CHALLENGES IN CREATING AND IMPLEMENTING A UNIT

ON PROOFS AND QUADRILATERALS BASED ON

WORKED EXAMPLES PRINCIPLES

A Thesis

by

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in

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DEDICATION

I would like to dedicate this work to my dad and my uncle Jim. Two of the calmest and coolest minds I ever met.

I would also like to dedicate this work to my wife, who constantly gives me support and encouragement in all my wheelings-and-dealings.

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ABSTRACT

CHALLENGES IN CREATING AND IMPLEMENTING A UNIT ON PROOFS AND QUADRILATERALS BASED ON WORKED EXAMPLES PRINCIPLES

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The concept of a worked example is simple. It is an expert's problem solving strategy that students use to study and learn. Previous research has repeatedly shown that using worked examples in the classroom can drive student learning in many content domains. Effective worked examples vary in structure depending on the learning goal. Some examples are created for students to study to learn about the process used in solving a problem, while others require students to complete missing steps or justifications in the problem solving process. This study explored the effectiveness of a unit based on the principles of worked examples to introduce and teach proofs to high school geometry students. In creating the unit, research was conducted into cognitive load theory and the principles of effective worked examples. Students' posttest scores were used to determine if students learned better with the worked examples unit when compared to students who learned the same material through a traditional teaching method. Student surveys were used to determine if the worked example unit improved students' self-confidence in math class. It was found that the unit did not improve student posttest scores or alter their self-confidence. While the unit was shown to be ineffective, this study gives insight into how a practicing teacher can apply the research on worked examples principles to make reasonable changes to their classroom materials and instruction methods with the aim of improving students' understanding of geometric proofs.

CHAPTER I

INTRODUCTION

Identification of Problematic Situation to be Investigated

In many high school classrooms, teachers and students use a textbook to guide the curriculum for the school year. Texts offer basic information, examples and practice for students. There are drawbacks from solely using textbooks, however. For example, students can get lost in the text if the examples provided are not thorough enough or students might need help on topics that are not fully explained. Teachers tend to recognize that supplemental material and exercises will be needed, so when planning ahead they will take material from other sources or create their own that they feel will be beneficial for student learning. The principle goal of any teaching or training is that what is learned survives beyond the short term and into a later time when that knowledge or skill is required in a real-world setting (Bjork, 1994).

How can a teacher design supplemental materials that do this effectively? There are numerous studies conducted every year attempting to understand what constitutes effective classroom material. A tool proven to be effective is a collection of worked example exercises (Sweller & Cooper, 1985; Tarmizi & Sweller, 1988; Chi, Bassok, Lewis, Reimann, & Glaser, 1989; Ward & Sweller, 1990). My initial goal is to apply the research on effective worked examples and create a geometry unit based solely around worked examples.

Before proceeding with the description of this project it is necessary to discuss what worked examples are and take note of their features. Figure A below is a worked example from an algebra class along with a traditional prompt (Carroll, 1992). The example is completed with descriptions of what is involved in each step and is paired with a second incomplete example asking the young mathematician to come up with her own explanation to arrive at the solution.

Worked Example with Prompt
When multiplying two binomials exactly the same except for one pair of signs, the product is a binomial of the form $x^2 – y^2$, which is also called a difference of squares.

Multiply: $(x + 4) (x – 4)$
$x^2 + 4x – 4x – 16$ Distribute the terms
$x^2 – 16$ Combine like terms

Multiply: $(x + 3)(x – 3)$
$x^2 – 9$ Here is another completed problem.
What steps do we take to arrive at this solution?

Traditional Prompt
Multiply: $(x – 5)(x + 5)$

Figure A

Worked Example Compared to Traditional Prompt

The initial phase of this project will be designing the unit based on these principles. There are different aspects to consider when introducing and using worked examples: (1) using research to create effective worked examples, (2) the challenges of introducing worked examples, (3) student engagement during the introduction and use of worked examples, (4) student perception of themselves in math class, and (5) the changes in student work before, during, and after the worked example unit. Before proceeding further it is necessary to elaborate on these aspects and why they deserve attention.

1. Using Research to Create Worked Examples

There have been numerous studies conducted on the effectiveness of worked examples in topics ranging from mathematics and physics to art history and music instruction. While the research is thorough and supported by empirical data, applying the information can be challenging because of how the articles are written, and the examples used in studies are described rather than displayed. The language used by researchers can be difficult to decipher by a practicing teacher. From my initial experience reading research articles, the use of special terms and phrases seems equivocal in that they do not hold much information if the reader is not versed in that style of writing. There has been a shift away from teachers using such terms because the models for addressing educational concerns have been more political rather than scientific in nature. Stanovich & Stanovich note that many teachers rely on the latest fad as the best way to teach, please a principal, or address local school reform (2003). These fads are usually not described with the language of empirical science. For me, it was necessary to become familiar with the education research language in the articles in order to understand the aims and methods of the research. Doing so was not a major hurdle, but it did take some re-readings to become acculturated with the lexicon. With the research decoded, the challenge became creating worked examples based on the research.

Just as research has shown studying worked examples improves learning, it would be useful to study different worked examples in order to know what effective worked examples look like. In their writings, researchers give examples and descriptions of the effective examples used in their studies, but the worked examples' structures vary from study to study depending on the learning goal of the example and what features of examples they are experimenting with.

For a practicing teacher, a challenging aspect of worked examples is knowing what features of the example to direct students' focus to give them an optimum learning experience. A second challenge for teachers is knowing how to modify or construct examples from a given curriculum that does not address a certain topic adequately. A way to address these challenges is to understand the eight key principles of worked examples described later in this paper. When appropriately applied these eight principles can help focus on the mathematics that students need to learn in each lesson (Gerjets, Scheiter, & Catrambone, 2004). The eight principles defined by Shen and Tsai (2009) and Atkinson, Derry, Renkl, and Wortham (2000) are listed in Table 1 below and will be elaborated on in this paper.

Table 1

Worked Examples Principles
Imagination
Completion
Fading
Process
Presentation
Media
Timing
Self-Explanation

2. Introducing Worked Examples

Introducing something new to a classroom of students can be challenging. It takes time for students to become acculturated to new terms, visuals, ideas, and skills (D'Amato, Lux, Walz, Kerby, & Anderegg, 2007). In a math class in which some students already feel out of place, introducing worked examples will take some getting used to. It is also important that students use them for their intended purpose. In each example there will be particular items for students to attend to, and they will be required to use the examples in new ways – taking time to reference the steps, understanding which steps are not shown, mentally holding the goal of the example while working through the exercise, etc. Introducing new tools is not daunting, but care must be given upon introduction in order for students to be successful with them.

3. Student Engagement During the Introduction and Use of Worked Examples

In any lesson or classroom activity, students must be engaged for learning to occur. Time on task and purposeful practice increases achievement. For students to be engaged, the instructions within each example must be clear so that the core mathematical concepts of the exercise will be clear. Improperly designed material can lead students away from the learning goals (Caldwell, Darling, Payne, & Dowdy, 1999). For example, the material can be worded or structured in such a way that too much effort is placed on deciphering what the goals of the material are. For some tasks complete directions are easily written. Other tasks that are more complicated might require extra verbal instructions, demonstrations, or visuals.

4. Students' Perception of Themselves in Math Class

After creating a worked example unit, I want to observe students' perception of their understanding of mathematics in order to determine how they value their continued effort in the class (Garriott, Flores, & Martens, 2013). Geometry as a subject does not receive the same attention in high school graduation requirements as Algebra in that many California districts require that all graduating seniors be proficient in Algebra. Geometry is overshadowed by Algebra 2 in that passing the latter is an A-G University of California and California State University high school prerequisite, as well as a predictor of first year college success. At our school, students typically progress up to Geometry by receiving a grade of D or better in the prior math course. Upon entering Geometry, which is the second course in the traditional secondary mathematics track, there will be a wide range of ability, so some students fall behind. Students who fall behind in the Geometry curriculum will likely struggle in Algebra 2.

By creating this unit in a Geometry course, I would like to observe how students perceive themselves in class before and after using worked examples. Will this unit have an effect on how students perceive their ability in the class? Will students feel more prepared and capable in math class?

5. Quality of Student Work

An issue that arises in nearly every class at any compulsory level is the quality of student work. Does it follow a logical progression? Does it make sense? Does it contain the necessary information? Is it legible? These key questions must be attended to by the teacher to determine students' graded performance in class. If the activity calls for peer grading, then the peer needs to be able to answer the same questions. What constitutes quality work can be confusing to students due to the relatively subjective nature of quality (Grainger, Purnell, & Zipf, 2008). Similarly, work that one teacher accepts might not be accepted by another teacher. As students progress from one class to the next, they take the skills and habits they learn with them. Therefore, class time should be spent analyzing completed assignments and acknowledging what type of work is acceptable.

Acculturation was mentioned above in regards to learning new vocabulary, and quality is another facet of school work that requires students to learn something new and use it with fluency. Just like any skill, practice makes perfect. Practice comes from doing, observing, and applying what was observed. A helpful teaching strategy to use is modeling (not to be confused with mathematical modeling) because students see exactly how their work should look upon submission (Haston, 2007). It can also aid in organizing their thought process, which is a skill that students need, to be successful anywhere. How effective are worked examples as a tool for modeling work? What effects will the unit have on student work?

Summary of Chapter One

While worked examples are not a cure-all for every situation that arises in high school education, research has shown that properly constructed examples are effective during initial knowledge and skill acquisition, and appear to be able to address the problematic areas of engagement, self-perception, ability, and learning the material.

Exactly how they are effective and to what extent are the research questions of this project. I will begin by constructing the geometry unit based on research of the key principles behind effective worked examples. Once created and implemented it will be possible to learn how the unit affects the issues above.

Statement of the Research Problem

The research question for this project has three parts:

1) Does using worked examples in a high school Geometry unit help students understand geometry proofs better?

2) Does using worked examples improve student quality and duration of their work?

3) Does using worked examples improve students' self-confidence in their ability to do math?

This project adds to the discussion on worked examples by determining if a high school geometry unit on proofs constructed around worked examples principles helps student learn mathematics. In previous studies on worked examples, focus was placed on one type of worked example and whether or not using it was beneficial. I wanted to determine if a unit that consisted of different types of worked examples improves student learning. In this project I conducted a quantitative study using a paired sample design with the independent variable being the worked examples unit and the dependent variable being the students achievement determined by their assessment scores.

Before continuing, it is necessary to explore the current theories behind worked examples. In the next chapter we will review the literature and research behind worked examples so as to give a clear picture of where to begin planning the unit.

CHAPTER II

LITERATURE REVIEW

Secondary mathematics classes, and mathematics classes in general, have maintained a consistent structure over the past decades. Students attend class to experience a lecture or some component guided by the instructor followed by classwork or homework through which students are supposed to struggle (Carroll, 1994). At this point they are required to break the exercises apart to match up with the concepts and procedures offered by the instructor in order to find the solution. This is a difficult task for an inexperienced learner or an experienced learner attending to a new concept for the first time. What has proven to be effective instructional guides are worked example exercises (Sweller & Cooper, 1985; Tarmizi & Sweller, 1988; Chi et al., 1989; Ward & Sweller, 1990). **Worked examples** are defined to be a model of an expert's problem solving strategy. In domains such as algebra or electrical engineering, worked examples are step-by-step solutions with explanations, justification, or reasons for each step.

Learners can actively participate in problem-solving activities for an extended period of time without any learning taking place (Kirschner, Sweller, & Clark, 2006). Some form of guidance is needed in order to move forward. Students learn more by studying many worked examples rather than by solving many problems (Ward & Sweller, 1990). Examples contain unsaid sequences and actions that can be unclear to students in the early stages of learning a concept (Chi et al., 1989). A study of the effectiveness of worked examples in algebra by Sweller and Cooper (1985) was based on the idea that attention shifts from the solution to the relations between problem states and actions taken at each step. As students studied worked examples they were able to see what steps they needed to take to arrive at the solution. Other studies have described worked examples as instructional guides that lead the student along the problem solving path (Hilbert et al., 2008). However they are described, research indicates that some students prefer to use worked examples because they feel they are more useful (LeFevre & Dixon, 1986).

The way a learner uses worked examples depends upon its structure and intended goal. In a worked example with the goal of knowledge retention, the structure could focus the learner's attention on key elements within the example for the learner to study. A follow up exercise could serve as a posttest to determine how much the learner retained after studying the example. Follow-up exercises could also be used to find if the learner is able to apply the knowledge from the example (Florax & Ploetzner, 2010; Tarmizi & Sweller, 1988).

I am going define **partially worked examples** as something different from worked examples. A partially worked example might ask the learner to explain the reason for a certain step, or to complete a missing element in the example. In doing so learners are involved in the problem solving process in the context of the worked example content. Renkl, Atkinson, Maier, and Staley (2002) instructed learners to complete missing elements of a mostly complete worked example. After learners were given feedback on whether or not their responses were correct, they were given time to study this correct completed example. Renkl et al. (2002) found that learners performed better on a posttest than groups who studied complete worked examples. The structure and features of effective worked examples and partially worked example will be elaborated on later in this chapter.

