USING RATIO TABLES TO ENCOURAGE
PROPORTIONAL REASONING

A Thesis
Presented
To the Faculty of
California State University, Chico

In Partial Fulfillment
of the Requirements for the Degree
Master of Science
in
Mathematics Education

by
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Fall 2009
USING RATIO TABLES TO ENCOURAGE

PROPORTIONAL REASONING

A Thesis

by

Rita M. Nutsch

Fall 2009

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DEDICATION

In loving memory of my mother and father, Leona and Edwin Nutsch, who instilled in me a true love of learning and encouraged me, by example, to be a life-long learner. Their work ethic and desire for knowledge were to be admired and emulated.
ACKNOWLEDGMENTS

I would like to thank my graduate advisory committee members: Dr. Deborah Summers, for the attention to detail and help with editing; Dr. Nancy Carter’s invaluable assistance with the statistics; and a special thank you goes to Katy Early, not only for her amazing editing ability, but the encouragement and friendship that blossomed along the way.

Above all, a sincere thank you goes to my chair, Dr. William Fisher. Without him, this thesis would not exist. Thank you for the support and especially for your belief in me, even when I did not believe in myself. You are a true friend and mentor.
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ABSTRACT

USING RATIO TABLES TO ENCOURAGE
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It is essential that proportional reasoning is developed at the middle-school level to allow students access to higher mathematics. Because of the importance of proportional reasoning, every effort should be made to assure its careful development. A quick scan of textbooks on the California state adoption list for middle schools shows little emphasis placed on understanding proportionality beyond showing the use of the cross-multiplication algorithm. Proportional reasoning is a sophisticated thinking process that goes far beyond the ability to solve routine proportion problems using cross-multiplication.

The purpose of this study was to explore the relationship between student use of ratio tables and their conceptual understanding of proportional reasoning. A ratio table is a chart of two or more rows that is used as a tool to help record and organize equivalent ratios. This study addressed the following questions: How does the use of ratio tables affect the conceptual understanding of routine and non-routine proportional reasoning
problems? and How does the use of ratio tables influence the quality of verbalization, written or oral, of a student’s proportional thinking?

All students were given a pretest and seven randomly chosen students were interviewed to assess conceptual understanding of proportional reasoning. Each problem required a written explanation of the student’s thinking while solving the problem. After conducting a unit on proportional reasoning in which ratio tables were introduced as an organization tool, a posttest and subsequent interviews were administered to the same pre-algebra students to answer the stated research questions.
CHAPTER I

INTRODUCTION

Background

National and statewide emphasis is being placed on algebraic reasoning. Research shows a key prerequisite to algebraic reasoning is proportional reasoning (California Framework, 2000; National Council of Teachers of Mathematics [NCTM], 2000). Ben-Chaim, Fey, Fitzgerald, Bendetto, and Miller (1998) claim “proportional reasoning is at the heart of middle grade mathematics” (p. 249). Yet, my experience as a middle-school mathematics teacher and a professional development provider shows that students and teachers alike have great difficulty with proportional reasoning concepts. The standardized California mathematics textbooks (California State Department of Education, 2000) for middle grade students do little to alleviate these misunderstandings; in fact, little emphasis is being placed on the understanding of proportionality beyond the use of the standard cross-multiplication algorithm. However, research has shown this often taught method only seems to confuse students and discourage their proportional reasoning. Further, students rarely, if ever, generate this method naturally in a problem solving situation (Lesh, Post, & Behr, 1988). Lamon (1993) suggests tapping into middle grade students’ rich store of informal knowledge and common sense making tools to investigate insightful methods of solving proportion problems rather than just applying an algorithmic solution. Patterning is part of an elementary student’s mathematical background...
(California Framework, 2000). Ratio or pattern tables build on this prior conceptual knowledge and could be a powerful tool to encourage proportional reasoning (Lanius & Williams, 2003; Sharp & Adams, 2003).

Statement of the Problem

This study considered the value of solving routine and non-routine proportional reasoning problems using ratio or pattern tables as a tool to enhance conceptual understanding of proportional reasoning.

Purpose of the Study

The purpose of this study was to explore the relationship between student use of ratio tables and conceptual understanding of proportional reasoning. The ratio table provides an alternative solution technique for proportion problems that can utilize both additive and multiplicative strategies. It is a simple tool that organizes numbers and keeps track of operations and results. After introducing students to ratio tables as an organizational tool, there will be an exploration of the following research questions.

1. How does the use of ratio tables affect students’ conceptual understanding of routine and non-routine proportional reasoning problems?
   • How does the use of ratio tables affect appropriate application of proportional reasoning as opposed to non-proportional reasoning or inappropriate use of additive reasoning?
• Are ratio tables equally effective for students when applied to routine (missing-value proportional problems), and non-routine proportional problems (proportion problems using ratios as the critical factor in comparing quantities)?

• Do ratio tables confine students to solving only integer-based proportional problems?

2. How does the use of ratio tables influence the quality of verbalization, written or oral, of a student’s proportional thinking?

• What details are brought forth in written or oral descriptions of proportional reasoning when there is evidence of use of a ratio table?

Answers to these research questions would prompt further consideration of the use of ratio tables in the mathematics classroom to encourage proportional reasoning. The time spent developing proportional reasoning in most mathematics textbooks does not correspond to the importance of students’ understanding and development of this concept. Teaching of the cross product algorithm does not ensure a students’ ability to reason proportionally. Neither does evaluating a student on his or her ability to correctly follow the cross multiplication algorithm guarantee that a student is reasoning proportionally. Therefore, this study could show that ratio tables are a beneficial method to be used as an organizational tool to help students develop a stronger sense of proportional reasoning.

Limitations of the Study

The author was the only teacher conducting the study and it was conducted with only one class. A single group of students from a small rural Northern California
middle school were involved in this study. This study had a testing threat due to multiple administrations of the same test on the same population.

Definition of Terms

*Cross multiplication algorithm:* \(\frac{a}{b} = \frac{c}{d}; \implies ad = bc\)

*Routine proportion problems:* Routine questions may be viewed as those for which students may be expected to execute a rehearsed procedure consisting of a limited number of steps. Routine proportion problems would have one given ratio and a second ratio with one term missing.

*Non-routine proportion problems:* A non-routine problem cannot be solved immediately after selecting the appropriate algorithm. Proportion problems that cannot be classified as routine are considered non-routine.

*Proportional reasoning:* Proportional reasoning focuses on describing, predicting or evaluating the multiplicative relationship between two quantities.

*Ratio or pattern table:* A table used to systematically organize work and where the operations of multiplication, division, addition, and subtraction are combined to produce equivalent ratios until some target quantity is achieved.

*Standardized method of teaching proportional reasoning:* Methods presented in California standards-based textbooks which are limited to variations of the common cross multiplication algorithm.
CHAPTER II

REVIEW OF LITERATURE

Proportional reasoning can very well be considered the capstone of elementary mathematics and the cornerstone of algebra and higher mathematics (Lesh et al., 1988). It is essential that proportional reasoning is developed at the middle-school level to allow students access to higher mathematics. Proportional reasoning is equally useful in many everyday contexts including recipe conversions, gas consumption, map reading, scale drawings, speed, scaling, comparison shopping, and monetary conversions. Although the importance of proportional thinking and its many uses are easily confirmed, only about one half of the adult population is competent in proportional reasoning (Lamon, 1999).

