if the students took the pre-test and the post-test with the same
motivation levels. Although the scores of the absent and low-motivated students were eliminated from the data and the statistical test results were basically reasonable to analyze, there was a concern about the reduced number of samples to make good generalizations about the total population. In other words, more data would be helpful to confirm and generalize that the deductive reasoning training would improve American students’ mathematics performances.

The survey should have been taken with the students’ desk numbers, so their differences in gender, ethnicity, and native language could have been statistically tested with the students’ test scores. The statistical test results could answer the question of whether or not deductive reasoning practices would affect students depending on their socio-cultural backgrounds.

An experienced Japanese teacher to give students lectures may be recommended for a future research. To provide the five algebraic word problems for the students, Mrs. Jensen was worried if the students would struggle with understanding the word problems because they had never learned word problems before. As mentioned in Chapter I, American teachers do not want students to suffer in solving mathematics problems, so this was expected. However, after knowing her hesitation to let her students struggle in solving mathematics problems, it felt like proceeding with the experiment was not the right thing to do. Oppositely, Japanese teachers believe that struggling in solving mathematics problems is a necessary path to improve students’ skills. A Japanese teacher would not mind providing students difficult mathematics problems, and this may give them more opportunities to experience deductive reasoning training in mathematics
classrooms. However, it may not be easy to find a Japanese instructor who is able to teach mathematics in English.

It is important to note that reforming quality education in elementary schools, reexamining the quality of teaching or instructors’ passion to teach students mathematics, and reconsidering instructional programs for non-native English speakers as well as changing the learning styles in mathematics classrooms in middle schools may be suggested to improve American students’ mathematics performances. While observing the CJHS seventh graders in the mathematics classrooms, it was recognized that there were factors other than the missing deductive reasoning practices in American mathematics classrooms which resulted in students who failed mathematics classes. For example, the CJHS seventh graders were learning fractions, exponents, multiplying power, and dividing power when the researcher observed the classrooms. The students are supposed to learn these basic mathematics concepts in elementary schools, but they seemed to have started middle school without understanding those concepts. These students’ tendencies of lacking basic mathematics skills may be caused by failure to improve those skills while they were in elementary schools (V. Jensen, personal communication, January 21, 2009). Without these skills, the students were having a hard time stepping up to the next level of mathematics, so Mrs. Jensen was teaching the basics to the students at that time. At this point, these students have already lagged behind Japanese junior high school students. It would be very difficult for American seventh graders to catch up with the level of Japanese seventh graders’ mathematics skills.

As far as elementary school mathematics education is concerned, that American students have less positive attitudes toward learning mathematics may be
rooted back in their elementary school mathematics learning experiences. For instance, students may not have enjoyed learning mathematics in elementary school or have had bad experiences with mathematics such as failing mathematics exams. Moreover, students who failed to understand basic mathematics concepts could be a result of some instructors who lack passion to teach. Teachers may not have reached students and might not have helped them gain their mathematics skills. Quality teaching or teachers’ awareness of the reason why students were failing mathematics classes may need to be reexamined.

Also, many of the students who speak English as a second language seem to attend public schools in the United States, and teaching mathematics in English could be making them fail mathematics classes. Mrs. Jensen (personal communication, January 21, 2009) mentioned that in her class, there were many English language learners who did not understand mathematics problems written in English. This might be seen in other classes or in other schools in the United States. Reforming instructional programs for English language learners in mathematics classrooms may be needed.

In addition to changing the learning styles in mathematics classrooms of middle schools, quality education in elementary school, teaching in elementary school, instructors’ passion to teach students mathematics, and instructional programs for the English language learners may need to be reformed. Deductive reasoning training can be one of the ways to improve American students’ mathematics performances, but this may probably work more effectively if other problems that have caused the students’ underperformances in mathematics are solved.
Therefore, in addition to implementing deductive reasoning training in mathematics classrooms, other problems causing American students’ underperformances in mathematics, such as students’ socio-cultural backgrounds, quality education in elementary school, instructors’ passion to teach, and instructional curricula for non-native English speakers, would be recommended to research and resolve. Reforming these factors as well as changing the learning styles in American mathematics classrooms would be essential for American students to achieve better mathematics performances.