Cognitive Load Theory and Long-Term Memory

Paas, Renkl, and Sweller (2004) describe why worked examples are effective in terms of cognitive load theory (CLT). Cognitive load is the term referring to the amount of working memory directed on a task. CLT is concerned with maintaining learning in a complex cognitive domain, such as mathematics, by controlling the cognitive demand placed on the learner. The theory assumes a limited working memory and a nearly unlimited long-term memory (Tuovinen & Sweller, 1999). Long-term memory is now viewed as the central, dominant structure of human cognition (Kirschner et al., 2006). Everything we see, hear, and think about depends on and is influenced by our long-term memory. Learning occurs when long-term memory is altered. Working memory is limited to only a few novel elements at any given time, and is what we use to process a task.

CLT explains the structure of long-term memory with regards to schema construction. The theory of schemas comes out of Jean Piaget's working on understanding how children learn. A schema is a set of mental representations of our experiences grouped together that we use to understand the world (Piaget, 1936/1952). When a learner comes upon something new, whether it be a fact or an experience, she will make sense of it by connecting it with what she has previously encountered. It is these connections between schemas that are central to human cognition. In terms of mathematics, a schema could be a group of problems requiring similar solutions (Cooper & Sweller, 1987). When a learner makes connections between these groups of problems solution paths are more easily recalled and utilized. It is like connecting ideas together on a string. When one end of the string gets pulled the other ideas tied to it are pulled as well. Studying worked examples offers the optimization of cognitive demand because emphasis is placed on the underlying principles of the problem as opposed to extraneous pieces of information. The theory behind the use of worked examples is that with attention given to the important information, intra- and inter-connections of schemas can be constructed in the long-term memory.

Researchers differentiate between novice and expert problem solvers in their schema construction. Experts have acquired and built relevant schemas while novices have not. Experts' schemas create quick, automatized connections allowing them to place various problems into certain categories which helps generate cognitive moves to arrive at solutions (Tarmizi & Sweller, 1988). Expert problem solvers use of appropriate schemas helps them navigate the problem and apply, for example, mathematical rules without thinking about the rules' validity. This quick retrieval and application is called rule automation. While schemas can be constructed relatively quickly, rule automation takes time and practice (Ward & Sweller, 1990).

Since many secondary students are novice problem solvers, it is necessary to consider structuring worked examples and guided instruction so as to build on appropriate pieces of information that students have available. They have not had much exposure to the material so their schemas are not as strongly built or easily accessed (Kirschner et al., 2006).

CLT assumes three types of cognitive load that need to be considered when constructing learning materials (Pass et al., 2004): **extraneous**, **intrinsic**, and **germane**.

Extraneous cognitive load, also called ineffective load, is imposed by components that do not contribute to understanding the principles or goal of the learning material or activity. Extraneous load does not contribute to schema construction or automation.

Intrinsic cognitive load is the load imposed by the information elements and their interactivity. This means it depends upon the difficulty of the task and the working memory of the learner. This can also be thought of as inherent cognitive load and cannot be changed. It exists in an exercise or activity no matter how it is presented. For example, take a problem such as, *If I have 10 quarters and 3 dimes, how much money do I have?* The imposed intrinsic load is knowing the value of these coins, how to add values together, and how to read the problem.

Germane cognitive load is the load imposed by the processing of schema construction and automation. In other words, this is the load imposed when we learn. In an appropriately constructed exercise, germane load is high, allowing learners to process the underlying principles in order to construct schemas or add to existing ones. This is where connections are made, long-term memory is altered, and learning happens.

Early thoughts on optimizing cognitive load focused on reducing extraneous cognitive load which was thought to be the only impediment to learning (Pass et al., 2004). In some learning activities, however, extraneous load is bound to germane load. For example, increasing the variability of the worked examples studied and practice problems attempted would be considered extraneous since the variety of tasks mask the underlying principles. By including instructional support that asks learners to make comparisons and find similarities between the problems, extraneous load is lessened because students attend to the differences. In some cases reducing intrinsic cognitive load can be more effective if the intrinsic load of a problem involves complex sequencing of processes (Gerjets et al., 2004). Breaking up the problem into smaller modules creates smaller meaningful solution elements that can be more easily understood, thereby improving learning. The load imposed by optimizing intrinsic, extraneous, and germane cognitive loads leads to meaningful learning experiences.

The Worked Example Effect

When students struggle through an exercise, they are required to search for the important pieces of information and ignore the rest. CLT claims this search demands more from working-memory, which makes problem solving more difficult (Gerjets et al., 2004). Studying a worked example demands less from working memory because search within the example is reduced or eliminated and directs working memory resources (i.e., attention) to learning the essential relations between problem-solving steps or schemas. Students learn to recognize which steps are required for particular problems, which is the basis for the acquisition of problem solving schemas. When comparing students who have solved traditional problems to students who studied worked examples, those who studied worked examples were able to perform better on similar tasks as well as transfer tasks that required them to apply their knowledge in a new context. Performing well in similar and transfer tasks as a result of instruction based on well designed worked examples is called the **worked example effect**. (Cooper & Sweller, 1987; van Gog, Kester, & Paas. 2010).

Split Attention Within Worked Examples

Sweller and Cooper (1985), Cooper and Sweller (1987), and Ward and Sweller (1990), looked at worked examples in the domain of Algebra. Their studies found that pairing worked examples with student exercises increased student performance on assessments.

In a study conducted by Sweller and Cooper (1985), participants were instructed to study worked examples like those in Figure B below until they felt they understood them and any questions they had were answered. It is worthwhile to note that their definition of worked examples did not include the justification of each step. After studying worked examples, participants were given similar practice problems where they were asked to solve for *a*, and feedback was given on whether or not their answers were correct. These participants performed better on a posttest compared to those who did not receive the worked examples to study. Sweller and Cooper determined that studying these worked examples helps process the mathematical operations between the problem solving steps. The important structural component of the examples is that all of the important details pertinent to learning are contained within the example itself.

$$c(a+b)=\frac{af}{a} \qquad\qquad a=d+ac$$

$$c(a+b)=f \qquad\qquad a-ac=d$$

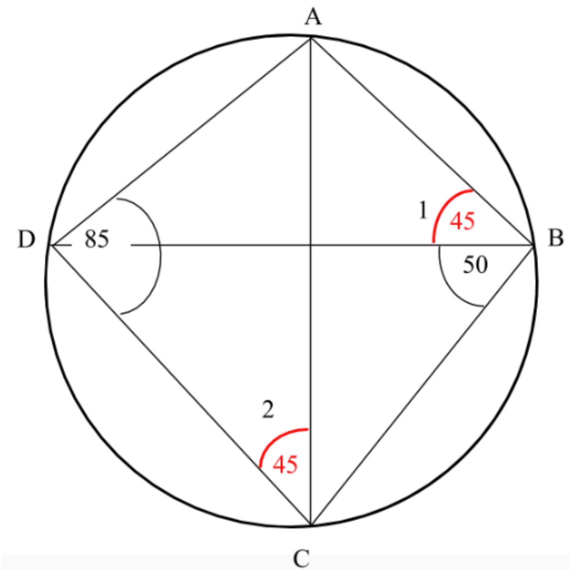
$$a+b=\frac{f}{c} \qquad\qquad a(1-c)=d$$

$$a=\frac{f}{c}-b \qquad\qquad a=\frac{d}{1-c}$$

Figure B

Worked Examples for Participants to Study

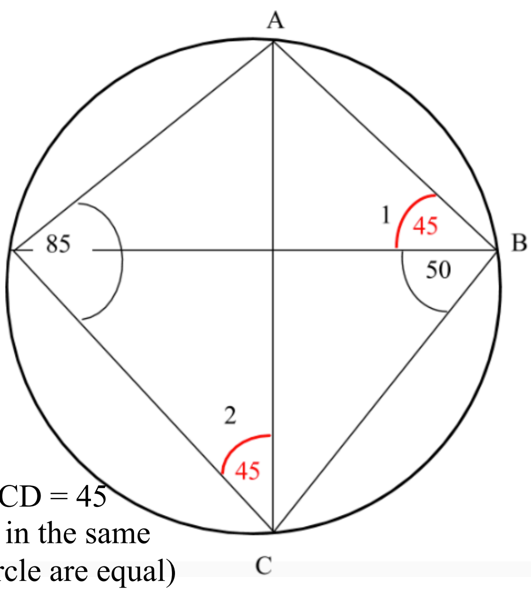
Tarmizi and Sweller (1988) studied the use of worked examples in geometry to determine whether or not the placement of information in a completed example that a learner needs to attend to has an affect on learning. Their control group studied examples in which the steps were given separate from the diagram, while the experimental group received the same diagram but the steps given inside the diagram. See Figure C for examples used in this experiment. In this experiment groups were instructed to study the completed example where the goal was to find the value of $\angle$ACD. As this was a completed example, the 45º angle values were printed red to indicate they were calculated rather than given, while the 85º and 50º angle values were printed in black to indicate they were given pieces of information rather than calculated.



1. ∠ABD = 180 − ∠ADC − ∠CBD (opposite angles of a cyclic quadrilateral sum to 180)
∠ABD = 180 − 85 − 50 = 45 degrees

2. ∠ABD = ∠ACD = 45 degrees (angles in the same segment of a circle are equal)

Control Group



1. ∠ABD = 180 − ∠ADC − ∠CBD (opposite angles of a cyclic quadrilateral sum to 180)
∠ABD = 180 − 85 − 50 = 45 degrees

2. ∠ABD = ∠ACD = 45 degrees (angles in the same segment of a circle are equal)

Experimental Group

Figure C

Geometry Worked Example from Tarmizi and Sweller (1988)

It was found that the experimental group performed better on a posttest than the control group. Tarmizi and Sweller hypothesized that splitting students' attention between the two created extraneous cognitive load. In other words, separating the calculation steps from the diagram took away from goal of understanding how to solve for angle ACD. The reason the load is heavier in their geometric worked examples than algebraic worked examples is that students had to connect values in the diagrams to values in the equations. By including the steps within the diagram, the mathematical principles behind the example are made more clear.

Florax and Ploetzner (2010) investigated split-attention and looked at the variables of spatial proximity, text segmentation, and picture labeling in the domain of human biology. There were five groups in their study, each of which received differently structured images. All of the images contained the same picture of a synaptic gap where the varying element was how it was labeled and how the text was presented. Four groups were assigned to the four different combinations of text structure (continuous or segmented) and picture labeling (unlabeled or labeled). A fifth group was given a worked example with spatially integrated text and a labeled picture, similar to that described above in the work of Tarmizi and Sweller (1988). Students' retention and comprehension of the material was measured in a posttest.

They found that while spatial proximity is important, a labeled diagram paired with text that is separate from the diagram yet segmented (i.e., numbered or bulleted) can also effectively facilitate learning. In comparing all five groups, the integrated text and picture group and the segmented text and labeled group received higher scores on a posttest measuring comprehension and retention. However, of those two, the segmented text group comprehended and retained more than the integrated text group. Florax and Ploetzner concluded that other formats of texts and pictures can facilitate learning as much as integrated formats depending upon how extensive the texts. In terms of CLT, this study demonstrates how one factor, such as spatial proximity, does not always serve as extraneous cognitive load. Effectively structuring worked examples depends upon what learners are being asked to do and what type of information is involved.

Worked Examples and Goal Free Exercises

While research has given evidence supporting the effectiveness of studying worked-examples, there is research that goal-free specific tasks and minimal guidance instruction are effective instruction tools. Lim, Dixon, and Moore (1996) tested the effects of studying non-goal specific geometry questions instead of geometry worked example exercises on high school students. In their study Lim et al. gave students an instruction sheet with four mathematical principles on an instruction sheet: supplementary angles sum to 180º, vertical angles are congruent, corresponding angles are congruent, and the sum of angles within a triangle sum to 180º. Participants were presented with this sheet to study, which contained two example diagrams of each principle, and each item was read out and explained by an instructor. Following instruction, Group 1 was given a set of questions with a specific goal to find the value of ‘x’, while Group 2 was given the same figures, but was given the goal free task to find as many angles as possible from the given information.

Their results showed that students who completed non-goal specific tasks were able to more quickly solve the posttest. Students in the worked example group had statistically similar posttest results, albeit slower. Despite the results slightly favoring the goal free method, it is worth noting some aspects of the methodology in order to understand what took place. The treatments in the Lim et al. study were comprised of three phases, only the second of which differed between groups. Phase 1 involved both groups discussing the four mathematical principles needed to solve the posttest questions. Phase 2 had one group use non-goal specific problems to develop schema (e.g., “Find as many angles as you can.”), and the other group was given conventional worked examples. Both groups were given the same "Find x" exercises in phase three before the posttest.