Proportional reasoning is a complex thinking process that is difficult to define. Piaget and Imholder (1975), identify proportional reasoning as a focus on describing, predicting or evaluating the relationship between two quantities. Lesh, Post, and Behr, (1988) continue the definition describing proportional reasoning as including the recognition of situations in which a proportional or multiplicative relationship is, or is not, relevant. It involves the understanding that both component measures in a ratio may change but in such a way that the original ratio relationship remains invariant. Proportional thinkers recognize differences between proportional versus non-proportional...
situations (Lamon, 1999). In addition, proportional thinkers develop a wide variety of strategies for solving proportions, most of which are based on informal strategies rather than prescribed algorithms. Students need several years worth of opportunities to reason in multiplicative ways before formalized introduction of algorithms (Van De Walle, 2007).

Although the National Council of Teacher of Mathematics Curriculum Standards confirmed the extreme importance of taking the time to develop proportional reasoning, state adopted middle-school math textbooks do little to enhance this understanding beyond the use of the cross-multiplication algorithm. A quick scan of the California middle-school standardized textbooks (from the 2001 adoption list) will show only the use of this cross-multiplication algorithm to solve proportional problems, encouraging mechanical manipulation of symbols. Lamon (1995) noted that proportional reasoning has typically been taught in a single chapter of the mathematical textbook, introducing symbols before sufficient groundwork has been laid for student understanding. The California State Framework (2000), also, gives only examples of solving proportional problems using the cross multiplication algorithm. The majority of textbooks encourage the use of formulas before building understanding, moving students to a rote manipulation of symbols, which discourages proportional thinking (Larson, 2001). Formally setting up proportions using variables and applying the cross-multiplication rule should be delayed until students have had an opportunity to build on their informal knowledge and develop an understanding of the essential components of proportional reasoning (Van De Walle, 2007).
Middle-grade students have a rich store of informal knowledge and common sense. Lamon (1993) suggests investigating these insightful methods of solving proportion problems rather than just applying an algorithmic solution. Finding patterns is an ability students have been encouraged to explore informally and formally throughout their grade school mathematics classes (California Framework, 2000). A knowledge of techniques to solve proportion problems, such as building patterns, can be very valuable in developing generalized thought patterns (Post, 1987). Being able to generalize thought patterns is a building block to the understanding of higher mathematics.

Ratio tables build on this prior conceptual knowledge; for this reason, they can be a powerful tool to encourage proportional reasoning (Sharp & Adams, 2003; Lanius & Williams, 2003). These tables appear to be a natural bridge between elementary patterning concepts and generalized proportional thinking. Ratio tables are individualized constructions that allow students to record their unique thought processes. Two examples of ratio tables (Figure 1) are exhibited below to gain an understanding of the ideas involved (Lamon, 1999). These tables allow students to construct efficient strategies at their own pace, building an opportunity to create a more generalized conceptual understanding of proportional reasoning. As students’ reasoning develops, so too does their ability to solve increasingly complex problems. Corresponding to their increasing ability to solve difficult problems, students’ strategies for solving problems also get more complex and more mathematically sophisticated.
A party planning guide said that three pizzas will serve about seven people. How many pizzas are needed for 350 people?

*Figure 1.* Ratio table examples.

Ratio tables can be described by the following five characteristics:

1. The table consists of two (or more) rows and two or more columns, with numbers in the cells.

2. The rows have a label, indicating the meaning of the numbers and specifying, if needed, the units used.

3. There is no preference on what to choose as the upper or lower row.

4. The ratio between the numbers in the columns is the same for all columns; this can be used to calculate an empty place in a column.

5. To get the numbers of a column, the numbers of another column can be multiplied or divided by a certain number; proportionately adding or subtracting are possible as well (Broekman, Van der Valk, & Wijers, 2000).

In summary, assuming that proportional reasoning is one of the most important building blocks of understanding of higher mathematics as many researchers claim,
there seems to be too little emphasis put on conceptual understanding of proportional reasoning in the middle school. The author believes that ratio tables are a natural step towards students’ understanding of proportionality. Construction of a ratio table may compel students to slow down and analyze what they are doing, rather than mechanically and sometimes randomly plugging numbers into the cross-multiplication algorithm. While additive reasoning usually precedes multiplicative reasoning in development, children (and adults) who do not possess proportional reasoning capability often inappropriately substitute additive for multiplicative processes (Baxter & Junker, 2001). Although ratio tables are only one of many tools needed to help students develop proportional reasoning, they may help encourage students to think more clearly about making comparisons based on a multiplicative relationship rather than just an additive one, which is a primary element to reasoning proportionally.
CHAPTER III

METHODS

Design of the Investigation

The study began with a pretest designed by the author, combined with selected student interviews that assessed further conceptual understanding of proportional reasoning. The pretest and interviews took place in early October (2008), before any formal teaching of proportional reasoning from the seventh grade standardized California textbook had taken place. The pretest included routine and non-routine proportional problems. Each problem was selected for a specific reason. The problems and characteristics for which they were chosen follow:

- **Missing Value Problem; integer valued:**
  1. Find the cost of 6 pieces of candy if 2 pieces cost 8 cents and if the price of the candy is the same no matter how many are sold.

- **Missing Value Problem; non-integer valued:**
  2. If 3 boxes of cereal are on sale for $6.90, and a daycare provider needs 17 boxes, how much will she pay? Explain your work.

- **Percent problem taken from the California Standards Test Released Test Questions from the seventh grade California Department of Education (2001) STAR (Standardized Testing and Reporting) mathematics test:**
3. A sweater originally cost $37.50. Last week Moesha bought it at 20% off. How much was deducted from the original price?

A. $7.50  B. $17.50  C. $20.00  D. $30.00

- Missing value; integer valued; pictorial, used extensively in proportional reasoning research (as cited in Khoury, 2002).

4. Here is a picture of Mr. Tall and Mr. Short. (See picture on complete test in Appendix A) Mr. Short is six paper clips in height. If he is measured in large buttons, he is four large buttons in height. Mr. Tall is similar to Mr. Short but is six large buttons in height. Predict the height of Mr. Tall if you could measure him in paper clips. Explain your response.

- Non-routine; Comparison of Quantities, used extensively in proportional reasoning research (as cited in Wachsmuth I., Behr M., & Post T., 1983) (Figure 2).

5. Which pitcher will have the strongest lemonade flavor or will they taste the same? Explain.

- Missing value, non-integer valued, taken from the California Standards Test Released Test Questions from the seventh grade California Department of Education (2001) STAR (Standardized Testing and Reporting) mathematics test.

Figure 2. Lemonade problem.
6. The distance a spring stretches varies directly with the force applied to it. If a 7-pound weight stretches a spring a distance of 24.5 inches, how far will the spring stretch if a 12-pound weight is applied? Show all of your work.

A. 3.4 inches  B. 19.5 inches  C. 42 inches  D. 294 inches

• Non-integer missing value problem with extensive national data available. This problem was taken from the eighth grade NAEP (National Assessment of Educational Progress, 2007) test (Figure 3).

7. Both figures below show the same scale. The marks on the scale have no labels except the zero point.

The weight of the cheese is \( \frac{1}{2} \) pound. What is the total weight of the two apples?

Total weight of the two apples = _________ pounds.

Figure 3. NAEP problem.

• Cross multiplication problem taken from the California Standards Test Released Test Questions from the sixth grade California Department of Education (2001) STAR (Standardized Testing and Reporting) mathematics test (Figure 4).

A complete copy of the test is available in Appendix A.

After all students took the pretest, the seven randomly chosen students were interviewed. The interview was used to help establish the levels of proportional
understanding, or lack of proportional understanding. The students were given the opportunity to support their proportional thinking verbally during the interview. The interview protocol can be found in the Appendix B.