Conclusions

From the experiment, it was confirmed that deductive reasoning training helped American students improve their mathematics performances. Learning mathematics deductively is applicable to American students as well as to Japanese students. Although this does not mean that deductive reasoning practices would work better than the American traditional learning style, the deductive reasoning training is potentially important to facilitate American students to understand the meaning of algebraic concepts better and thus perform mathematics better. Deductive reasoning training can be a good teaching tool in mathematics classrooms in American middle schools.

However, missing deductive reasoning training in American seventh grade mathematics classrooms is not the only element that made American middle school students underperformed in mathematics. The students’ backgrounds such as gender, education level of the students’ parents, ethnicity, and primary language also influenced the students’ ways of and attitudes toward learning mathematics. This may imply that
there are many factors that may have caused underperformances of American middle school students in mathematics. Unfortunately, it is still questionable if deductive reasoning training could solve the problems of the students’ background influence on learning mathematics.

The Japanese middle school students relatively enjoy learning mathematics more than the American seventh graders. This could be the result of Japanese students’ satisfaction with their accomplishments after struggling through the trial and error process of mathematics word problem solving. In this sense, deductive reasoning practices may possibly help American students enjoy learning mathematics and improve their mathematics skills as Japanese students do. However, the other differences of the attitudes toward learning mathematics are more likely caused by the cultural differences between the two countries.

Deductive reasoning training is essential in learning mathematics because it requires deductive thinking. As John Dewey (1910, 1916) explained that experiencing was learning, and that deduction was one of the learning phases, experiencing mathematics problem-solving and finding solution methods by students, which mean learning mathematics deductively, are important in learning mathematics. However, deductive reasoning practices have been ignored in the current American mathematics education. This was found in a TIMSS video study (U.S. Department of Education, 1999) which showed that American students did not practice deductive reasoning at all while 61% of the Japanese lessons were deductive reasoning practices as mentioned in Chapter I. This tendency has occurred because American mathematics curricula have focused on teaching students generalized algebraic concepts, which is learning mathematics
inductively. However, about 50 years ago, mathematics education pointed out that middle school students should practice deductive reasoning in mathematics classrooms. It may be time to reconsider deductive reasoning practices in American seventh graders’ mathematics classrooms to help them improve their performances in mathematics.
REFERENCES
REFERENCES


ent_storage_01/0000019b/80/1b/ba/ec.pdf


ent_storage_01/0000019b/80/36/c5/61.pdf


## INTEGERS TEST
### ADDITION
#### 100 PROBLEMS

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<td><code>-2 + 6</code></td>
<td></td>
</tr>
<tr>
<td><code>-8 + (-6)</code></td>
<td><code>-9 + 2</code></td>
<td><code>0 + (-7)</code></td>
<td><code>-2 + 3</code></td>
<td><code>-3 + (-9)</code></td>
<td></td>
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<tr>
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<td><code>-6 + (-5)</code></td>
<td><code>1 + (-5)</code></td>
<td><code>17 + (-8)</code></td>
<td></td>
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<tr>
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<td><code>-6 + 0</code></td>
<td><code>-5 + 14</code></td>
<td><code>11 + (-4)</code></td>
<td><code>-4 + 10</code></td>
<td></td>
</tr>
<tr>
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<td><code>12 + (-10)</code></td>
<td><code>4 + (-7)</code></td>
<td><code>-3 + 8</code></td>
<td><code>-6 + (-8)</code></td>
<td></td>
</tr>
</tbody>
</table>

**Corrected by:** __________________________
ACTIVITY PROJECT 1

DIRECTIONS: Neatly show ALL work for solving the following word problems.

1) A boy chooses a number and multiples it by 6 and then adds 10. He then subtracts
his number (the number he chose to start with) and divides the result by 5. He
notices that the answer he gets is 2 more than the number he started with. He says,
“I think that would happen, no matter whatever number I started with.” Is he
right?

Use \( n \) (unknown value) as the number which the boy chose to start with. Write,
and solve an equation to prove that he is right.

2) You are riding your bike from one town to another town. If you stop to rest every
13 miles, you make \( r \) number of stops and you still have 5 miles to go before you
reach the next town. But if you stop to rest every 15 miles \( r \) times, you still have 1
mile to ride your bike to the other town.

Using \( r \) as your unknown variable, write an equation and find the distance
between the two towns.