Lim et al. acknowledged that there could have been excessive cognitive load in the worked examples given because students were forced to integrate information from different sources (i.e., a diagram and a separate text). Integration of information within diagrams is a key feature of effective geometry worked examples that do not involve excessive text (Tarmizi and Sweller, 1988). Additionally, the information the students used in the non-goal tasks and worked example tasks was not new. The groups were allowed to reference the phase one material which functioned as a worked example. Both groups had been given the same direct instruction in the first phase that they could reference during phase two. In regards to the comparison between worked examples and goal free exercises, this study by Lim et al. does not discount the effectiveness of worked examples. While pursuing goal free exercises is a worthwhile endeavor, it is outside the scope of this paper.

Designing Effective Worked Examples

Worked-examples, when structured properly, are effective solution guides for learners attending to a novel concept or a complex task (Kirschner et al., 2006). As evidenced by the few examples described in this chapter, worked examples are not always similar despite serving the same purpose of helping students learn. As a teacher's learning goals vary from topic to topic, so should the type of worked examples. Variability of worked examples and example-problem pairs allows learners to understand how a particular skill or piece of knowledge can be applied in novel situations (Atkinson et al., 2000). So how can we go about designing worked examples? What factors or components must be included for worked examples to be effective? Shen and Tsai (2009) reviewed worked example empirical studies in order to generate the instructional design principles of worked examples. They found eight principles were consistent among effective examples, which can be used to guide worked example design. Each principle is not necessarily in every effective worked example as worked examples vary greatly from content domain and the learning goal, and no one principle can be considered better than others.

1. Imagination principle. Cooper, Tindall-Ford, Chandler, and Sweller (2001) gave two groups of students worked examples involving calculations on spreadsheets. The primary difference between the two groups was that one group was asked to study the worked examples before moving to the next example, while the other group was explicitly instructed to mentally review what they did in the completed example and reconstruct the process in their minds before moving on. The results showed that the imagination group retained more than the study-only group. Cooper et al. (2001) discuss the effectiveness of imagining in regards to schema construction and automation. The imagination strategy is most effective for those with more background knowledge of the material, and less effective for those with little or no background knowledge. The latter would benefit more from studying worked-examples in order to understand how each element within it relates to the other elements to construct the necessary schema. If the learner has more background knowledge the necessary schema is already constructed. By imagining the steps learners are automating the schema making it easier to recall.

2. Process principle. According to Gerjets, Scheiter, and Catrambone (2004) worked examples that are broken up into separate meaningful solutions, or modules as described by Gerjets et al., do not impose cognitive load at as high a level as an example focusing on steps, problem-type schemas, or category-specific procedures. They argue that examples requiring a series of complex steps create heavy cognitive demand. Working memory is taxed from holding pieces of information together such as the steps, the type or category of the problem, and the goal. The process principle is the creation of subgoals within the example such that working memory is not taxed by holding pieces of information together. This enhances learning by leading learners to understand the process of arriving at the solution and determine how the subgoals function with respect to the goal.

3. Presentation principle. Tarmizi and Sweller (1988) and Florax and Ploetzner (2010) found that the presentation of information within worked examples is important in regards to how much working memory is devoted to piecing together information with diagrams. Depending on the type of example (i.e., text heavy, text light, or simple computations) and the information contained within it, learners' attention can split causing extraneous cognitive load that impedes learning. Examples that involve more text should contain segmented text separate, yet close, to the diagram. Short text and computations can be included within the diagram.

4. Media principle. The media principle can be summarized as the joining of sight and sound within worked examples to manage the cognitive load imposed on working memory. Just as the integration of text and visuals can enhance learning and possibly split attention causing cognitive overload, introducing auditory information can also split attention. In other words, it is a balancing act between an appropriate amount of information displayed in diagrams, text, and sound. Mousavi, Low, and Sweller (1995) conducted a series of worked example studies in which they compared what they called visual-visual, visual-auditory, and simultaneous. Visual-visual refers to diagrams and text presented together visually, visual-auditory refers to diagrams presented with spoken or tape-recorded statements, and simultaneous is the combination of text, diagrams, and spoken statements. For simpler, shorter problems, the latter two scenarios provide the most effective support for learning since less load is placed on working memory to process the provided information. This is an example of an extraneous element used to improve working memory. It is worth noting, however, complex situations are more challenging to process in visual-auditory or simultaneous modes of presentation (Jeung, Chandler, & Sweller, 1997). Processing spoken statements in these scenarios puts more load on working memory.

5. Timing principle. Timing refers to the order in which problems and worked examples are provided to learners. Trafton and Reiser (1993) presented two groups of students six worked examples and six problems to solve. One group received alternating pairs of worked examples and problems, while the other group was given the six worked examples followed by the six problems. The group that received the alternating sequence performed better on a posttest. Atkinson et al. (2000) called these example-problem pairs.

6. Self-explanation principle. In a complex worked example, the amount of detail could be overwhelming for working memory impeding learning. Students learn from examples via the explanations they give while studying them (Chi et al., 1998). Prompts that ask students to describe, justify, or summarize aspects of the example improves student understanding of material and improves retention. Explanations generated by the learner rather than given to the learner are more effective because the former takes better advantage of the learner's prior knowledge (Catrambone & Yuasa, 2006). Expert problem solvers generate explanations while studying making connections with the necessary schema. Since worked examples are supposed to represent an expert's thought process, providing self-explanation prompts fosters expert problem solving abilities in students.

7. Completion principle. Examples that implement the completion principle remove steps in the solution path so that learners are required to complete it. Completing missing steps fosters self-explanations, which is another of the eight principles of worked example design.

8. Fading principle. Fading is a continuation of the completion principle in that learners are still asked to complete steps in a worked example, but they gradually progress to problem solving. What is being faded is the amount of missing components in the worked examples. Forward fading is leaving the first step incomplete, then the second, then the third, etc. Backward fading is the opposite. The solution is left missing, then the step prior to the solution is incomplete, etc. The last phase in forward and backward fading is just a problem statement or exercise. Renkl et al. (2002) studied the transition from a complete example to an increasingly more incomplete example to a problem to be solved. It was found that forward and backward fading were effective for similar types of problems, backward fading provided learners with a deeper understanding of the problem so that they could solve different types of problems. Effective fading depends on the learner's prior knowledge. Faster fading is beneficial to those with high prior knowledge, but not as effective for those with less or no prior knowledge. A slower transition from worked examples to problem solving is more beneficial to those with less prior knowledge.

Summary of Literature Review

It is all well and good if students can recite some facts or use a particular skill to pass a test. Students are more powerful if they have the ability to connect something in the world to their existing schemas. We want our students to make these connections with ease in order to navigate a new scenario that we could not do in the classroom. To take that a step further there are scenarios that do not even exist yet that our students will come across in the future. To prepare students for this we need to give them experiences to create the necessary schemas to broaden their understanding of the world. Including worked examples as part of instruction has been shown to create these schemas.

From the review of worked example literature, it is clear that worked examples have a place in the classroom. Many of the earlier studies assessed worked example effectiveness in mathematics and the sciences. Studies within the last decade are being conducted outside of the STEM fields, which shows the importance of studying worked examples and example-problem pairs as a generally instruction practice. With the variety of fields effectively using worked examples, there is not a universal template that can be used across all disciplines. Even within a discipline, the format can, and should, vary depending on the learning objectives of each lesson. Varying the worked examples and example-problem pairs allows the learner to see how a skill or piece of knowledge can be applied in novel situations (Atkinson et al., 2000).

Cognitive load theory argues that working memory can be easily overloaded by improperly designed exercises during initial knowledge acquisition hindering schema construction (Cooper and Sweller, 1987; Sweller, 1988). Limiting extraneous cognitive load early in the learning process can better direct attention to the core ideas thereby facilitating schema construction and automation (Sweller, 1988). Since worked examples are tools used to demonstrate an expert's problem solving thought process, researchers found that by studying worked examples students learn effective problem solving strategies, acquire the skills needed to arrive at the solution, and gain insight into the variety of situations in which those strategies and skills can be used. This can be facilitated more efficiently by studying many quality worked examples than solving many problems (Ward and Sweller, 1990). One way to determine a worked example's quality before giving it to students is by viewing it through the lens of the eight design principles which are based on worked examples that were shown to be effective (Shen & Tsai, 2009).

Bjork (2000) notes that in novel real world settings people need to know how to define the problem and infer what information can be used. Doing so is a skill that needs to be learned and practiced. Learning occurs by solving problems in varying situations and is aided by the guidance of a teacher or trainer. CLT explains learning in terms of schema construction by giving students experiences that balance germane and intrinsic cognitive load, and reducing extraneous load. Studies have shown that using worked examples, example-pairs, and applying the principles of effective examples improves student learning in the mathematics classroom. When a student has created strong schemas with meaningful connections that person will be better suited to think their way through novel situations. By constructing strong schemas in the learning environment, a person can then learn more from novel problems elsewhere since they will be able to filter out the information from the new context and attend to the core problem. This is the main academic goal teachers have for our students.

CHAPTER III

METHODOLOGY

Project Creation and Research Design

In order to evaluate a unit composed of worked examples, the unit must first be created. The curriculum used at Sonoma Valley High School is College Preparatory Mathematics (CPM). The textbook is text-heavy with exercises based on discovery and inquiry-led learning. It is a resource that needs additional explanation from an instructor should there be questions on the material since there are few examples to reference over the course of the book.

With the lack of example problems, students need supplemental material to support the computations they will be responsible for using. Each section within a chapter offers one or two core problems (usually with multiple parts to each problem) that address the topic. The worked examples for this project were created to meet the needs of the curriculum since they serve as the primary examples of the concepts.

The unit that is the focus for this project is titled "Proof and Quadrilaterals". It is the seventh chapter in the CPM *Geometry Connections* textbook (2007). For the two classes that were assessed in this project, this was the students' first full chapter in the second semester based on our math departments pacing schedule. It is important to note that SVHS was on a block schedule which means some sections will be taught on the same day in order to align with the schedule. Also, some sections are shorter than others, which allows them to be combined with other sections. As I created the worked examples, I needed to determine which units could be combined and which should be taught separately.

While "Proof and Quadrilaterals" focuses on proof and justification, the chapter also focuses on coordinate geometry. Prior to this chapter, students had practiced justifying triangle congruence with little formal organization of their reasoning and justification, and they had already done some calculations involving coordinates and slope. Chapter 7 is the first chapter that stresses organizing their proofs while using slopes and midpoints as justifications.

The reason I selected this unit was because this was a unit that I had struggled with teaching before. A main reason this is challenging is because writing a proof is a very different experience from learning rules of Algebra. Writing proofs in mathematics is a complex process that takes us on circuitous routes that can lead us to dead ends or right back to the beginning. It is only after our reasoning is sound that we present our proof in a linear manner to make sense of the path taken. It has been shown that many students around the world have difficulty with understanding and writing proofs. While students may have the knowledge necessary to solve a proof, they have trouble in areas such as finding a starting point or identifying correct arguments in regards to the proof (Hilbert et al., 2008). As this is similar to what I had observed in previous math classes during this chapter, I wanted to revisit it through the lens of the eight worked example principles.

For this project I conducted an unpaired sample study with the independent variable being the worked examples unit given to students and the dependent variable being the students' scores on the unit test. In addition to these scores I also evaluated students beliefs about themselves in math class by giving them a Likert-type survey early in the school year and after the unit was completed. See Appendix F for the surveys.

There were two Geometry classes that received the worked examples unit, of which 39 students took part in this study. Five students already took Geometry and repeated it this school year. Four students took Algebra more than once at the high school. See Table 2 below for a further breakdown of the students in this study by gender, grade level, and ethnicity. This information is included to give the reader a snapshot of my students. This information will not be used in the evaluation of this project.

Table 2

Breakdown of Student Population

	Grade Level		Ethnicity		Gender
Sophomore	30	White	14	Male	14
Junior	6	Latino	24	Female	25
Senior	3	Filipino	1		

My first task was to create worked examples for each section. During initial knowledge acquisition, Atkinson et al. (2000) found that alternating worked examples and problems was more effective than providing, say, four worked examples followed by four problems. For this project I reviewed the objective of each section and created example pairs around the concepts that address the objective. Depending on the size of the chapter and depth of the concepts there needed to be two or three example pairs to be used for each section which has been shown to be effective in terms of geometry proof worked examples (Mousavi et al., 1995; Hilbert et al., 2008). These examples were in addition to the homework problems from the textbook that also served as practice for each lesson.

Regarding the homework, the CPM curriculum makes use of spiraling, which means some of the exercises pertain to the chapter's key concepts and others pertain to material from previous units. I kept the homework sections the same for three reasons. The first reason was because the goal of this project was to determine the effectiveness of a high school geometry proofs unit based on worked examples principles. The second reason being that the students were familiar with them and appreciated revisiting previous material if they had had trouble with it. Lastly, I kept them was because our unit tests were made collaboratively as a department so I needed to keep the content delivered in my class (both new and old) to be the same as those in other classes. It should be noted that I created worked example-problem pairs for a few of the early homework sections to help get students acquainted with worked examples and how to use them.

Table 3 below lists each section in the chapter along with the objective. Something noteworthy about the CPM curriculum is that each section title is a question, which is used to guide student thinking. Each example was constructed based upon the principles outlined by Shen and Tsai (2009) and Atkinson et al. (2000). The eight principles are described in the literature review.