A unit was then taught covering proportional reasoning introducing ratio tables as an organizational tool. Ratio tables were used to encourage conversations about equivalence and grew from informal student strategies to a more generalized model, and as such, an important tool that students used to organize their proportional thinking.

An outline of the proportional reasoning unit that was taught can be found in Appendix C in PowerPoint format. It is divided into eight days, although the days do not have to be taught consecutively. Since percent problems are the first concept that is taught in the seventh grade curriculum that is conducive to ratio tables, this is where ratio tables are introduced.
The unit began with setting up ratios in a table format and finding equivalent ratios. It then progressed into a percent problem, “3 is what percent of 40?” This is the first time students experienced ratio tables, so tables were set up together with student-led interactions. After the idea of ratio tables was explained and demonstrated, two examples of student-generated ratio tables for “3 is what percent of 40?” are shown in Figure 5.

![Figure 5. Examples of student-generated ratio tables for “3 is what percent of 40?”](image)

After allowing students to work and share a few more sample problems in class, homework using missing value percent problems was assigned. The second day was spent with students sharing and defending their methods and/or ratio tables in their
groups and as whole class presentations. This helped students see there are different paths to the same outcome.

The third day, missing value problems were introduced with students generating tables and sharing answers. Each lesson had a set of homework problems for students to complete, and the following day answers were shared and defended within small groups and with the whole class.

Comparison problems were introduced next. One problem used from this section was, “I want to get some juice to take on my bike trip. I like both apple and orange. The orange juice pack holds more (12 oz) and it costs $1.70. The apple juice pack costs less, $1.10, but holds less (8 oz.). Which is the better deal?” This type of problems requires students to use two ratio tables and to generate ratios that are easily compared. The unit concluded with two days during which students worked on and discussed a variety of proportional situation problems, as well as non-proportional situations.

At the conclusion of this proportional reasoning unit using ratio tables as an organizational tool, the posttest was given to all pre-algebra students and the same seven students were interviewed.

Sample

Twenty-one students from a pre-algebra class in a rural Northern California school district participated in this study. They are heterogeneously grouped sixth- and seventh-grade students, including gifted, resource, English-Language Learners, and regular education program students. The classroom is arranged with students sitting in cooperative learning groups, four students per group. Students in the study filled out all
applicable permission forms which were signed by student and parents (Appendix E).

The only teacher involved in the study was the researcher.

All students in the class participated in the pre/posttest part of the study, but
only seven were randomly chosen to participate in a more thorough case study involving
interviews. Of the seven students, four were females, three were in sixth grade, four were
Hispanic and three Caucasian, one was in the gifted program, one in an Individualized
Education Plan for Attention Deficit Disorder, and one was an English Learner (high
level). Randomly selected, these students typified this pre-algebra class.

Treatment

The pretests and posttests were each scored twice. First, each was scored
using a general mathematics rubric that the author has used for many years covering all
areas of mathematics. This rubric is used by the author in the daily scoring of student
mathematics work. The tests were scored a second time using a proportional reasoning
rubric. The proportional reasoning rubric was adapted from research. (Baxter & Junker,

The criteria for determining the student’s level of general mathematical under-
standing for each problem was based on upon the following rubric levels.
Four Point General Mathematics Scoring Rubric

Level 0: Response indicates no appropriate mathematical reasoning or no response (nothing right)

Level 1: Response indicates some mathematical reasoning but fails to address the item’s main mathematical ideas (something is right about the problem) or there is only an answer with no work shown or described

Level 2: Response indicates some appropriate mathematical reasoning addressing the item but is not clearly communicated

Level 3: Response is correct and the underlying reasoning process is appropriate but not thoroughly and clearly communicated

Level 4: Response is correct and the underlying reasoning process is appropriate and is clearly communicated

The criteria for determining the student’s level of proportional reasoning for each problem was based on upon the following levels:

Five Point Proportional Reasoning Scoring Rubric

Level 0: No response

Level 1: Non-proportional reasoning

• Guess

• Randomly uses numbers, operations, or strategies

• Early attempts at quantifying which involve constant additive differences (i.e., $a - b = c - d$) rather than multiplicative relationships.
Level 2: Informal reasoning about proportional situations

- Uses pictures, models, or manipulatives to make sense of situations
- Makes qualitative comparisons

Level 3: Quantitative reasoning

- Still relies on counting up or down.
- Can only double, triple, halve, etc.

Level 4: Quantitative Reasoning/Multiplicative Reasoning

- Able to solve only problems that have an integer relationship between the ratios.
- The numbers in the target ratio have to be bigger than the numbers in the given ratio.

Level 5: Formal proportional reasoning

- Students are able to solve problems that involve both an integer and a non-integer relationship.
- Students recognize both between and within relationships in ratios
- Students are able to solve all three types of proportion problems:
  - Missing Variable
  - Numerical Comparison
  - Qualitative Prediction/Comparison Problems
- Minor arithmetic errors may exist, but do not detract from the correctness of the response or the reasoning (adapted from [Lamon, 1995, 2007; Steinthorsdottir, 2005; Baxter and Junker, 2002; Van De Walle, 2007; Khoury, H. 2002]).
Only seven students were randomly chosen to participate in a more thorough case study involving interviews. Interviews were conducted after the pretest and after the posttest. The interviews were used to help establish the levels of proportional understanding, or lack of proportional understanding and to gather information on how student were able to verbalize their proportional thinking. The interview protocol can be found in Appendix B.

Data Analysis Procedures

A paired difference $t$ test was used to compare posttest levels of each problem with pretest levels of the same problem for each student to determine whether significant change had occurred. The test is one-tailed as the author is only looking for improvement in levels of mathematical understanding and proportional reasoning. The data were tested to ensure it had a normal distribution.

Reliability was estimated using internal consistency by grouping questions in the test that measure the same type of proportional reasoning concept. This study has a testing threat due to multiple administrations of the same test to the same population. This has been controlled by creating a sufficient gap of time between the two administrations (the pretest was administered in October [(2008], the posttest at the end of April [2009]).
CHAPTER IV
RESULTS AND DISCUSSION

Presentation of the Findings

General Mathematics
Scoring Rubric Results

The overall results indicated that there was a significant difference between the posttest and pretest scores (posttest – pretest) in general mathematical understanding, \( t(20) = 7.4171, p = .0001 \) and the confidence interval provides a 95% chance that the post scores increased between 6.7 and 11.9 points (out of 28) from the pretest scores.

Table 1 shows the results from the general mathematics scoring rubric for each individual problem. Problem 8 is purposely omitted from this chart and was examined differently. The table shows there was a significant difference between problems 1 through 6, with a 95% confidence level. For problem 7, there is a significant difference less than 95% (94.7% confidence level).

Proportional Reasoning
Scoring Rubric Results

The overall results indicated that there was a significant difference between the posttest and pretest scores (posttest – pretest) in proportional reasoning \( (t(20) = 8.4821, p \leq 0.0001) \), and the confidence interval gives a 95% chance that the post scores increased between 8.7 and 14.4 points (out of 35) from the pretest scores. Table 2
Table 1

**Paired Difference General Mathematics Rubric**

<table>
<thead>
<tr>
<th>Problem #</th>
<th>Std. error mean</th>
<th>Lower</th>
<th>Upper</th>
<th>$t$</th>
<th>$Df$</th>
<th>Significance level (p-value)</th>
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<tbody>
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<td>0.22787</td>
<td>0.42492</td>
<td>1.3801</td>
<td>3.9704</td>
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<td>1.70518</td>
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<td>0.94640</td>
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<td>1.05502</td>
<td>2.0569</td>
<td>20</td>
<td>0.0265</td>
</tr>
</tbody>
</table>

shows the results from the proportional reasoning scoring rubric for each individual problem. Problem 8 is purposely omitted from this chart and was examined differently.