3) A running track is \( n \) (unknown value) miles long. You run 15 times around the
track for a total of 60 miles.

Using \( n \) as your unknown variable, write an equation and find how long the
running track is.

4) You babysit your neighbor’s children for \( x \) (unknown variable) hours. You make
\$7 each hour you work and you earned \$805.

Using \( x \) as your unknown variable, write an equation to determine the total
number of hours you worked.

5) Divide \$20.20 among three people so that the second will have \$1 less than the
first and the third will have \$2.70 more than the second person.

Using \( n \) as your unknown variable, write an equation, and find how much money
each of the people person will have.
ACTIVITY PROJECT 3

DIRECTIONS: Neatly show ALL work for solving the following word problems.

1) Divide $70.35 among three people so that the second person will have $2.00 more than the first person and the third person will have $3.50 more than the second person.

Using \(a\) as your unknown variable for the amount of money the first person will have, write an equation and determine how much money each of the three people will have.

2) You babysit your neighbor’s children for \(b\) (unknown variable) hours. You make $7.50 each hour you work and you earned $93.75.

Using \(b\) as your unknown variable, write an equation and find the total number of hours you worked.

3) You are riding your bike from one town to another town. If you stop to rest every 11 miles, you make \(r\) number of stops and you arrive at the other town. But if you stop to rest every 8 miles, you make \(r\) number of stops again, but you still have 6 more mile to ride your bike to the other town.

Using \(r\) as your unknown variable, write an equation and find the distance between the towns.

4) A running track is \(b\) (unknown value) mile long. You run 50 times around the track for a total of 350 miles.

Using \(b\) for your unknown variable, write an equation and find the length of the running track.

5) Nancy chooses a number and multiples it by 10 and then adds 63. She then subtracts her number (the number she chose to start with) and divides the result by 9. Nancy notices that the answer she gets is 7 more than the number she started with. Nancy says, “I think that would happen, no matter whatever number I started with.” Is she right?

Use \(n\) (unknown value) as the number which Nancy chose to start with. Write and solve an equation to prove that Nancy is right.
APPENDIX D
**Word Problem Worksheet**

During class, students will practice on one or two questions each category of the following. (Vallarie will pick questions to ask students to practice in class. “$n$” can be substituted by any letter)

- **“Find the Number” questions, using an unknown variable.**
  1) The sum of 3 and six times a number is equal to 42. Find the number.
  2) The sum of 5 and a number divided by 4 is 9. Find the number.
  3) The sum of a number and the difference between 16 and a number plus 5 is 21.
  4) The difference between 8 and a number is 2. Find the number.
  5) The difference between 55 and four times a number is equal to 21. Find the number.

- **Semantic Questions, using an unknown variable.**
  6) Eight times Sally’s age plus 3 times Sally’s age is the same as 21 more than 9 times Sally’s age. Use $n$ (unknown variable) as Sally’s age, write an equation, and find how old Sally is.
  7) The sum of 6 and three times Mike’s height equals 33. Use $n$ (unknown variable) as Mike’s height, write an equation, and find his height.
  8) Find Tom’s weight such that 5 more than one-half his weight is the three times his weight. To find Tom’s weight, use $n$ (unknown variable) as the number, and write an equation.

- **Multiple Ways of Solving the Problems**
  9) The gas tank on your mom’s car holds about 15 gallons. Gas costs $n$ per gallon. If your mom paid $10.00 for gas to fill the tank when it is empty, using $n$ (unknown variable), write an equation, and find how much gas costs per gallon.
  10) There is an $n$ inch rope. If you cut it into 7 pieces, each piece is 5 inches long. Using $n$ (unknown variable), write an equation, and find the length of the original rope.
11) Imagine a grocery store parking lot that will hold 1000 vehicles. When you go to buy groceries, there are 200 cars and \( n \) (unknown variable) trucks in the parking lot. The parking lot is \( \frac{3}{4} \) full. Using \( n \) (unknown variable), write an equation, and find how many trucks are in it.

12) The recipe for mint chocolate ice cream requires 2 \( \frac{1}{4} \) cups of cream for 6 people. You need ice cream for 8 people. Use \( n \) (unknown variable) as the number of cups of cream, write an equation, and find how much cream you will need.