Table 3

CPM Chapter 7 Outline

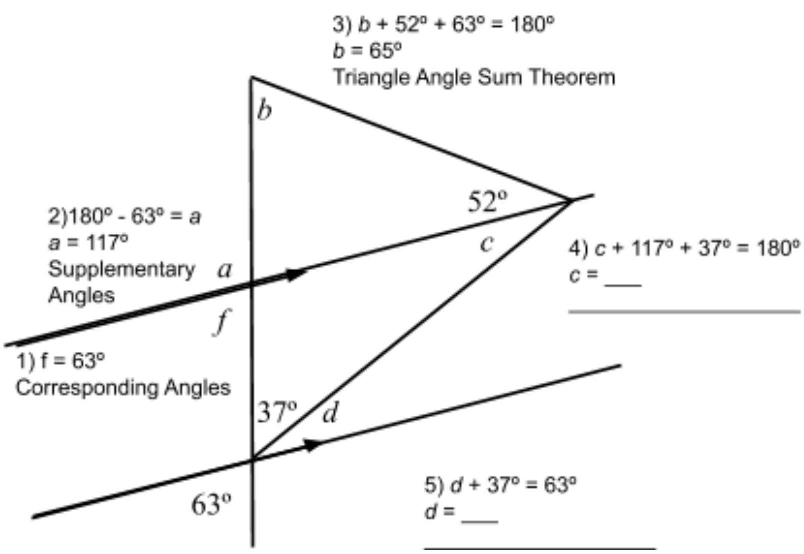
Section	Section Title	Objective
7.1.1	Does it roll smoothly?	Students explore the special properties of a circle: constant radius and diameter of a circle.
7.1.2	What can I build with a circle?	Students review shapes and their properties as they fold a circle to create a tetrahedron. Students begin to understand how area changes as a shape is enlarged proportionally.
7.1.3	What's the shortest distance?	Students analyze and solve "shortest distance" problems, and use their understanding of reflection and similarity.
7.1.4	How can I create it?	Students will use symmetry to study polygons.
7.2.1	What can congruent triangles tell me?	Students will be introduced to proofs and learn more properties of parallelograms and kites.
7.2.2	What is special about a rhombus?	Using congruent triangles, students will prove properties of rhombi.
7.2.3	What else can be proved?	Students continue to develop proofs and communicate a logical argument.
7.2.4	What else can I prove?	Students continue to use proofs to demonstrate additional properties of quadrilaterals and isosceles triangles.
7.2.5	How else can I write it?	Continue using proofs and practicing using logical arguments.
7.2.6	What can I prove?	Students prove new properties of triangles and quadrilaterals using auxiliary lines.
7.3.1	What makes quadrilaterals special?	Students investigate quadrilaterals for parallel sides and right angles. Students apply algebraic skills to shapes on a coordinate grid.
7.3.2	How can I find the midpoint?	Students will develop methods for finding the midpoint of a segment on a coordinate grid.
7.3.3	What kind of quadrilateral is it?	Students analyze quadrilaterals on a coordinate plane and identify them by type.

Not every principle was used in each worked example-problem pair. Trying to do so would have added extraneous cognitive load. As such, balancing extraneous cognitive load from the novelty of the examples while assessing the effectiveness of a unit based around worked examples was something I was cognizant of. Figure D is an example of a partially worked example-problem pair I created based on the eight worked examples principles. The example in Figure D below was created in which the follow principles are addressed:

- Imagination principle - Mentally create the angle relationships used in the worked example.
- Completion principle - It is acceptable to remove steps from the worked example for students to complete. In this case, all of the steps are removed in the paired example.
- Process principle – The worked example is broken up into shorter steps. It is structured such that the steps are not complex or require working memory to be taxed holding the angle values in their working memory.
- Presentation Principle - Steps are displayed within the diagram so that students' attention is not split between steps and the image.
- Timing Principle - The pair is presented with the worked example first followed with places for students to complete the work.
- Self-Explanation Principle - Students are asked to explain the reasoning behind one of the steps in the worked example.

Partially Worked Example to be completed by students

Use the relationships in the diagram below to find the values of *a*, *b*, *c*, *d*, and *f*. Name which geometric relationships you used. Complete the final calculation in steps 4 and 5 and give the geometric relationship used. Answer the prompts at right.



Describe why we might need the unmarked angles adjacent to angles *a* and *f*.

Where did we get the 117° angle from in step 4?

Paired Exercise

Use the relationships in the diagram below to find the values of *b*, *c*, and *d*. Calculate the missing angles in any order you choose. Name which geometric relationships you used.

Before you begin, imagine in your mind the relationships we used to find the missing angles in the previous example. Imagine what the angles look like for each relationship.

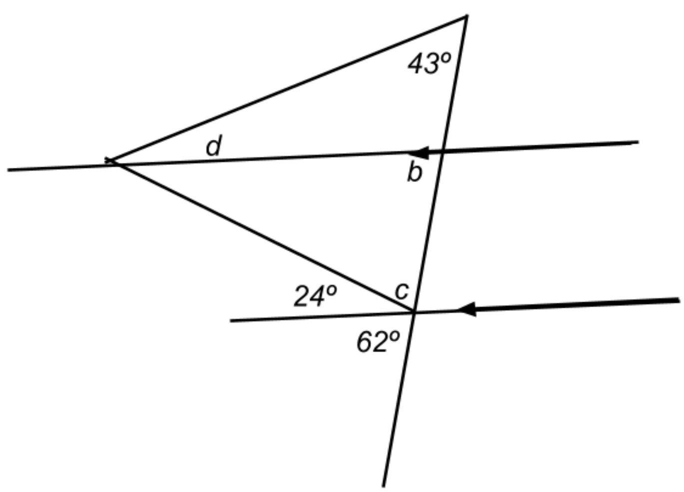


Figure D

Angle Relationship Partially Worked Example Problem Pair

Creating Worked Examples

When looking at the textbook exercises I considered how much time to allow for this unit compared to my colleagues who taught the same unit. Our department has a policy where we give our unit tests within a week of one another so I needed to keep myself to that timeline to stay within those parameters and to maintain that regular classroom practice consistent for my students. Maintaining the pace within the textbook was also critical because the homework sections spiral material so that students repeatedly get practice at previous concepts, which is an important component to the CPM program.

CPM lessons are set up such that there is an exploratory problem or series of exercises to address a particular concept. Within either of these, students are asked to follow along with a set of instructions or given a question to think on during the lesson. Some exercises were taken from the text and modified with the eight worked example principles. Others will not be used to allow for greater focus on the math content.

Note the examples of two column and paragraph proofs below (Figures E and F, respectively). Both have the text segmented to focus students' attention on key components of the proofs. The two column proof is well known and recognized for its organization (Weiss, Herbst, and Chialing, 2009). Its format naturally allows for self-explanation by requiring students to give a justification for their statements used in the proof. In the paragraph proof I wanted to model another way mathematicians provided their justification, while trying to keep the extraneous cognitive load low. I did not feel this was overwhelming but it did require students getting used to that type of example. In the examples that required more explanation, I left longer explanations off of the example and use that as a moment for a classroom discussion. See Appendix A for the worked examples I created for this unit.

Given: ray CA bisects ∠XCY and segments XC = YC

Prove: ∠X = ∠Y

Statement	Reason
1) Segment CA bisects ∠XCY, so ∠XCA = ∠YCA	1) Definition of bisect (*cuts an angle in two equal measures*)
2) XC = YC	2) Given
3) AC = CA	3) Reflexive property
4) ΔAXC = ΔAYC	4) SAS triangle congruence theorem
5) ∠X = ∠Y	5) __________

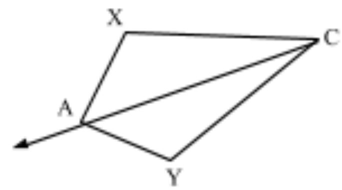
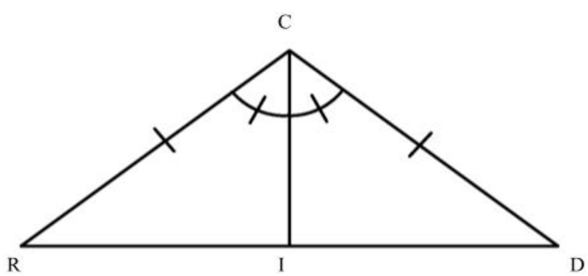


Figure E

Partially Worked Example Two Column Proof

Given the information in the diagram, prove segments RI and DI are equal.



We are given that segments RC and DC are equal to each other, ∠RCI = ∠DCI, and IC is of course equal to itself.

This means ΔRCI ≅ ΔDCI because of SAS ≅.

Since corresponding parts of congruent triangles are congruent and RI and DI correspond, then segments RI = DI.

Figure F

Worked Example Paragraph Proof

Introducing Worked Examples

To introduce students to worked examples I first asked them what they thought a worked example would be. The responses were very similar. A student came up to the board and wrote the equation below and its solution to demonstrate what she thought a worked example was. See Figure G for her work shown on the white board. Students agreed with her. For many students algebraic computations are what come to mind when they think of mathematics so this as an example is not surprising.

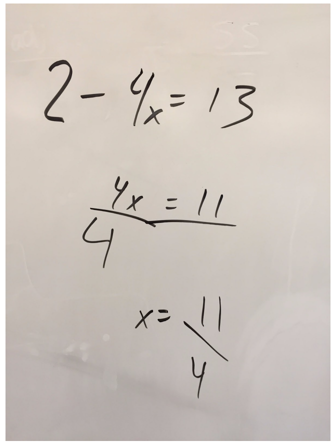


Figure G

Student Provided Idea of a Worked Example

This student provided example of what she believed constituted a worked example offered great discussion points for our upcoming work. We were able to discuss what features of it were helpful and what would make it more effective as a reference. We discussed how studying the steps taken allows us to understand the solution path taken and potentially find errors in the reasoning. In this particular case we noted the error of -4 becoming 4. Justification for the steps was not explicitly stated in this student's work which I pointed out because moving forward we would need to provide justification. I also used this time to point out to students that they needed to be aware of each piece in their upcoming worked examples as those were the key problem solving elements that they would be learning about.

Teaching the Unit

After our class discussion on the idea of a worked example, I told students that our worked examples would focus on the proving process and guide them to a solution. I began with the worked example from the Hilbert et al. (2008) study, which was shown to improve students proving skills and their knowledge about proving. In this study the participants were given a story of two fictitious students who explored a problem and their thinking process was described along the way. The problem solving process the two characters followed was exploring the scenario, making a conjecture, and then reasoning their way to a proof of their conjecture through appropriate justification. See Appendix B for this material from Hilbert et al. (2008). This process of exploring, conjecturing, and proving was noted as these general steps were referenced throughout the unit.

For the other sections in the unit, I applied the media principle regularly by having a student read aloud the titles and goals. They would study the worked examples and complete any missing steps or provide explanations (the completion principle and self-explanation principle, respectively). The first one or two examples were then used as our model to work from. Regarding the specific time given to students in class to study worked examples on their own, controlled studies have given varying amounts of time for their participants to study. Chi et al. (1989) found that in a controlled setting participants who performed better studied their examples for approximately 12 minutes while students who performed lower studied for approximately 7 minutes. As 12 minutes was not practical in my classroom due to classroom management concerns such as student behavior, and the limited number of instructional minutes we had, I kept this initial example study time between 5 and 7 minutes. I felt this allowed students quiet time to focus on the example and generate questions about the examples. It is worth noting that I still needed to remind students that this time was for studying as some students attempted to use this time to socialize and discuss off-task topics.

After students studied the example, we had a class discussion about the key elements in it and how they pertained to the learning goal. This discussion time was also used to answer questions. Once questions were answered students were given time to work on the paired exercises. If students asked me questions during this time I would direct their attention to the worked examples for reference and bring their table partners into this small discussion to help answer the question. As the class period neared the end I pulled the class back together to review the day's material and learning goal. I would frequently let students talk about their work or articulate realizations they had during class. This allowed students to leave class with correct responses for their exercises to practice on their assigned homework. Four short quizzes were given over the course of the unit to help keep the pace of the class moving in terms of important graded practice for developing student understanding. These quizzes were not used to evaluate the effectiveness of worked examples.

Data Collection

In accordance with the Human Subject Research Committee (HSRC), I asked for student and parent consent to collect and use their data for this study. See Appendix G for the Parent / Student Consent Letter, and Appendix H for the HSRC approval letter. Student tests were used to evaluate how effective the worked examples were in developing students' skills and understanding. For this project I looked at my students' mean score for the unit test and compared that with my previous year's students mean score of the same unit taught in a traditional teaching method. An unpaired sample *t*-test was performed on the two groups to determine if there was a significant difference in the students' mean score. See Appendix C for the test rubric and scored samples. These samples provided to show how they were used, but the tests used to show this were not student provided samples.