The table shows there was a significant difference between problems 1 through 6, with a 95% confidence level. For problem seven, there is a significant difference with less than 95% confidence (78.62%).

Problem 8 was taken from a released question from the California State Testing program for sixth grade mathematics. It is a multiple-choice question, indicating whether a student is capable of setting up a cross multiplication to solve the proportional problem. Data were released from the state in relation to statewide student accuracy.

These data were used to compare the author’s students with the statewide average. On the pretest, the author’s students had a 23% accuracy rate. On the posttest, they had a 52%
Table 2

**Paired Differences Proportional Reasoning Rubric**

<table>
<thead>
<tr>
<th>Problem #</th>
<th>Std. error mean</th>
<th>Lower</th>
<th>Upper</th>
<th>Test statistic</th>
<th>One tail significance level (p-value)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.80952</td>
<td>0.20531</td>
<td>1.41137</td>
<td>2.7948</td>
<td>20</td>
</tr>
<tr>
<td>2</td>
<td>1.71428</td>
<td>0.70520</td>
<td>2.72337</td>
<td>3.5437</td>
<td>20</td>
</tr>
<tr>
<td>3</td>
<td>3.47619</td>
<td>2.87198</td>
<td>4.08040</td>
<td>12.0011</td>
<td>20</td>
</tr>
<tr>
<td>4</td>
<td>0.71429</td>
<td>0.11830</td>
<td>1.31028</td>
<td>2.5000</td>
<td>20</td>
</tr>
<tr>
<td>5</td>
<td>2.19048</td>
<td>1.46189</td>
<td>2.91906</td>
<td>6.2714</td>
<td>20</td>
</tr>
<tr>
<td>6</td>
<td>2.23810</td>
<td>1.20104</td>
<td>3.27515</td>
<td>4.5018</td>
<td>20</td>
</tr>
<tr>
<td>7</td>
<td>0.38095</td>
<td>-0.237781</td>
<td>0.99969</td>
<td>1.2843</td>
<td>20</td>
</tr>
</tbody>
</table>

accuracy rate, compared to the state accuracy rate of 42%. Since this is a sixth grade state standard, the students’ pretest results were compared to the statewide results. The results indicated there was a 95% confidence level that the author’s students were significantly lower than the statewide results ($t(20) = -1.6889, p = 0.0496$) on the pretest. The posttest scores showed that although the author’s students were not significantly higher than the state average, there was less than a 95% chance they are different ($t(20) = -0.963, \ p = 0.167$).

Problem 3 was also taken from the released questions from the California State Testing program, but was from the seventh grade mathematics test. It is a multiple-choice test on percentages. On the pretest, the author’s students had a 14% accuracy rate; on the posttest, a 100% accuracy rate, compared to the state accuracy rate of 51%. The
results indicate that there was a significant difference between the author’s students’ posttest scores and the statewide percentages ($t(20)=4.4918, p = 0.000000$). So there is at least a $99.999\%$ confidence level that the students in the study improved over the statewide average.

Problem 7 was selected from the National Assessment of Educational Progress (NAEP) test. It is an eighth-grade level problem. On the pretest, the author’s students had a 19\% accuracy rate, and on the posttest, a 47.6\% accuracy rate, compared to the national accuracy rate of 49\%. The results indicate there was no significant difference between these students’ results and the national results ($t(20) = -0.1266, p = 0.89926$).

Discussion of the Findings

One goal of the study was to test whether the use of ratio tables affect the conceptual understanding of routine and non-routine proportional reasoning problems. This hypothesis was tested by comparing pretest and posttest scores using a proportional reasoning rubric, as well as interviewing the seven randomly selected students. The statistical analysis of research question 1, “How does the use of ratio tables affect the conceptual understanding of routine and non-routine proportional reasoning problems?” showed student use of ratio tables as an organizational tool did increase their ability to reason proportionally.

An in-depth item analysis of strategies students used to solve the problems showed there was an increased use of higher level proportional reasoning. More level 3 (Quantitative Reasoning), level 4 (Quantitative Reasoning/Multiplicative Reasoning), and
level 5 (Formal Proportional Reasoning), and less level 1 (non-proportional) and level 2 (informal proportional) strategies were being used when students chose to use ratio tables as a tool in their solutions. Approximately 30% of the answers were levels 3, 4, and 5 before ratio tables, over 70% were levels 3, 4, and 5 after the introduction of ratio tables (Appendix D).

This study showed the use of a ratio table lessens the use of inappropriate additive reasoning and increases multiplicative or appropriate proportional reasoning. Results from problem 5, the lemonade problem, showed a noticeable decrease of incorrect additive reasoning and an increase in proportional reasoning. On the pretest, all but five students used an inappropriate additive approach, thinking the lemonade would taste the same because the difference between the water and lemonade in each pitcher was one unit. Out of the five students who got a correct response, only one was able to verbalize why the answer was correct. After the introduction of ratio tables, the posttest showed a significant increase in how the students compared the flavors of the lemonade pitchers. Fifteen students correctly answered the question, with 11 of those able to completely and accurately verbalize their thinking process.

There were six missing values or routine-type proportional reasoning problems on the pre/posttest and two non-routine types of problems. This study shows that ratio tables are equally effective in helping students reason proportionally, whether solving a routine or non-routine proportional problem. Students showed an increase in the level of their proportional thinking on all problems, whether routine or non-routine (Appendix G).
Some literature alluded to the fact that ratio tables confine students to integer value proportional reasoning only, or problems that allowed students to use a building-up method (Steinthorsdottir, 2005). This study has shown that was not the case with this pre-algebra class. Students are capable of using a ratio table to solve both integer and non-integer proportional reasoning problems, as evidenced by their strategies and correct answers on the first three problems. The first problem was an integer value problem, while the second and third were non-integer value problems. All students except one were able to solve both an integer and a non-integer value proportion problem using a ratio table (Appendix D).

Results of Student Interviews

While the first research question was addressed more fully in the statistical analysis, the interviews and written work helped to explore the answer to the second research question, “How does the use of ratio tables influence the quality of verbalization, written or oral, of a student’s proportional thinking? What details are brought forth in written or oral descriptions of proportional reasoning when there is evidence of use of a ratio table?”

Problem 1 was a missing value problem with integer values. All but two of the case study students correctly answered the question on both the pretest and the posttest. Their pretest explanations were also fairly clear and most were easy to understand. Yet, the clarity of their explanations did improve and most used a more precise one-step method when they used a ratio table to organize their thoughts.
Selected Student Interviews

Problem 1 asked: Find the cost of 6 pieces of candy if 2 pieces cost 8 cents and if the price of the candy is the same no matter how many are sold.

DJ was a sixth grade Hispanic female student. DJ’s pretest interview:

DJ: It said two pieces cost eight cents, I thought, and both of them were eight cents.
Teacher: So did you find the total cost?
DJ: I put 16 cents. I don’t remember why.

DJ’s posttest interview:

DJ: Two pieces cost eight cents, and another two pieces cost eight cents, so that means four pieces cost sixteen cents and another two pieces for eight cents added to that makes 6 pieces for twenty-four cents.

MF was a seventh grade EL (English learner) Hispanic male student. MF’s pretest interview:

MF: At first, I figured out that two candies equal eight cents. Since we want six pieces, I timed six times eight and that equal forty-eight and that would be what I would get for how much it would cost, and then I divided it by two because its two pieces of candy. Then I got twenty-four.