- An Unknown Variable appears on both the right and the left of an equation

13) Three monkeys and two giraffes eat a combined 45 ounces of food in a week. A single giraffe eats two ounces less than twice what a monkey eats each week. Use \( n \) (unknown variable) as the amount that a single monkey eats in a week, write an equation, and find how much a single monkey eats in a week.

14) If the girls in the class were split into three equal groups, the size of each group would be the same as if 18 girls had left the class. Use \( n \) (unknown variable) as the number of the girls in the class, write an equation, and find how many girls are in the class.

15) There are 5 apples and 4 pears. They cost $9.10 total. An apple costs $1.05 less than twice the cost of a pear. Find how much an apple costs.

16) A New York Junior High School consists of 365 students. The total number of girls is 39 fewer than the total number of boys. Find how many girls and boys are at this school.

17) The sum of two numbers is 91. The larger number is 1 more than four times the smaller number. Find the numbers.

18) Nick has $8.20. Alex has $6.00. After both of them buy 5 cookies, Nick will have three times more money than Alex will have. Use \( n \) (unknown variable) as the cost of a cookie, write an equation, and find how much one cookie costs.

19) The contents of a box weigh a certain amount. If you remove 15 pounds from the box, its contents will weigh 5 pounds more than if you had halved their weight. Use \( n \) (unknown variable) as the original weight of the box, write an equation, and find how much the box’s contents originally weighed.

- Two Conditional Statements Questions

20) Andy wants to put his football cards into boxes. If he puts 48 cards each box, he has room for 4 more cards to put in the last box. If he puts 35 cards each box, he
has 165 cards left over. Use $n$ (unknown variable) as the number of boxes he has, write an equation, and find how many football cards he has.

21) Mt. Lassen is a popular field trip destination. This year, the 7th graders’ class at Chico Junior High School (CJHS) planned trips to Mt. Lassen. If CJHS rents 7 buses, 31 students cannot get on the buses. If CJHS rents 9 buses, there are 29 empty seats in the last bus. (Suppose all seats on each bus are filled by students.) Use $n$ (unknown variable) as the number of seats in a bus, write an equation, and find how many students there are in the 7th graders’ class at CJHS.

22) Chico Junior High School has a band. All members of the band are assigned to play instruments. If half of the band members are assigned to play woodwinds and 8 members are assigned to play percussions and triangles, there are 36 members who are not assigned any instrument. If $\frac{3}{4}$ of the band members are assigned to play woodwinds, 18 members are assigned to play percussions and triangles, there are 4 members who are not assigned any instrument. Use $n$ (unknown variable) as the number of the band members, write an equation, and find how many total band members there are.
APPENDIX E
ACTIVITY PROJECT 2  
(Assignment from Luna)  

DIRECTIONS: Neatly show ALL work for solving the following word problems.  
(Candy will be given to you based on the neatness and accuracy of your work!)

1) In a classroom, a teacher is about to provide colored papers to all students. If the teacher distributes 3 papers to each student, 20 papers will be left over. But, if the teacher distributes 5 papers to each student, the teacher lacks 2 pieces of papers to give students.

Using \( n \) as your unknown variable for the number of students in the classroom, write an equation and find the total number of colored papers the teacher has to give to her students.

2) There are 1,246 pieces of candies. This is 4 more than twice the number of cookies.

Using \( x \) as your unknown variable, write an equation and find the total number of cookies.

3) Jessica spent $80 on books. This was $10 less than 5 times what she spent on her lunch.

Using \( n \) as your unknown variables, write an equation to determine how much money Jessica spent on her lunch.

4) A group of boys and girls totals 260. There are 4 times as many boys as girls.

Using \( x \) as your unknown variable to represent the number of girls, write an equation to find the number of boys and the number of girls in the group.

5) The sum of two numbers is 84. One of the numbers is 12 more than the other number.

Using \( n \) as your unknown variable for one of the numbers, write an equation and determine what the two numbers are.
Survey Questions

There are 18 questions. Please check a box for the following questions or statements.