Homework was analyzed during the school year to determine the changes in student work quality and the number of problems attempted. I used a five point rating scale to determine the quality. See Appendix D for the homework evaluation rubric and scored samples. Similar to the unit test scored samples, the homework in the appendix is not student provided work. In addition to the rating scale, the number of problems attempted by each student was tallied for each assignment to see if they would attempt more problems after studying worked examples. The purpose of this analysis was to replicate the findings of Tarmizi and Sweller (1988) and Sweller and Cooper (1985) in which they found students completed more work when studying worked examples.

Students were given a Likert-type questionnaire before Chapter 7 and at the end of the project unit that asks them about their perception of their ability in math and the class as a whole. The purpose of this survey (included as Appendix F) was to determine the effects the unit had in altering student self-confidence in math class. I did the paired analysis on questions 3, 6, 9, 19, 21, and 25 from the post-unit survey.

CHAPTER IV

RESULTS

In order to determine the effectiveness of the worked example unit as it was implemented, multiple modes of evaluation were used. Below are the results of the statistical analyses and tests.

Chapter Tests Analysis

An unpaired samples *t*-test was conducted to compare the experimental and control groups' pretest score means before learning with worked examples. At the 0.05 significance level, there was not a significant difference in the control group's pretest score mean (M=14.90, SD=4.32) and the experimental group's pretest score mean (M=13.67, SD=3.21); $t(86) = 1.48$, $p = .1422$.

An unpaired samples *t*-test was conducted to compare the student test scores after learning with worked examples and student test scores by learning the same material via a traditional teaching method. At the 0.05 significance level, there was not a significant difference in the worked examples unit (M=12.28, SD=4.42) and traditional method (M=14.49, SD=3.91) conditions; $t(86) = -1.27$, $p = .1041$.

Homework

To determine whether or not the unit of worked examples had an effect on students' homework quality, I noted the difference in quality before, during, and after the worked example unit. Classwork and homework were analyzed throughout the school year in order to determine the changes in student work quality and the number of problems attempted. I used a five point rating scale to determine the quality. In addition to homework quality, the number of problems attempted by each student were tallied for each assignment. Students' mean number of problems attempted were compared to before and after the unit.

I did not run any statistical tests on this data, but there did not appear to be any difference in the number of problems attempted. For example, students who attempted three out of five problems before the unit still completed three out of five problems during and after the unit.

Survey Analysis

The purpose of this survey is to determine the effects the worked example unit had in altering student perceptions about themselves in math class. In the surveys students were asked about their self-confidence in math class. The responses were on a Likert-type scale. Students read a statement and responded "Always", "Most of the time", "Once in awhile," and "Never", or "Very confident", "Confident", "Not really confident", or "Not at all confident". Students were given the survey before the unit and then again one week after completion of the unit. Of the 39 students participating in this study, all students returned the post-unit survey while 30 students returned both pre- and post-unit surveys so it is these 30 students' responses that were used in this analysis. To analyze this data, I performed a one-tailed paired sample *t*-test on the items that pertained to their self-confidence in math class to compare their responses before and after the unit. I did the paired analysis on questions 3, 6, 9, 19, 21, and 25 from the post-unit survey (see Appendix F), which pertained to student confidence. At the 0.05 significance level, there was not a significant difference in the confidence responses on the survey before ($M=12.6$, $SD=3.24$) and after ($M=13.2$, $SD=3.62$) the unit; $t(29) = 1.607$, $p = .088$.

CHAPTER V

EVALUATION AND DISCUSSION

Initial Findings

Initial data analysis of this study indicates that a worked examples unit on geometric proofs as it was created and implemented was not effective at developing students' ability to prove in geometry. The different assessments used to determine the effectiveness of the unit are evidence that students did not increase in their ability to create and complete a proof. Analysis of the data shows the control and experimental groups began Chapter 7 with statistically equal understanding based on the pretest. After learning from the worked examples unit, students' assessment scores were significantly lower on the content delivered during the worked examples unit. While these data show that the worked examples unit did not improve students' understanding of the material, these findings give us insight into how to implement worked examples into regular classroom instruction.

Cognitive Load and the Worked Examples Unit

The teaching methods I implemented before and after the project unit were more effective than those during the worked examples unit. These methods include problem-based instruction, direct instruction on certain topics, and class discussions. The worked examples unit was like a pothole on the students' learning trajectory during the school year. There was evidence students were learning during the school year then this unit came along causing them much cognitive struggle and did not help their learning of this material. The novelty of the style of classroom exercises in addition to the topic of proof was too much for the classes as a whole.

While creating these examples, I wanted them to be as robust as they could be which caused me to over think them and then structured them so that they imposed heavy cognitive load. As Kirschner, Sweller, and Clark (2006) note, a worked example can be structured in such a way that it imposes heavy cognitive load. When creating the worked examples for the unit, I had to make sure they were keeping true to the eight design principles of worked examples described in Chapter 2 while keeping the focus of each example on the content. Looking back I found that I attempted to highlight more connections than appropriate within some of the examples. My reason for including these is because making connections is integral for schema construction (Piaget, 1936/1952). Intrinsic cognitive load is high in the early stages of learning something new because schemas have yet to be constructed (Paas et al., 2006). The examples themselves imposed intrinsic load from the math content, and paired with the extra connections there was little to no germane load. Students participated in class and completed the classwork, but that does not mean any significant learning occurred. As Kirschner et al. (2006) note, a learner can engage in problem solving and still learn almost nothing. In keeping with the spirit of worked examples, maintaining the focus on one goal per example would have provided sufficient germane cognitive load, which is the load imposed by schema construction and the type that directs student learning.

Within each example pair, I struggled with making each proof similar enough to make logical jumps for students but also different enough so that students could not just copy the worked example. Learners need examples to be different enough to develop their skill at searching through a series of problem to see the commonalities between them, thereby creating schemas (Atkinson et al., 2000). This was a challenge because devising proofs consists of more than following a rote set of steps. Mathematicians go back and forth on a problem rather than in a straight line. It is when the mathematician arrives at the solution does he/she write the proof linearly (Weiss et al. 2009). The mathematician tinkers with ideas and attempts different strategies while proving a statement. While worked example pairs have their place, this study suggests that a unit of proofs is not an effective method to develop students' ability to prove in geometry.

Class Closure and Schema Construction

Class discussions were another area where I think students were overwhelmed with new material causing cognitive overload. This is partially due to me focusing on too many details of our day's work rather than focusing on the bigger ideas of the day. Looking back on these discussions, I attempted to elaborate on many details of the day such as solution methods, and specific claims and their justifications. In addition to the cognitive load of the material, my attempts at clarification likely provided extraneous cognitive load. There were times I referenced examples from previous class periods because I wanted students to be aware of these connections. However, since I was referencing something that students had not integrated into their schema structure, I believe this was also an impediment to gains in student learning. Properly closing a class is just as important as the lesson itself. A lesson could be amazing with connections made between many topics, but if the learning objectives for the activity are unclear and the main points are not understood by students, they will not retain the key ideas of the lesson. This is an area I continue to push myself as a teacher because I know this has big benefits for my students.

Where I have changed my practice dramatically since this project is keeping my voice down during this class closure time and bringing student voices up. During the unit I found myself doing more of the talking while making the connections described above, rather than make the students more involved in this important time in class. Students' schema structures were not altered because I was not allowing them time at the end of the day to process what they had learned.

Student Surveys

Not only was this project intended to determine the effectiveness of a mathematics unit based around worked examples principles, but also to observe the effects it had on students' beliefs about their capability in mathematics. Based on student responses in the survey, students' self-confidence did not change as a result of this unit.

In addition to the questions pertaining to student self-confidence, students were asked to respond to the follow open-ended questions: "What was the most challenging aspect of proofs?" and "What did you find helpful or unhelpful about the worked examples that we used in class?" To the question about the challenging aspects of proofs, 27/39 of students responded that giving the reasons to justify their statements was the most challenging part of proofs. Many student responses did not give more specifics to what was troubling about giving reasons, but a few student did give more specific responses. For example, one student responded with, "the reasons such as SSA, ASA", and another student responded with, "Trying to prove something when you know that it's true." This gives evidence that students did not have enough prior knowledge of the material leading up to the unit in order to provide justification. Students knew something was true, but did not have the words to say why it is true.

To the question pertaining to what was helpful or unhelpful about the worked examples, 20/39 students responded that they found the examples helpful to refer back to when they were given a similar problem. This gives evidence that more than 50% of students attempted to use the examples as intended, but as evidenced by their scores, the examples were not an effective learning tool.

Changes to Classroom Culture

Before beginning the unit, it was common to see students give up on problems in class and on homework if solutions were not immediately clear, which made me think students would appreciate the further guidance on more problems during this unit. Since worked examples require students to follow a given process I thought they would spend more time on problems and I would see students improve. Information that would have been given through a familiar process, like a short lecture, was instead given through these examples, which were new to students. The novel format of the worked examples acted as extraneous cognitive load, which helps explain students' confusion and low assessment scores. Since this project was based on a single unit of worked examples, the introduction of the worked examples themselves in addition to the new, challenging material was too much for students to process. The mere mention of proofs in Geometry class often gets a sigh of disdain from students. By compounding that feeling with new instructional tools in the form of worked examples, students were overwhelmed. To address this, I would make two changes to my classroom structure leading up to the unit on triangle congruence proofs to help counteract this.

The first change is to create a norm of holding classroom discussions, which was an important piece that was left out of this project. Since self-explanation and discussion is important for worked examples to be effective, the practice of self-explanation and frequency of classroom discussions should be part of the classroom culture. Discussion should involve regularly giving students space to share their thinking with another student to help them formulate their thinking into language. This is best implemented at the start of the school year so that students know it will be a regular part of class. When students explain their thinking to another student both students benefit (Atkinson et al., 2000). Creating such a culture in the classroom is necessary for students to feel comfortable explaining their thinking with one another (Hufferd-Ackles, Fuson, and Sherin, 2004). CPM exercises ask students to explain or justify their reasoning (see Figure H). However, I had not asked students to share or explain their thinking with each other as thoroughly as needed prior to the worked example unit to create an environment that fosters that practice.

> This problem reminds Bradley of problem 3-93, *You Are Getting Sleepy…,* in which you and a partner created two triangles by standing and gazing into a mirror. He remembered that the only way two people could see each others' eyes in the mirror was when the triangles were similar. **Examine** your solution to problem 7-19. Are the two triangles created by the speaker wires similar? **Justify** your conclusion. (*Bold print was included in the original text.*)

Figure H

Example of CPM exercise

The second classroom structure change to make is to keep class closures focused on what the students took note of during the lesson. It is important for us as teachers to help them make connections, but the focus of the end of class wrap-up is on what they students are leaving with and how that pertains to the day's learning objective. As mentioned above, I did much of the talking during discussion about what connections existed within each days' work, but it should have been the students offering their observations and connections they made during the work.

Introduce and Use Worked Examples Effectively

Based on these results and looking back at this unit through the lens of using worked examples to teach geometric proofs, I would make the following key changes to their use and construction. First and foremost, at the beginning of the school year we need to establish and maintain norms for students to articulate arguments and ideas so that justifying their arguments becomes a regular part of class. Students need to regularly prove a variety of types of statements. That way, when geometric proofs arise it is merely another way to provide justification. When proofs and proving statements are more or less limited to a few units in geometry class teachers are doing students a disservice. Students are missing out on being exposed to different problem solving strategies and the way a mathematician thinks. Math is more than solving for *x* or finding the height of a triangle with a given area and base. It is about making sense of the way numbers and concepts relate with each other. Focusing on congruence proofs, which was the focus of this unit, greatly limits students' understanding of what proofs are. By practicing proving different types of statements, students will just be learning another type of proof during this unit.

When I created these geometric worked examples I found myself putting too much in each example that impacted the cognitive demand of the material. Since novices pay too much attention to the context of the problem, not deep conceptual structures (Atkinson et al., 2000), all of the extra pieces were preventing schema construction. This gets at the idea that teachers need to be cognizant of what is reasonable for students to be able to do and take away from a given example. When novice problem solvers, such as high school geometry students, are first given worked examples to learn from they should be fairly straightforward in terms of their learning goals.

An area for improvement for worked example use in terms of the eight principles is to take advantage of verbal and written self-explanation. Self-explanation is key to effective worked examples because that allows students to process more of the information in front of them (Hilbert et al., 2008; Renkl, 2002; Ward & Sweller, 1990). I checked for understanding during these lessons by asking open-ended questions or questions that required more than a one word answer, or asking the class a specific question that would be answered by individual students. To ease students' anxiety, a responding student was always able to ask another person for help. Students were also prompted within some examples to explain a particular step or idea, but these students' self-explanations were not followed upon routinely or thoroughly over the unit as much as they could have been to increase understanding of the examples. When giving students self-explanation prompts in their class work, we should follow up on the prompts in a class discussion within the class period to drive home the key topics of the exercise.