MF’s posttest interview:

MF: If two pieces cost eight cents, then I multiplied the number of pieces by three and the cost by three and know that six pieces would cost twenty-four cents.

Problem 2 was a missing value problem with non-integer values: If 3 boxes of cereal are on sale for $6.90, and a daycare provider needs 17 boxes, how much will she pay? Explain your work.
Only two of the interviewed students got the answer correct on the pretest. On the posttest, five of the seven students arrived at the correct answer. All seven had a correct process, but two made multiplication errors. One of the students that arrived at a correct response both times was a gifted sixth grade male student named WB. While he was able to get the answer correct both times, the organization of his thoughts and how he described his procedure changed dramatically from the pretest to the posttest.

WB had work all over his paper and this is how he described his pretest work:

WB: I got this answer by first dividing 17 by 3. Then I divided 6.90 by 3 to find how much each cereal box was because of the remainder. Then I multiplied 6.90 by 5 to get part of the answer. I added 4.60 to the product of 6.90 and to get the answer of $39.10.

WB used a ratio table to set up his work and this is how he described his posttest work:

WB: 3 boxes cost $6.90. I multiplied both by 5 and know that 15 boxes cost $34.50. Therefore I know that 18 boxes would cost $41.40. (WB showed adding two ratios). I divided the original price of 3 boxes for $6.90 by three and found the cost of 1 box was $2.30. I subtracted the cost of one box from $41.40 and found 17 boxes cost $39.10.

Two other interviewed students, both seventh graders, got an incorrect answer and used an incorrect process on the pretest, but both were able to use a ratio table to correctly solve the same problem on the posttest. Their work is shown.

MF was a seventh grade EL (English learner) Hispanic male student. Figure 6 illustrates his pretest work.
Figure 6. MF’s pretest work for problem 2.

Figure 7 illustrates MF’s posttest work.

Figure 7. MF’s posttest work for problem 2.

DW was a seventh grade male Hispanic regular education student.

Figure 8 illustrates his pretest work.

Figure 8. DW’s pretest work for problem 2.
Figure 9 illustrates DW’s posttest work.

Not only did DW get the answer correct on the posttest, he did it with a unitizing method, showing proportional reasoning at the highest level.

Problem 5, the lemonade comparison problem, showed the students’ flexibility in the use of ratio tables. Students set the table up using many different approaches. Some used part-part and some part-whole and yet were still able to appropriately use proportional reasoning to compare the strength of the lemonade. On the pretest, none of the seven interviewed students was able to correctly answer the problem. On the posttest, six of the seven correctly set up the proportion, but two of those six incorrectly described their conclusions. (They struggled with which would be the stronger solution.) In interviews, both of these students reached correct conclusions as they talked about how they chose their answer. EM, a seventh grade Caucasian female student, was
the only one still unable to correctly solve this problem. She was also the only student who chose not to use any ratio tables on her posttest.

DW’s pretest and posttest responses to this problem were compared. On the pretest, he used an incorrect additive approach. On the posttest, he used a ratio table to set up a part to part comparison, comparing water to lemonade. He incorrectly identified the stronger mixture, but, in his interview, he corrected himself when he was trying to explain how he chose the stronger solution. Figure 10 illustrates DW’s pretest work.

![Figure 10. DW’s pretest work for problem 5.](image)

DW’s pretest interview:

**DW:** Because right here, in the first one, it has three lemonade mix and the other one has four, and then this one has two lemonade and three water. So you add four and two it equals six, and three and three equals six. So six and six are equal.
Figure 11 illustrates DW’s posttest work.

![Image of DW’s posttest work for problem 5.](image)

Figure 11. DW’s posttest work for problem 5.

DW’s posttest interview:

DW: So I put two different ratio tables for each one. Then I put water and the lemonade mix on top and then I put how much there was for the first jar. There was three water and two lemonade. Then here there was four water and three lemonade. Then I tried to get the same number right here to see which one would be greater in the top. So then I multiplied by two and by three and that equals six and six. Then I multiplied the top by the same number to and I got eight and nine. And if there was six lemonade mixes, there would be nine water and if there was six water mixes on this, there would be eight waters, so this one would have more.

Teacher: So more water makes it stronger?

DW: Oh yeah. Yeah. I think it was this one because it has less water and it has the same amount of mix. [DW changed his answer to jar 2]
KT was a sixth grade Hispanic female student. She also used an incorrect additive approach on the pretest, but then was able to compare using a part to part (lemonade to water) comparison on the posttest. Her pretest interview:

KT: They were the same because they each had one more cup of water than they did of lemonade mix.

Figure 12 shows KT’s posttest work.

![Figure 12. KT’s posttest work for problem 5.](image)

KT’s posttest interview:

KT: The lemonade, I put the lemonade mix on top and the water on the bottom and it had two lemonade mix and then three waters, so I put two over three. Then for No. 2, it had three lemonade mix and four water, so I tried to get the same bottom number and I just times by three, and then up here I times it by four, and I got 12. Then the lemonade mix on No. 2 was nine, and on No. 1, it was eight. So this one had more flavor. [She pointed to the second pitcher]
NP is a seventh grade Caucasian female enrolled in the regular education program, but on an IEP (Individualized Education Plan) for ADD (Attention Deficit Disorder). She also used an incorrect additive approach to the comparison problem on the pretest, but, on the posttest, she used a part-to-whole comparison and was able to identify the stronger mixture correctly. Figure 13 shows NP’s pretest work.

![NP's Pretest Work](image)

*Figure 13. NP’s pretest work for problem 5.*

NP’s pretest interview:

NP: We had to find which cup of lemonade would be stronger, and I said I thought they would be the same because in each glass, there would be one more cup of water than the lemonade. So I thought they would be the same.
Figure 14 shows NP’s posttest work.

![Figure 14. NP’S posttest work for problem 5.](image)

In the analysis of question 2, “How does the use of ratio tables influence the quality of verbalization, written or oral, of a student’s proportional thinking?,” the study found students using ratio tables have greater depth to their written or oral explanations. Ratio tables appear to allow students to organize their thoughts in such a way that it is easier for them to explain their proportional reasoning. Generally, the author recognized that student interviews were necessary in attempting to understand student thinking on the pretest work. When students organized their work into a ratio table on the posttest, for the most part, their reasoning was clear and they were confident that the ratio table spoke for itself; students felt there was not much to explain that was not self-evident in the table. Students who were properly setting up a ratio table, taking time to read and analyze
where to put the numbers, and labeling, were more likely to correctly use proportional reasoning.

Most of the students in the study showed that ratio tables significantly helped them to reason proportionally. Pre- and posttest proportional reasoning levels were assigned to each student by the author. Using the Proportional Reasoning Rubric described earlier, students were assigned a level using a checklist of their proportional reasoning skills. A level 5 indicated they had shown the ability to solve integer, non-integer, routine, and comparison proportional problems. At the end of the study, nine of the 21 students were assigned level 5. Four students were between levels 4 and 5: while they could solve both integer and non-integer proportion problems, they struggled with the comparison problems. Six students were between levels 3 and 4, as their work showed they still had some difficulties not only with the comparison problem, but also in working with some of the non-integer problems. Only one student was lower than level 3, and even she showed growth, moving from a level 1 proportional thinker on the pretest (see chart in Appendix D).

The students in this study were able to outscore their peers when compared to state and national averages. Although these students were not directly taught how to set up the cross multiplication method of solving a proportion problem, when presented with the state test problem (problem 8), which directly asked them to set up a cross-multiplication problem, they were still able to vie with the statewide average. On the percent problem taken from the state test, these students significantly outscored their
statewide counterparts, having a 100% accuracy rate, compared to the statewide 51% accuracy rate.