1) Are you a boy or girl? □ Boy □ Girl

2) Which group describes you best? (You may mark more than one.)
   □ Alaska Native/ American Indian
   □ Black/African American
   □ Mexican/ Hispanic/ Latino
   □ Asian/ Pacific Islander
   □ White – Not Hispanic
   □ Other _______________

3) Is English your first language? □ Yes □ No
   (Your first language)__________________

4) Which one (highest one) did your mother graduate?
   □ High School □ Community College □ 4 year College □ other
   □ I don’t know

5) Which one (highest one) did your father graduate?
   □ High School □ Community College □ 4 year College □ other
   □ I don’t know

Please indicate how much you agree or disagree with the following statements.

6) I like math.
   □ strongly agree □ agree □ maybe □ disagree □ strongly disagree

7) Learning math is important.
   □ strongly agree □ agree □ maybe □ disagree □ strongly disagree
8) I understand math frames that I wrote in class.

☐ strongly agree ☐ agree ☐ maybe ☐ disagree ☐ strongly disagree

9) Math is one of my favorite subjects.

☐ strongly agree ☐ agree ☐ maybe ☐ disagree ☐ strongly disagree

10) What kind of math do you use outside of class?

☐ addition ☐ subtraction ☐ multiplication ☐ division ☐ measuring

---

**How often do you do the following statements? Please check a box for each statement.**

11) I try to think how to use math in everyday life.

☐ always ☐ almost always ☐ sometimes ☐ almost never ☐ never

12) I try to work math problems/homework until they are solved.

☐ always ☐ almost always ☐ sometimes ☐ almost never ☐ never

13) I make a note for math problem solving on my frame sheets or bindernotes so that I can look back and see how to solve similar math problems in the future.

☐ always ☐ almost always ☐ sometimes ☐ almost never ☐ never

14) I try to figure out an easier way to solve math problems.

☐ always ☐ almost always ☐ sometimes ☐ almost never ☐ never

15) I set a specific time to study math at home.

☐ always ☐ almost always ☐ sometimes ☐ almost never ☐ never

16) I read the math textbook before coming to class.

☐ always ☐ almost always ☐ sometimes ☐ almost never ☐ never

17) I review frames before beginning math homework.

☐ always ☐ almost always ☐ sometimes ☐ almost never ☐ never
After getting math quizzes/tests back, I review mistakes that I made, for example, misunderstanding math questions, calculation errors, or careless errors.

☐ always  ☐ almost always  ☐ sometimes  ☐ almost never  ☐ never

Thank you for your participation!
APPENDIX G
**Grading Scale**

<table>
<thead>
<tr>
<th>+ points</th>
<th>- points</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Deductive Reasoning + 4 points</td>
<td>• Inductive Reasoning – 4 points</td>
</tr>
<tr>
<td>• Correct Answer + 4 points</td>
<td>• Incorrect Answer – 4 points</td>
</tr>
<tr>
<td>• Used Variable + 4 points</td>
<td>• Not Used Variables – 4 points</td>
</tr>
<tr>
<td>• Correct Equation + 4 points</td>
<td>• Incorrect Equation – 4 points</td>
</tr>
<tr>
<td>• Equation Solved Correctly + 4 points</td>
<td>• Equation Solved Incorrectly – 4 points</td>
</tr>
<tr>
<td>Total: + 20 points each problem</td>
<td>Total: - 20 points each problem</td>
</tr>
</tbody>
</table>
4) A running track is \( b \) (unknown value) mile long. You run 50 times around the track for a total of 350 miles.

Using \( b \) for your unknown variable, write an equation and find the length of the running track.

\[
350 \div 50 = 7
\]

\[
\frac{7}{50} \div \frac{7}{350} = \frac{b}{7}
\]

Check:

\[
\frac{50}{7} \times \frac{7}{350} = \frac{b}{7}
\]

\[
b = 7 \text{ miles long}
\]

5) Nancy chooses a number and multiplies it by 10 and then adds 63. She then subtracts her number (the number she chose to start with) and divides the result by 9. Nancy notices that the answer she gets is 7 more than the number she started with. Nancy says, “I think that would happen, no matter whatever number I started with.” Is she right?

Use \( n \) (unknown value) as the number which Nancy chose to start with. Write and solve an equation to prove that Nancy is right.

\[
n \times 10 + 63 - n \div 9 = n + 7
\]

\[
20 + 63 - 2 \div 9 = 2 + 7
\]

\[
83 - 2 = 2 + 7
\]

\[
81 \div 9 = 2 + 7
\]

\[
n = 2 + 7
\]

\[
n = 9
\]

Nancy is right.
4) A running track is \( b \) (unknown value) mile long. You run 50 times around the track for a total of 350 miles.