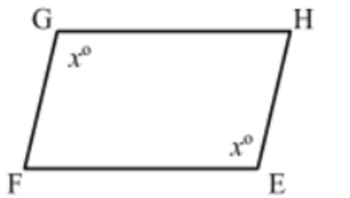
A change for the proofs in this unit in terms of the eight principles is to integrate the process, completion, and self-explanation principles in guiding students towards writing their own proofs. Students learned about angle relationships relatively recently before this unit and then in this unit they are expected to use apply them in their proofs. As students have little skill with applying this prior knowledge, when they are asked to do so their working memory is taxed leaving little to no germane load. Common general strategies I offered students to complete proofs of this type were to "look for angle relationships" or "consider what we know about that type of shape". I would address the development of these strategies in two ways. First, I would give students examples that develop this general strategy in students by allowing them to focus on a smaller number of geometric properties. Tarmizi and Sweller (1988) gave students a short lecture on two properties of circles followed by a series of geometric worked example problems.

For the worked example exercises, students were instructed to give one of the two properties as justification for a step in the worked examples. This narrowed students' selection down to a manageable amount of new information so as not to cause cognitive overload. By making this change to the worked examples I would help my students develop their justifications for their statements.

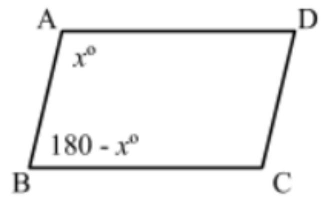
The second way I would address the development of general strategies is by starting with more concrete rather than abstract worked examples in terms of their properties. Learners develop schemas by understanding specific instances of a scenario and then move to generalities of the scenario (Piaget, 1936/1952). Something that was lacking in my worked examples unit and in the lead up to it was giving students practice at applying properties of quadrilaterals in specific instances. Figure I below is a modified example based on the work of Tarmizi and Sweller (1988) that addresses this. Students are given a selection of properties to learn about, followed by a sample problem that asks them to give a property as justification.

By provided students with this type of practice, they would get a better understanding of geometric properties that they can use to explore a problem to complete a proof. In making this change, students are given practice at developing strategies for subgoals, such as "look for angle relationships" and "consider what we know about parallelograms". An important component of this change in the worked examples is to have shorter proofs that ask students to complete portions of the examples that ask why a certain statement can be made. By reducing students' choices and keeping the initial tasks short, their working memory is focused on the application of the choices as opposed to searching from a long list of possibilities.

Properties of Parallelograms (Given to students to study)



Opposite angles in a parallelogram are congruent.



Adjacent angles in a parallelogram are supplementary.

Partially Worked Example Exercise
(Given to students to complete using one of the properties above.)

Given: ABCD is a parallelogram and ∠A = 117º
Prove: ∠C = 117º

Statement	Reason
1) ABCD is a parallelogram	1) Given
2) ∠A = 117º	2) Given
3) ∠C = 117º	3)

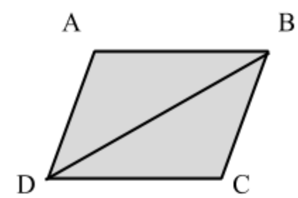


Figure I

Modified Worked Example

The changes mentioned above are important to address students' ability to apply quadrilateral properties and give justifications. Regarding the proving process, Hilbert et al. (2008) noted students need guidance on it, as well. Another change I would make is to help students see the steps of the proving process more clearly in the examples. I modified the task I used from the work of Hilbert et al. so that the story is shorter and more focused on the proving process of *Explore*, *Conjecture*, *Prove*. This is the proving process as described in the CPM textbook so I would use that to be consistent for students. See Appendix E for examples I altered based this discussion.

Worked examples clearly have a place in math class. However, they should be used throughout the school year rather than on only one unit. It is too much to introduce, implement and have student academic performance gains in one worked examples unit. They are incredibly useful tools to scaffold material and problem solving strategies. There is no need to limit them to one unit. In regards to using worked examples to teach more formal proofs and mathematical thinking, we should vary the types of statements to prove or justify. In doing so students will become better acquainted with constructing viable arguments in mathematics.

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APPENDIX A

Appendix A

Worked Example Unit

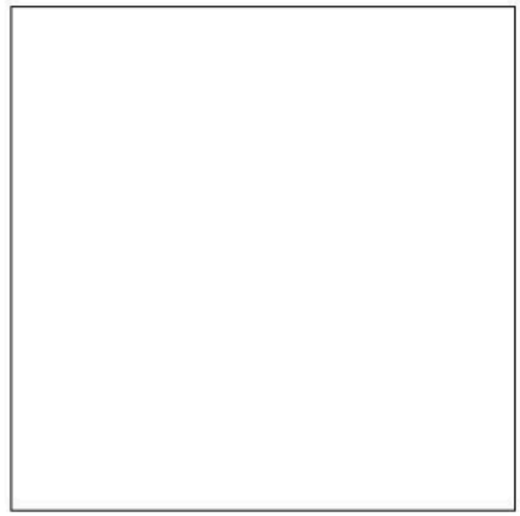
To give a understanding on how these worked examples and partially worked examples were used in the classroom, examine lesson 7.1.1 below. Part A was completed by students and then used as a reference for part B and C. This completed example was to serve as an example of an expert problem solver's solution. In the class wrap up at the end of the day, we discussed how the tasks in this lesson followed the proving process students learned about in the examples from Hilbert et al. (2008). In this lesson the focus was on the exploration of the scenario rather than on conjecturing or proving. See Chapter Three for more information on how this worked example unit was implemented.

7.1.1 Circles and Reuleaux Curves

A Shape's Width: Introducing the Reuleaux Triangle (Pronounced "Ree-low")
Goal: Explore different 2 dimensional shapes and their widths and note their characteristics
Question: Imagine a manhole cover. Why are manhole covers circular? What if the cover was square instead? Triangular?

Part A
1) Cut out the circle, equilateral triangle, and Reuleaux triangle.

2) Measure and label the height and width of the **square** at right. Draw a line **within** the square that is longer than the height and width. Approximate the length of this line.



3) Why do you think manhole covers are not square?
(Mr. Wizard Video)

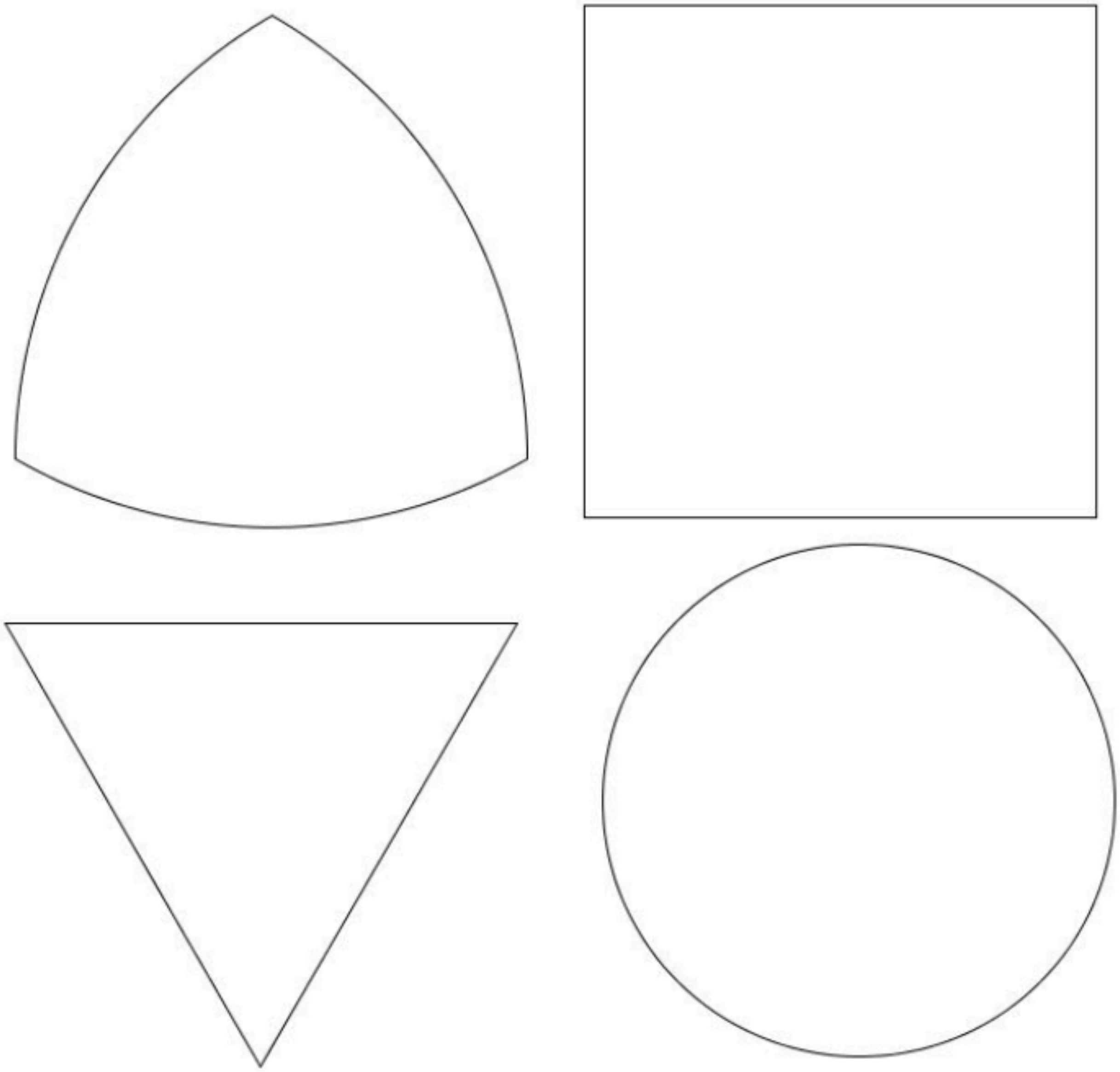
Part B)
4) Using our square above as a guide, we want to explore the widths of the circle, equilateral triangle, and Reuleaux triangle.

5) Is there another shape that could be used as manhole cover? What makes these "manhole covers" unique?

Part C)
6) Give two differences between a circle and a Reuleaux triangle.

7) Give two similarities between a circle and a Reuleaux triangle.

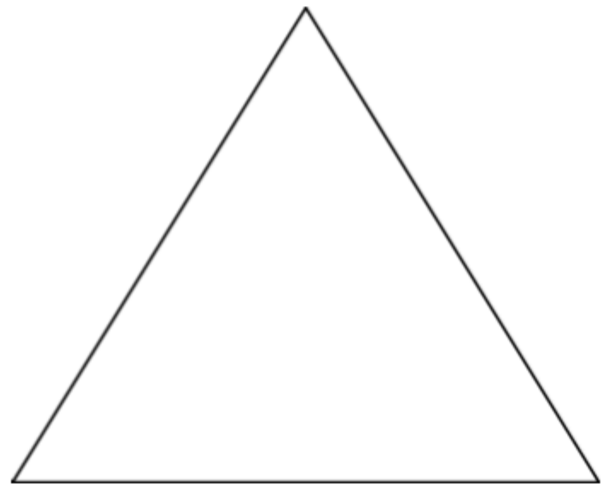
Some 2 dimensional shapes



<u>7.1.2 Building tetrahedra and proportional area</u>

Name ________________ **Period** _____

Goal: Find the area of an equilateral triangle.



Question: What are the measurements we need to find the area of our triangle?

<u>Example</u>

1) Measure a length of our triangle at right. This is a base.

2) Divide the length of our base by 2 in order to find the midpoint.

3) Draw a height of the triangle by connecting the midpoint to the opposite vertex. Calculate this height using one of the trigonometric ratios.

4) Use the lengths of the base and height to find the area.

(base • height)/2 = Area

Why does the following calculation ALSO give us the area of our triangle?

½ • base • height = Area

<u>Find the area of each face of your tetrahedron.</u>

1) Measure a length of one triangle in your tetrahedron. This will be the base of one face.

2) Divide the length of our base by 2 in order to find the midpoint..

3) Draw a height of the triangle by connecting the midpoint to the opposite vertex. Measure this segment as the height of our triangle.

4) Find the area of each triangular face.

Goal: Create a tetrahedron using a compass, circle, and straight edge and explore some of properties of the *Platonic Solid*

Part 1 - Create an equilateral triangle from a circle.
Part 2 - Fold the triangle into a tetrahedron.

Example [have students note that steps 1 through 5 give us the triangle, and step 6 give us the tetrahedron]
1) Use a compass to draw a circle with your desired radius. Cut out this circle.

2) Fold your circle in half. What is a name for this fold?

3) Fold your circle in half again so that your folds are perpendicular.

4) On the circle, label the endpoints of one *fold* A and B. Fold the circle so the point A touches the center. Label this second crease C and D. [cross out *fold* and write diameter]

5) Fold the circle twice to form creases BC and BD. You should now see triangle BCD.

6) Fold triangle BCD so that points BCD come together

Exercise #1
Following the same steps as in the example, create four tetrahedra using circles with a radius of _______.

Exercise #2
Once you have four completed tetrahedra, put them together so that they form a large tetrahedron like the figure below.