The average level of proportional reasoning on the five-point rubric before ratio tables were introduced was about level 2: students were using informal reasoning about proportional situations. After introduction of ratio tables, this level rose to almost level 4, indicating that students had advanced to using quantitative multiplicative reasoning. Every student’s proportional reasoning level increased (Appendix F).
CHAPTER V

SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

Proportional reasoning is considered to be the capstone of elementary mathematics and the cornerstone of algebra and higher mathematics (Lesh et al., 1988). Most mathematics textbooks place little emphasis on this important concept. The purpose of this thesis was to examine whether introducing ratio tables would encourage students to reason proportionally. The research was conducted in a rural middle school in Northern California, in a pre-algebra classroom consisting of sixth and seventh grade students. A pretest was given in the fall; two weeks of instruction and student-led discussions on ratio tables followed in the spring, with a posttest given at the end of the unit. To gather more in-depth understanding of students’ proportional thinking, student interviews were conducted with seven randomly chosen students from the pre-algebra class.

The results obtained offer convincing evidence that ratio tables do affect the development of proportional thinking in pre-algebra students in a positive manner. Ratio tables did help the students in this study reason proportionally. Ratio tables are a powerful tool for developing and encouraging proportional reasoning. While the majority of textbooks only introduce proportions with cross multiplication, it should be pointed out that premature use of rules encourages students to apply rules without thinking and, thus, the ability to reason proportionally often does not develop (Van De Walle, 2007).
Therefore, ratio tables can and should be used as an organizational tool to foster students’ development of proportional reasoning.

This conclusion is based on a limited sample population. The data were collected from only 21 students. When the study was planned, the author had two pre-algebra classes with over 60 students. The class schedule was changed over the summer and only one pre-algebra class was available for the study. Thirty-two students were given the initial pretest. At semester (in January), class schedules were changed so that when the posttest was given, although there were still 32 students, only 21 were the same students who had been given the pretest. A larger sample size might provide stronger evidence or change the conclusion.

A recommendation would be to expand the study beyond the scope of this one classroom. Another recommendation would be to give a follow-up test to the same 21 students to find out how they scored after time had passed from the initial lesson on ratio tables. From experience as a teacher using ratio tables for several years, students seem to be able to recall how to set up a ratio table and solve a problem proportionally much longer than they are able to remember how to set up a cross multiplication problem, as was evidenced by the fact that not one student tried to use a cross multiplication method on the pretest, although they were taught this method the previous year.

Proportional reasoning is a powerful mathematics topic that warrants more emphasis in the middle school years. Ratio tables are a method that can easily be implemented into any classroom to encourage students to develop their proportional reasoning
ability. Any future studies which look at methods that are directed at helping students reason proportionally are highly encouraged.
REFERENCES
REFERENCES


APPENDIX A
PRETEST AND POSTTEST QUESTIONS

Missing Value Problems:

1. Find the cost of 6 pieces of candy if 2 pieces cost 8 cents and if the price of the candy is the same no matter how many are sold. Show all of your work and explain.

2. If 3 boxes of cereal are on sale for $6.88, and a daycare provider needs 17 boxes, how much will she pay? Explain your work.

3. A sweater originally cost $37.50. Last week, Moesha bought it at 20% off.

   How much was deducted from the original price?
   - A $7.50
   - B $17.50
   - C $20.00
   - D $30.00

4. Here is a picture of Mr. Tall and Mr. Short. Mr. Short is six paper clips in height. If he is measured in large buttons he is four large buttons in height. Mr. Tall is similar to Mr. Short but is six large buttons in height. Predict the height of Mr. Tall if you could measure him in paper clips. Explain your response.
5. Which pitcher will have the strongest lemonade flavor or will they taste the same? Explain

6. The distance a spring stretches varies directly with the force applied to it. If a 7-pound weight stretches a spring a distance of 24.5 inches, how far will the spring stretch if a 12-pound weight is applied? Show all of your work.

A 3.4 inches  
B 19.5 inches  
C 42 inches  
D 294 inches
7. Both figures below show the same scale. The marks on the scale have no labels except the zero point.

The weight of the cheese is \(\frac{1}{2}\) pound. What is the total weight of the two apples?

Total weight of the two apples = _______ pounds.

8. A farmer harvested 14,000 pounds of almonds from an 8-acre orchard. Which proportion could be solved to find \(x\), the expected harvest from a 30-acre orchard?

A \[ \frac{8}{14,000} = \frac{x}{30} \]

B \[ \frac{8}{14,000} = \frac{30}{x} \]

C \[ \frac{30}{14,000} = \frac{x}{8} \]

D \[ \frac{30}{14,000} = \frac{8}{x} \]
INTERVIEW PROTOCOL

This interview protocol will be used for both the pre and post interview. The interview will be an additional tool, used after the test to help establish the levels 1 through 4 of proportional understanding, or lack of understanding. Students that have missing written work, or written explanations that are hard to follow will be given the opportunity to support their proportional thinking verbally by being interviewed.

Depending on what the student written work already shows, any or all of the following questions may be used to get students to verbalize their thoughts about their problem-solving process. To gain insights into students’ conceptualization, probing questions will be ask without correcting computational or problem solving errors.

- Tell me about the process you used when solving this problem.
- Why did you start with _____ (multiplying, dividing, subtracting, or whatever operation was first used)?
- What were you thinking when you did this step?
- What does that stand for?
- Why did you pick this strategy to solve this problem?
- Why did you change your mind?
- How did the ratio table you used help you to solve the problem?
- What patterns were you using?
- How confident are you that your answer is correct?
These questions will be used to allow more accurate ratings of the student level of proportional reasoning.
APPENDIX C
LESSON PLANS FOR UNIT ON RATIO TABLES

Slide 1

Using Ratio Tables to Encourage Proportional Reasoning

A Unit to Help Teach Ratio Tables

Slide 2

Ratio Tables Defined:

1) The table consists of two (or more) rows and two or more columns, with numbers in the cells.
2) The rows have a label, indicating the meaning of the numbers and specifying, if needed, the units used.
3) There is no preference on what to choose as the upper or lower row.
4) The ratio between the numbers in the columns is the same for all columns; this can be used to calculate an empty place in a column.
5) To get the numbers of a column, the numbers of another column can be multiplied or divided by a certain number; proportionately adding or subtracting are possible as well.

(Broekman, 2000)

Slide 3

DAY ONE:

- Introduction of Ratio Tables
- Using Ratio Tables to Find Missing Values in Percent Problems.

Slide 4

3 is what percent of 40?

7 ½ %

Allow a few minutes for students to think about how they would construct their own ratio table. Then, as a class construct a ratio table, allowing the student’s input to guide the construction.

If it doesn’t come up, show how equivalent ratios could be added within the table to generate other equivalent ratios.

For example, if a ratio table is being constructed that needs 2 ½ times an
Let’s try another percent problem.

12 is what percent of 27?

<table>
<thead>
<tr>
<th>part</th>
<th>12</th>
<th>27</th>
<th>100</th>
</tr>
</thead>
</table>

One possible solution.

One more problem:

3 is what percent of 22?

Amount, sometimes for students it is easier to multiply by 2 and then by \( \frac{1}{2} \) and then to add the two ratios. Most students will be able to do both of these operations mentally, whereas multiplying by \( 2 \frac{1}{2} \) is a more difficult task.

Usually, I first introduce ratio tables with percent problems. Seventh grade students have been exposed to different strategies to solve percent problems, although ratio tables may be new to them.