Using \( b \) for your unknown variable, write an equation and find the length of the running track.

\[
b = \text{length of running track}
\]

\[
50 \cdot b = 350 \\
50 \cdot 7 = 350 \\
350 \div 50 = 70
\]

5) Nancy chooses a number and multiplies it by 10 and then adds 63. She then subtracts her number (the number she chose to start with) and divides the result by 9. Nancy notices that the answer she gets is 7 more than the number she started with. Nancy says, “I think that would happen, no matter whatever number I started with.” Is she right?

Use \( n \) (unknown value) as the number which Nancy chose to start with. Write an equation to prove that Nancy is right.
APPENDIX J
ACTIVITY PROJECT 3

DIRECTIONS: Neatly show ALL work for solving the following word problems.

1) Divide $70.35 among three people so that the second person will have $2.00 more than the first person and the third person will have $3.50 more than the second person.

Using \( a \) as your unknown variable for the amount of money the first person will have, write an equation and determine how much money each of the three people will have.

\[ \begin{align*}
1: & \quad \text{1st person has} \\
2: & \quad 2 + a \\
3: & \quad 3.50 \times (2 + a)
\end{align*} \]

\[ \begin{align*}
70.35 & = 3.50 + (2 + a) + (3.50 \times (2 + a)) \\
70.35 & = 3.50 + 2 + a + 7 + 3.50a \\
70.35 & = 3.50 + 2 + (a + 7 + 3.50a) \\
70.35 & = 3.50 + 2 + 4a + 7 + 3.50a \\
70.35 & = 1.50 + 5a + 14 + 3.50a \\
70.35 & = 1.50 + 8.50a + 14 \\
70.35 & = 15.50 + 8.50a \\
70.35 - 15.50 & = 8.50a \\
54.85 & = 8.50a \\
a & = \frac{54.85}{8.50} \\
a & = 6.45 \\
\end{align*} \]

The first person has $6.45.

The second person has $22.95. The third person has $26.45.

2) You babysit your neighbor’s children for \( b \) (unknown variable) hours. You make $7.50 each hour you work and you earned $93.75.

Using \( b \) as your unknown variable, write an equation and find the total number of hours you worked.

\[ \begin{align*}
b: & \quad \text{hours worked} \\
7.50b & = 93.75 \\
\frac{7.50b}{7.50} & = \frac{93.75}{7.50} \\
b & = 12.5 \\
\end{align*} \]

You worked 12.5 hours for 12 hours and 30 minutes.

3) You are riding your bike from one town to another town. If you stop to rest every 11 miles, you make \( r \) number of stops and you arrive at the other town. But if you stop to rest every 8 miles, you make \( r \) number of stops again, but you still have 6 more miles to ride your bike to the other town.

Using \( r \) as your unknown variable, write an equation and find the distance between the towns.

\[ \begin{align*}
r: & \quad \text{number of stops} \\
11r & = 8r + 6 \\
\frac{3r}{3} & = \frac{6}{3} \\
r & = 2 \\
\end{align*} \]

The distance between the two towns is 22 miles.
4) A running track is \( b \) (unknown value) mile long. You run 50 times around the track for a total of 350 miles.

Using \( b \) for your unknown variable, write an equation and find the length of the running track.

\[
\frac{50b}{50} = \frac{350}{50} \\
\text{Check:} \\
50b = 350 \\
50(7) = 350 \\
350 = 350 \quad \text{(OK)}
\]

\( b = 7 \) miles long.

5) Nancy chooses a number and multiples it by 10 and then adds 63. She then subtracts her number (the number she chose to start with) and divides the result by 9. Nancy notices that the answer she gets is 7 more than the number she started with. Nancy says, “I think that would happen, no matter whatever number I started with.” Is she right?

Use \( n \) (unknown value) as the number which Nancy chose to start with. Write and solve an equation to prove that Nancy is right.

\[
\frac{(10n + 63) + n}{9} = 7 + n \\
9n + 63 + n = 7 + 9n \\
9n + 63 = 7 + 9n \\
\]

(Yes)