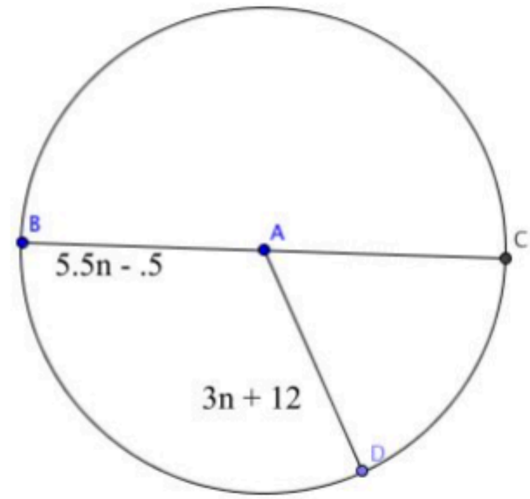
How many faces of the small tetrahedron would it take to cover one face of the large tetrahedron? What would the area of each face be?

Review & Preview

Radii and Diameters of circles.

Goal: We want to determine the radius and diameter of the circle below.

This is an exercise that involves circle principles and creating algebraic equations.



Example

1) **Using the information in the diagram, write and solve an equation to find the value of the variable.**

Since radii in a circle are equal, we can set up our equation as below and solve it for n.

$5.5n - .5 = 3n + 12$

2) **Use that variable to find the diameter and radius of the circle.**

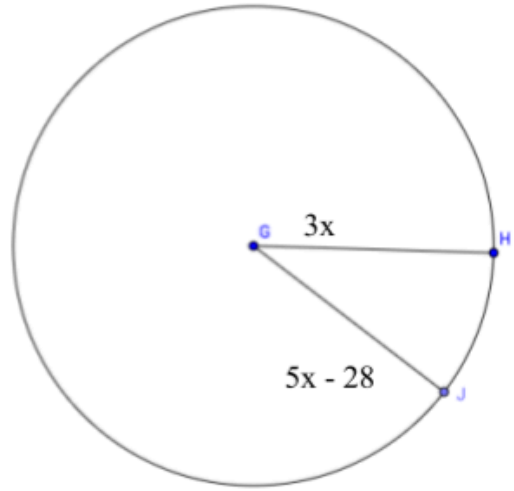
Since all radii are equal, we can use *n* in either expression to find the radius.

$3(5) + 12 = 15 + 12 = 27$ units.

3) **What is the relationship between a circle's radius and diameter?**

Exercise #1

1) **Using the information in the diagram, write and solve an equation to find the value of the variable.**



2) **Use that variable to find the diameter and radius of the circle.**

Exercise #2

What is the radius and diameter of the circle if:

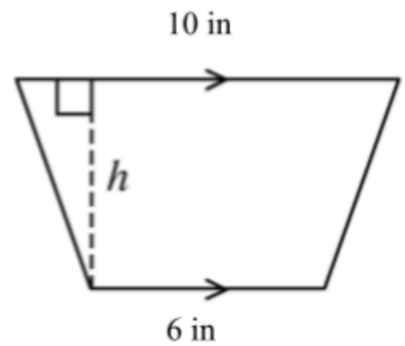
GH = 2n + 8

GJ = 3n – 1

Review & Preview
Area of Trapezoids

Goal: The area of the trapezoid at right is given. What is its height?

This exercise involves area formulas and substitution.



Area = 56 in^2

Example
1) Recall the area formula for a trapezoid.

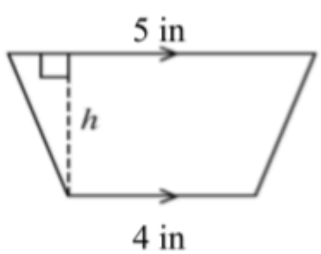
$$A = \frac{h(b_1+b_2)}{2}$$

2) Substitute the information we have from the figure into our formula and solve for *h*.

$$56 = \frac{h(10+6)}{2}$$

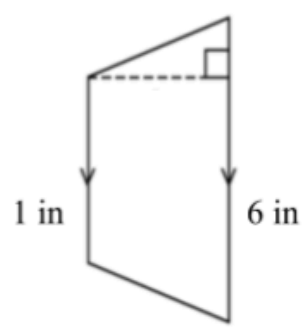
Use the steps from above to find the height of each trapezoid.

Exercise #1



Area = 50 in^2

Exercise #2



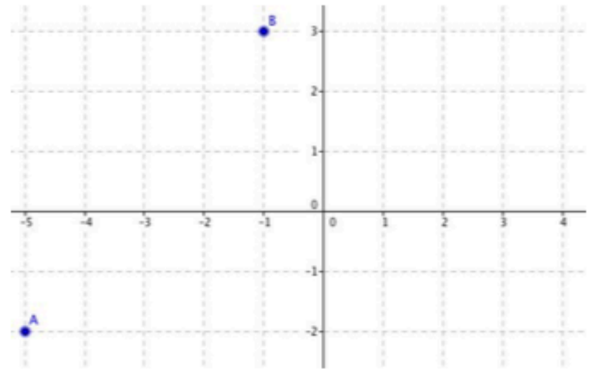
Area = 10 in^2

Review & Preview

Midpoints and Segment Lengths.

Goal: Find the length of a line segment and locate its midpoint.

These exercises require us to find midpoints algebraically. We will use an image to understand the concept.



Example

Part 1 - Find the length of segment AB using the Pythagorean Theorem.

1) Plot points A(-5, -2) and B(-1, 3) and draw in a right triangle using these points.

2) Find the horizontal and vertical distances between the points by taking the difference between the x-values and y-values.

Horizontal distance
-1 – (-5) = -1 + 5 = 4

Vertical distance
3 – (-2) = 3 + 2 = 5

3) Use the Pythagorean theorem to find the length of segment AB.

$AB^2 = 4^2 + 5^2$
$AB^2 = 16 + 25$
$AB^2 = 41$
$AB = \sqrt{41} \approx 6.40$ units

Part 2 - Find the midpoint coordinates of segment AB

1) Take the average of the x-values and y-values. The result will be the coordinates of our midpoint.
A(-5, -2) and B(-1, 3)

Average of x-values
$\frac{-5+(-1)}{2} = \frac{-6}{2} = -3$

Average of y-values
$\frac{-2+3}{2} = \frac{1}{2}$

The coordinates are:
(-3, ½)

Exercise #1

Given the points C(2, 3) and D(4, -2), find the length and midpoint of segment CD.

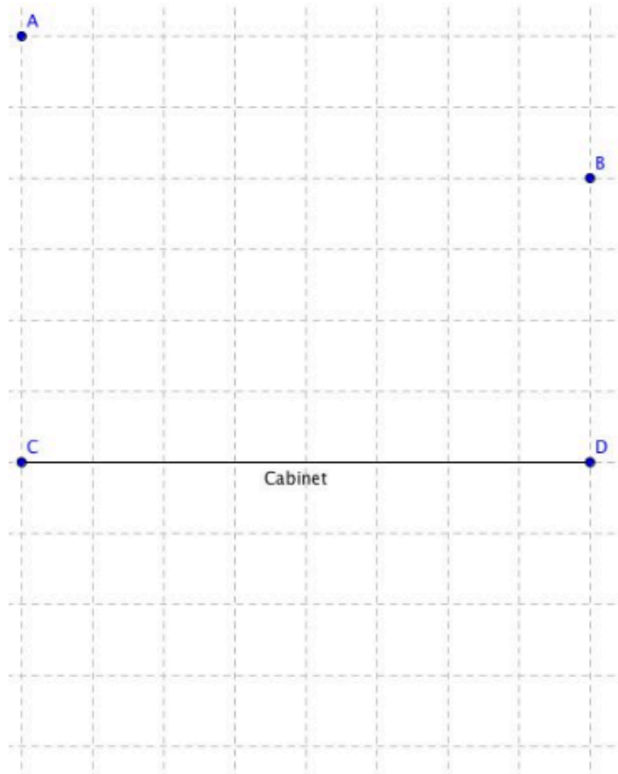
Exercise #2

Given the points E(12, -35) and F(34, 75), find the length and midpoint of segment CD.

7.1.3 What's the shortest distance? (Similar Figures are the root of many solutions)

Goal: Explore the following questions:
What is the shortest distance between two points?
What does the shortest distance have to do with similar triangles?

Goal: Place the stereo on the cabinet minimizing the amount of wire connecting each speaker to the stereo.
Part 1 - Calculate the total amount of wire.
Part 2 - Find the optimum location of the stereo using similar triangles.



Example

Part 1

1) Reflect speaker B across segment CD and mark point B'.

2) Connect point B' with speaker A.

Mark the intersection of segments AB' and CD as point P. This is where our stereo should be located.

3) Use the Pythagorean Theorem to find the length of wire by using the right triangle with segment B'A as its hypotenuse.

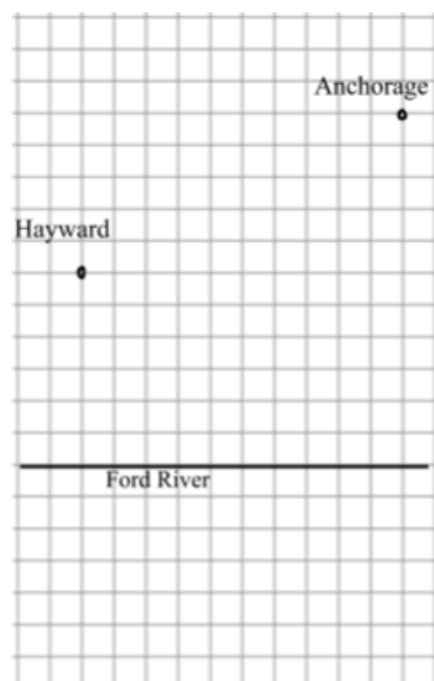
4) Why do you think this process gives us the shortest length of wire?

Part 2
We want to find the location of our speaker using our desired amount of wire. We can find this location by using proportions of similar triangles.

Why does this method work when the point B' is only a reflection, and not the actual location of the speaker?

Exercise #1

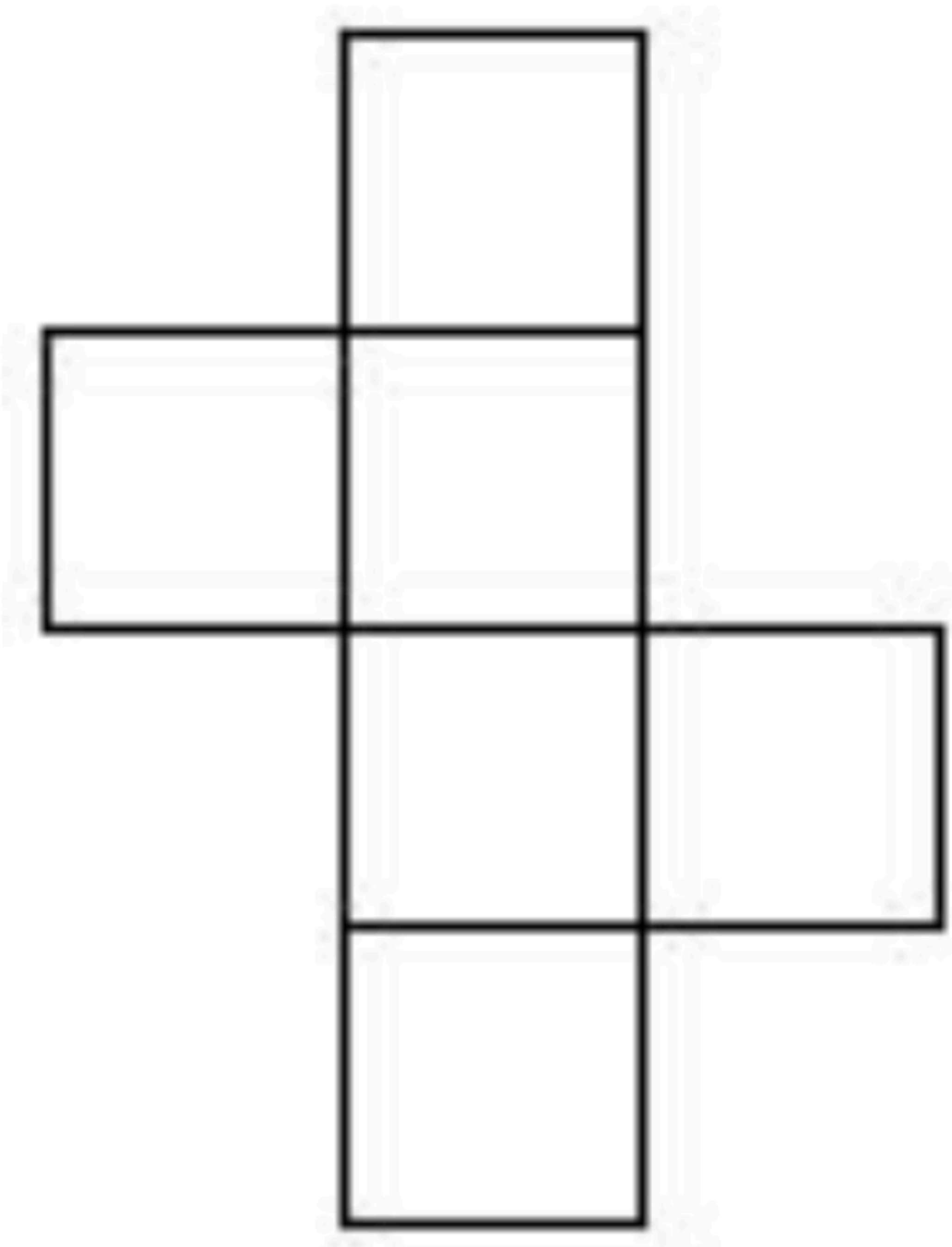
Two towns, Hayward and Anchorage, want to build a bridge across the Ford River. Since bridges are *extremely* expensive, the towns decided to split the total cost of the bridge and each road leading the bridge. In order to minimize cost, they need to find the location of bridge that *minimizes the amount of road.* We need to find this location on the map at right AND determine the amount of road needed to reach the bridge.