It is very important to allow students the opportunity to find their OWN path to the answer. Part of the power of ratio tables lies in the fact that students are generating their own tables, using their own strategies. Some strategies will definitely be more advantageous than others. That is why student sharing and discussion must take place. Students will see other’s methods and assimilate these strategies with their own, hopefully arriving at an efficient method.
Slide 8

Student Questions?

Click here to jump to the properties slides.

Slide 9

Homework Problems:

1) What percent out of 30 is 17?
2) 30 is what percent of 27?
3) 15% of 35?

These problems need to be completed by students for homework. Tell them to be prepared to share and defend their answers.

Slide 10

DAY TWO:

- Students share and defend their answers using the ratio tables they generated for homework.

Allow students to share within their groups and then have students prepare transparencies of their methods to share with the whole class. Advantages to certain strategies should be shared.

Slide 11

DAY THREE:

- Missing Value Proportion Problems
- Using Ratio tables to find missing values in proportion problems.

This would not have to be taught directly after day 2, it is the third lesson in this sequence of lessons.
Slide 12

Missing Value Proportion Problem:

If we measure Mr. Peterson’s height in basketballs, he is 6 balls tall. He is also as tall as a pile of 15 boxes. His son, Matt, is as tall as 4 basketballs. How many boxes will be as tall as Matt?

<table>
<thead>
<tr>
<th>Basketball</th>
<th>6</th>
<th>2</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boxes</td>
<td>15</td>
<td>5</td>
<td>10</td>
</tr>
</tbody>
</table>

Matt’s height = 10 Boxes

Lamon (2005)

Once again, allow the students to generate the table and share their methods. Students may set the table up in different fashions. This example model is only ONE way to set up the table. For example students might generate a table comparing 6 basketballs to 4 basketballs, and 15 boxes to ? boxes.

Slide 13

Missing Value Proportion Problem:

If it took Jane 3/4 hour to paint a wall that was 12 ft by 12 ft, how long will it take to paint another wall that is 15 ft by 16 ft?

<table>
<thead>
<tr>
<th>Hours</th>
<th>3/4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Square feet</td>
<td>144</td>
</tr>
</tbody>
</table>

1 1/4 hours

Lo (2004)

This problem will take a bit more effort for students to set up. Even though this is a missing value problem, more than three values are given, forcing students to decide which values will make up the ratios. This problem requires students to think before putting the numbers in the table. First they will have to figure the area to find out how long it takes to paint a square foot. Plus they will have to work with a fraction within a ratio, or students might change hours to minutes so they don’t have to work with a fractional value.

There are numerous ways to solve this problem. The discussion and student interaction will be valuable.

Remember, the solution shown is only ONE possible way to use a ratio table to generate the answer.

Slide 14

Homework Problems:

1) In 3 weeks, a horse eats 10 pounds of hay. How much will he eat in 5 weeks?
2) You watch TV for 12 1/2 hours per week. How much TV do you watch in a year?
3) If a box of detergent contains 80 cups of powder and your washing machine recommends 1 1/4 cups per load, how many loads can you do with one box?

Explain your answers. Be prepared to defend and share your answers.

Lamon (2005)

These problems need to be completed by students for homework. Tell them to be prepared to share and defend their answers.
Slide 15

**DAY FOUR:**

- Students share and defend their answers using the ratio tables they generated for homework.

Allow students to share within their groups and then have students prepare transparencies of their methods to share with the whole class. Advantages to certain strategies should be shared.

Slide 16

**DAY FIVE:**

- Comparing
- Using Ratio tables to compare quantities.

I have separated the lessons by days, but the days do not need to be consecutive. Although the homework discussion needs to follow the day of the lesson.

Slide 17

Comparing Problem:
Which of the following cartons has more brown eggs?

<table>
<thead>
<tr>
<th># brown eggs</th>
<th># eggs in carton</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>12</td>
</tr>
<tr>
<td>7</td>
<td>18</td>
</tr>
</tbody>
</table>

Students will need to set up two different ratio tables and come up with values that they are able to compare. These problems will bring about a good opportunity to discuss multiplication reasoning/additive reasoning.

Students using additive reasoning will say the carton of 18 contains more brown eggs. Urge students to notice the fact that, although one carton has less brown eggs, it also has less eggs in the carton. Encourage students to think beyond additive to multiplicative reasoning.

Slide 18

Brown Egg Problem Continued:

<table>
<thead>
<tr>
<th># brown eggs</th>
<th># eggs in carton</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>12</td>
</tr>
<tr>
<td>7</td>
<td>18</td>
</tr>
</tbody>
</table>

Some students may stop after writing the two ratios and notice that 7 out of 18 is less than half, while the other carton has exactly half brown eggs, therefore the carton of 12 contains more brown eggs.

Others may change the first carton to 18 eggs, and be able to see that if both cartons contained 18 eggs, the first carton would have more brown eggs.

Still other students may change both to cartons of 36 and compare.
Slide 19

Brown Egg Problem Continued:

<table>
<thead>
<tr>
<th># brown eggs</th>
<th># eggs in carton</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>12</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>7</td>
<td>18</td>
</tr>
</tbody>
</table>

9 brown out of 18 eggs is greater than 7 brown out of 18 eggs.

Some students may stop after writing the two ratios and notice that 7 out of 18 is less than half, while the other carton has exactly half brown eggs, therefore the carton of 12 contains more brown eggs.

Others may change the first carton to 18 eggs, and be able to see that if both cartons contained 18 eggs, the first carton would have more brown eggs.

Still other students may change both to cartons of 36 and compare.

Slide 20

Brown Egg Problem Continued:

<table>
<thead>
<tr>
<th># brown eggs</th>
<th># eggs in carton</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>12</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>7</td>
<td>18</td>
</tr>
<tr>
<td>18</td>
<td>36</td>
</tr>
</tbody>
</table>

18 brown eggs out of 36 eggs is greater than 14 brown eggs out of 36 eggs.

Some students may stop after writing the two ratios and notice that 7 out of 18 is less than half, while the other carton has exactly half brown eggs, therefore the carton of 12 contains more brown eggs.

Others may change the first carton to 18 eggs, and be able to see that if both cartons contained 18 eggs, the first carton would have more brown eggs.

Still other students may change both to cartons of 36 and compare.

Slide 21

Comparing Problem:

I want to get some juice to take on my bike trip. I like both apple and orange. The orange juice pack holds more (12 oz) and it costs $1.70. The apple juice pack costs less, $1.10, but holds less (8 oz.). Which is the better deal?

Cost $1.70

<table>
<thead>
<tr>
<th>Ounces</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>$1.70</td>
</tr>
<tr>
<td>8</td>
<td>$1.10</td>
</tr>
</tbody>
</table>

Better Buy!!

Students will need to set up two different ratio tables and come up with values they are able to compare. These problems will bring about a good opportunity to discuss multiplication reasoning/additive reasoning.

Once again (like the egg problem) there are many ways to accurately compare the two units.
Homework Problems:

1) A car traveling at a speed of 65 mph and a truck was traveling on the same road at a speed of 60 mph. They came to a hill and both vehicles slowed down by 15 mph as they climbed the hill. Which vehicle had a harder time maintaining its speed?

2) Jo has two snakes, String Bean and Slim. Right now, String Bean is 4' long and Slim is 5' long. Jo knows that two years from now, both snakes will be fully grown. At her full length, String Bean will be 7' long, while Slim's length when he is fully grown will be 8'. Over the next two years, will both snakes grow the same amount?

Be prepared to explain and defend your answers.

These problems need to be completed by students for homework. Tell them to be prepared to share and defend their answers.

DAY SIX:

- Students share and defend their answers using the ratio tables they generated for homework.

DAY SEVEN:

- A variety of Proportional Reasoning Situations.

I have separated the lessons by days, but the days do not need to be consecutive. Although the homework discussion needs to follow the day of the lesson.

Some problems to think about:

1) A 16 oz box of Bites cereal costs $3.36 and a 12 oz box of Bits cost $2.64. If both cereals are of equal quality, which cereal is the better buy? Explain.

Students will need to set up two different ratio tables and come up with values they are capable of comparing. These problems will bring about a good opportunity to discuss multiplication reasoning/additive reasoning.
2) The Science club has four separate rectangular plots of land for experiments with plants.
1 foot by 4 feet
7 feet by 10 feet
17 feet by 20 feet
27 feet by 30 feet
Which rectangular plot is most square? How do you know your answer is correct?

(Van de Walle, 2007)

3) A person who weighs 160 pounds on Earth will weigh 416 pounds on the planet Jupiter. How much will a person weigh on Jupiter who weighs 120 pounds on earth? Explain your work.

(Van de Walle, 2007)

4) At the local college, five out of every eight seniors live in apartments. How many of the 35 senior math majors are likely to live in an apartment? Explain your answer.

(Van de Walle, 2007)

5) Cheese is $4.25 per pound. How much will 12.13 pounds cost. Explain how you solved this problem.

(Lamon, 1999)

6) 13% of 62 is what number? Show how you arrived at your answer.

Students should plan ahead and mentally devise a plan to help them solve problem number 5.

These problems need to be completed by students for homework. Tell them to be prepared to share and defend their answers.
DAY EIGHT:
• Students share and defend their answers using the ratio tables they generated for homework.

Some students strategies will become very efficient, to the point of using two steps, dividing to get a unit ratio, then multiplying to get some known quantity, similar to the cross multiplication algorithm. Other students will continue to build tables they may not be the most efficient, but are generated with understanding at their skill level. The advantageous, efficient strategies need to be pointed out and shared with the whole class.

Rest of Year:
Ratio tables will continue to be presented throughout the rest of the year whenever applicable.

It is understood that ratio tables are only one tool of many tools needed to encourage students’ proportional thinking. Yet, while ratio tables are easy for teachers to implement they may result in encouraging powerful proportional thinking.

This should be a reminder to seventh grade students about the identity property. Students should be familiar with the identity property, but review is often needed and valuable to do because this property is such an important part of ratio tables.

An equivalent ratio is found by multiplying (or dividing) both parts by the same number.

To be able to use ratio tables to their full extent, it is useful to explore adding ratios to get equivalent ratios. Allow students to guess what the answer will be, then discuss fraction addition versus ratio addition.
First, remind students they are adding RATIOS, not fractions. Remind them a ratio is a relation between two amounts.

One way to explore this is to think about 3 out of 6 as being copies of 1 out of 2. Instead of doing 3 copies of 1 out of 2 right away, we could do 1 copy and then 2 copies and add to get the same answer as if we had made 3 copies right away. By distributive property, we can show this to be true. Seventh grade students should be familiar with the distributive property.

Although adding equivalent ratios together produce equivalent ratios, it might be beneficial at this point to talk to students about the misconception that adding a value of one to a ratio will also produce an equivalent ratio.

For example: 1 out of 2 plus 2 out of 2 is not equivalent to 3 out of 4. Or, ½ + 2/2 does not equal ¾.

Allow a few minutes for students to think about how they would construct their own ratio table. Then, as a class construct a ratio table, allowing the student’s input to guide the construction.

If it doesn’t come up, show how equivalent ratios could be added within the table to generate other equivalent ratios.

For example, if a ratio table is being constructed that needs 2 ½ times an amount, sometimes for students it is easier to multiply by 2 and then by ½ and then to add the two ratios. Most students will be able to do both of these operations mentally, whereas multiplying by 2 ½ is a more difficult task.
Both methods will give the same answer. This shows the power of ratio tables, students are able to construct ratio tables at their own pace, building at their own skill level.

Slide 37

Multiplying by 2 ½ the first time.

<table>
<thead>
<tr>
<th>part</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>total</td>
<td>40</td>
</tr>
</tbody>
</table>

X 2 ½

We still get 7 ½ %

Slide 38

One more problem:

<table>
<thead>
<tr>
<th>part</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>total</td>
<td>22</td>
</tr>
</tbody>
</table>

3 is what percent of 22?

Slide 39

Another way to think about the addition:

\[
\frac{12}{88} + \frac{\frac{1}{2}}{11} + \frac{3}{22} = \frac{12 + \frac{11}{22} + \frac{3}{22}}{88 + 11 + 1} = \frac{13 + \frac{7}{11}}{100} = \frac{13}{100} \cdot \frac{7}{11} = \frac{7}{11}
\]

Slide 40

To Return to DAY ONE HOMEWORK, click here
## FIVE POINT PROPORTIONAL REASONING

### SCORING RUBRIC

<table>
<thead>
<tr>
<th>5 Point Proportional Reasoning Scoring Rubric</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student</td>
</tr>
<tr>
<td>---------</td>
</tr>
<tr>
<td>LT</td>
</tr>
<tr>
<td>LG</td>
</tr>
<tr>
<td>TF</td>
</tr>
<tr>
<td>JG</td>
</tr>
<tr>
<td>GG</td>
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<tr>
<td>LG</td>
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<tr>
<td>NS</td>
</tr>
<tr>
<td>TI</td>
</tr>
<tr>
<td>MI</td>
</tr>
<tr>
<td>SV</td>
</tr>
</tbody>
</table>
Dear Parent / Guardian,

I am your students’ current mathematics teacher. For the past two summers I have been working on a master’s degree in Mathematics Education at CSU Chico. As part of my work I will be conducting a research study this year. I will be using assessments of students’ proportional reasoning anonymously for a research study I am doing in order to fulfill the requirements for a master’s degree at CSU Chico. There are no anticipated risks or benefits to students resulting from this study, nor can anyone be identified from their work. Although these assessments of students’ proportional reasoning skills are required coursework, their participation in this research is voluntary and confidential. Based on the results of their proportional reasoning assessments, some students may also be asked to participate in an audio-recorded one-on-one interview with me at school. The interview will take place during your students’ regular math class. They can choose to opt out of the study with no penalty and without affecting their grade. Thank you for your time and continued support.

Sincerely,

Rita Nutsch
**Parent Permission**

I, _____________________________, give my permission for: (Please check one or both)

Parent / Guardian (please print)

☐ My student’s results to be included in the research project described.

☐ My student to participate in a one-on-one interview with Ms. Nutsch if chosen. I understand that if my student is chosen for an interview I will be notified at least one week prior to the interview and his/her identity will be protected in the study.

_________________________  _________________________  __________
Student’s Name    Parent / Guardian’s Signature  Date

**Student Permission**

I, _____________________________, give my permission for: (please check one or both)

Student (please print)

☐ My results to be included in the research project described.

☐ Me to participate in a one-on-one interview with Ms. Nutsch if chosen. I understand that if I am chosen for an interview I will be notified at least one week prior to the interview and my identity will be protected in the study.

Please return this sheet to Ms. Nutsch and keep the first page for your records.
AVERAGE PROPORTIONAL REASONING LEVEL
BEFORE AND AFTER INTRODUCTION
OF RATIO TABLES

Average Proportional Reasoning Levels Before and After Introduction of Ratio Tables

Student

Average Before Ratio Tables
Average After Ratio Tables
AVERAGE PROPORTIONAL REASONING

LEVEL BY PROBLEM NUMBER

Average Proportional Reasoning Level by Problem Number