With the exception of the map, do all work on another sheet of paper.

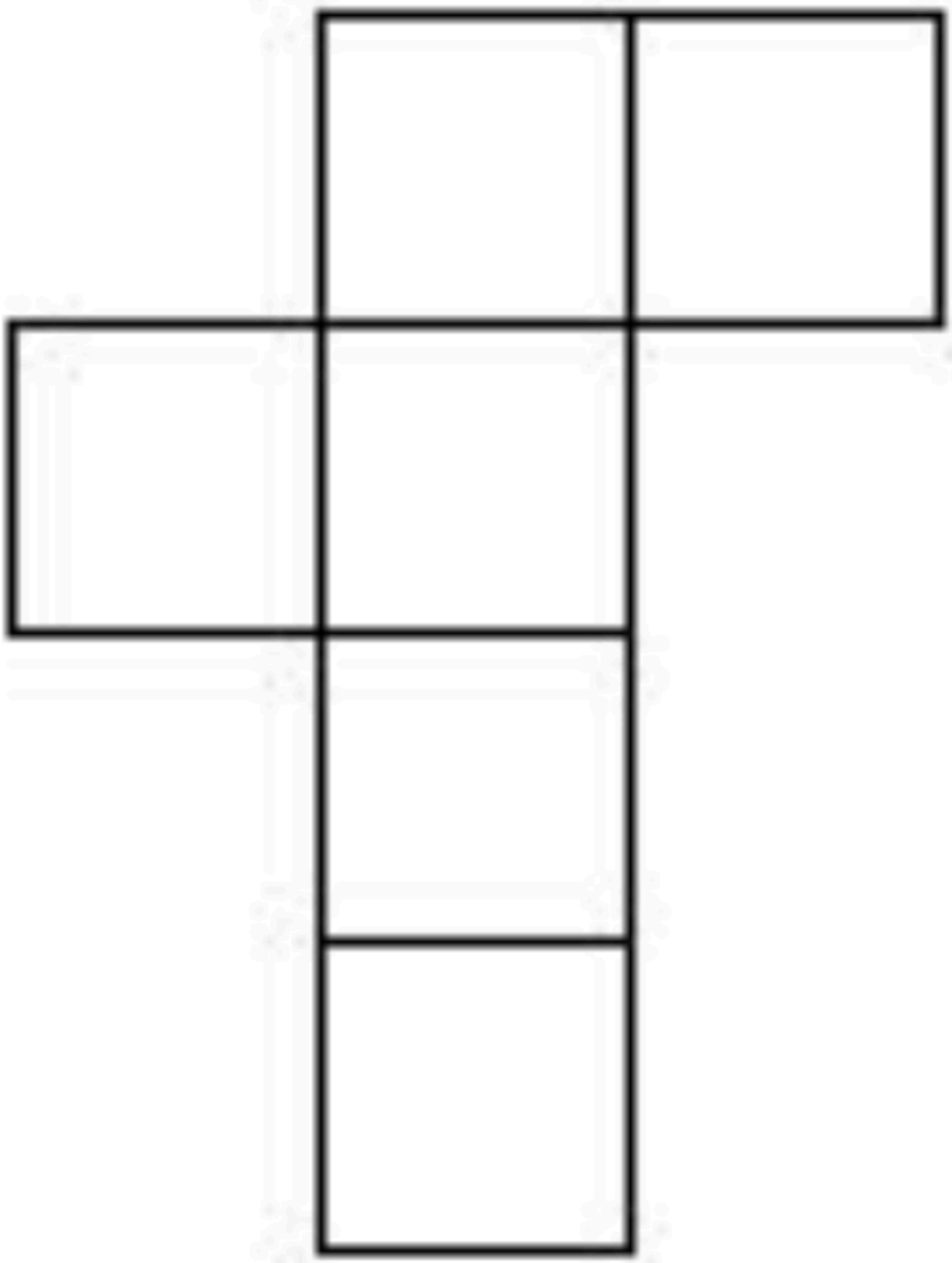
Exercise #2

Cut out the cube net below and create a cube. Pick any two vertices of the cube that are opposite each other. Determine the shortest path between the points without going through the cube. Show all of your work on the cube.



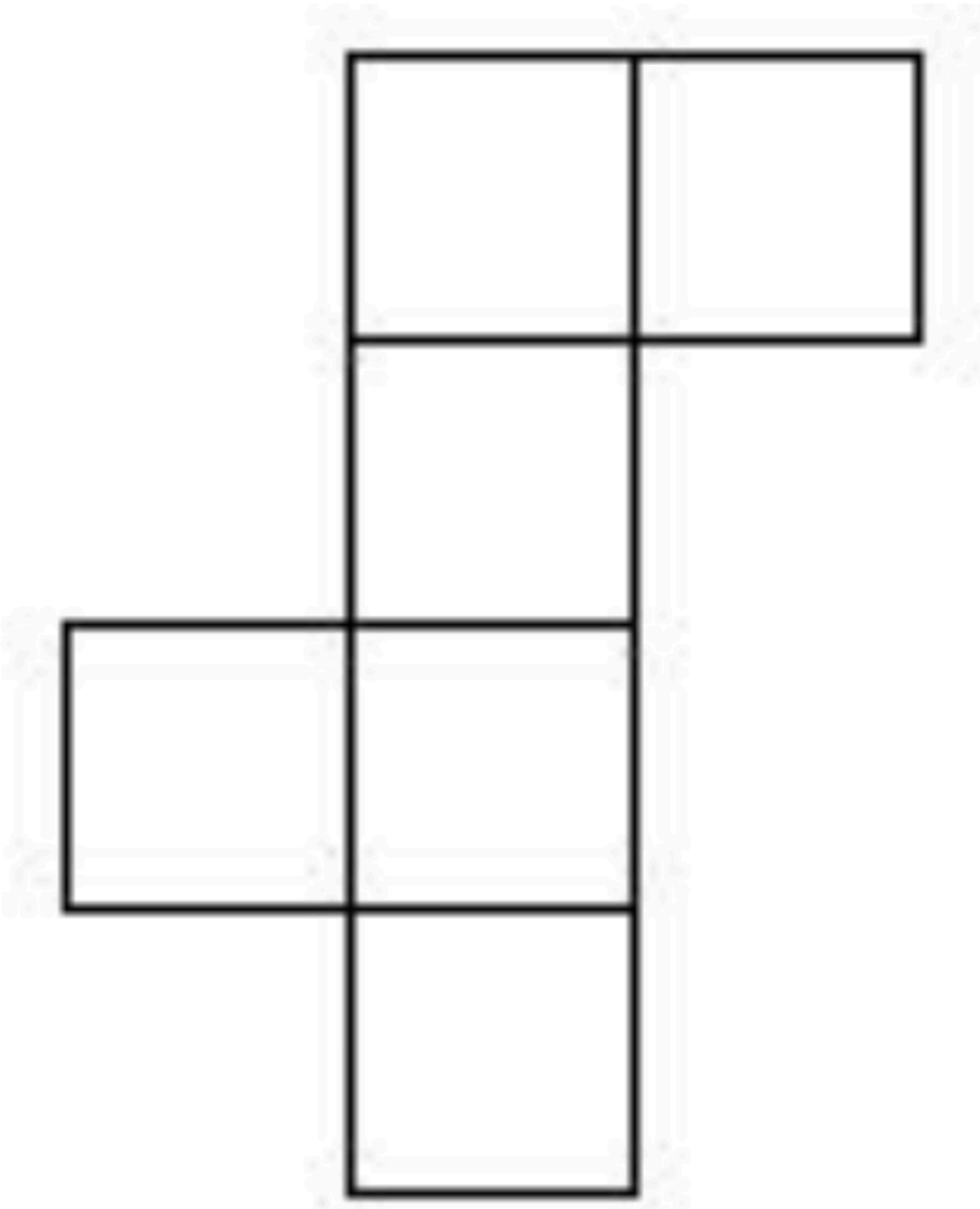
Exercise #2

Cut out the cube net below and create a cube. Pick any two vertices of the cube that are opposite each other. Determine the shortest path between the points without going through the cube. Show all of your work on the cube.



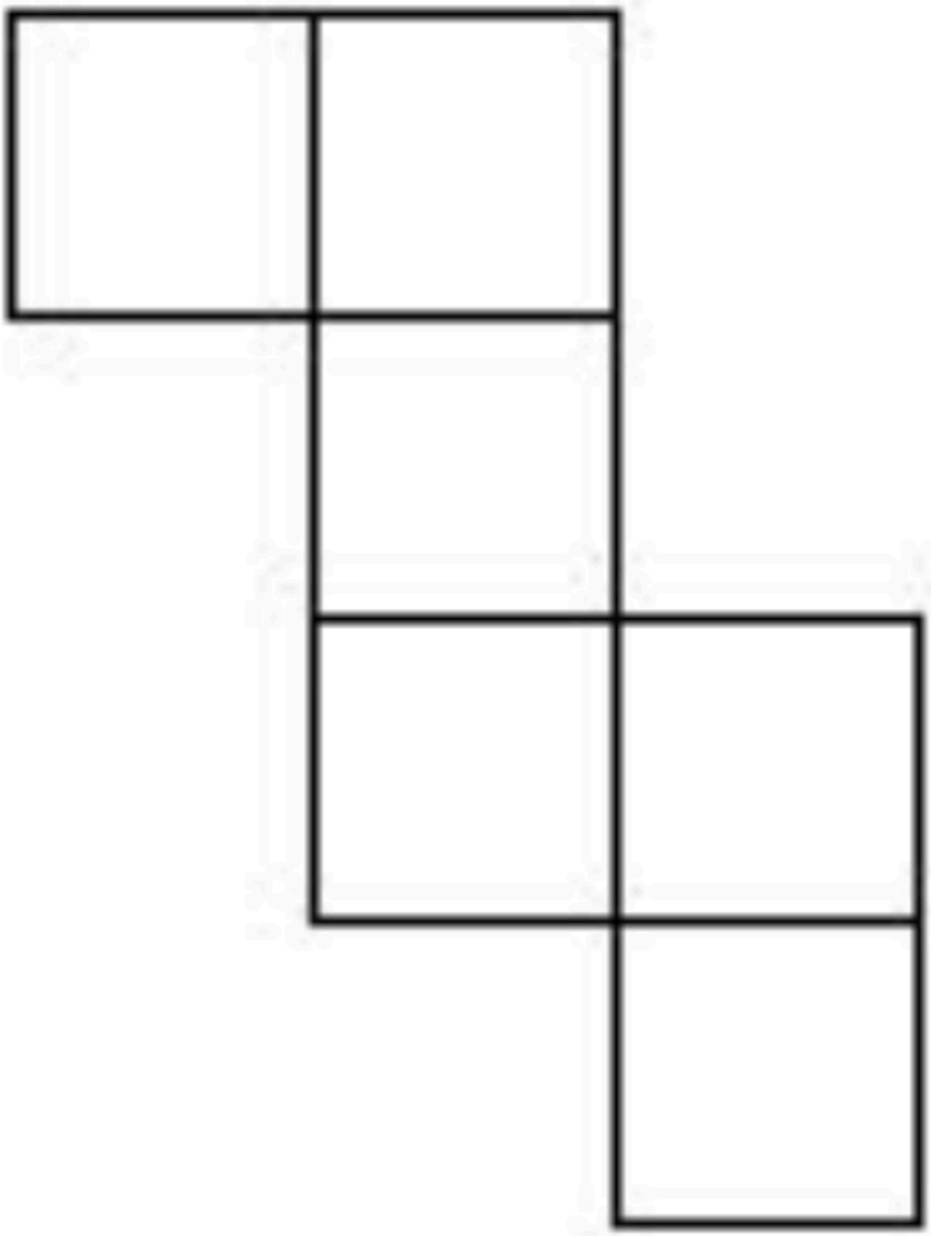
Exercise #2

Cut out the cube net below and create a cube. Pick any two vertices of the cube that are opposite each other. Determine the shortest path between the points without going through the cube. Show all of your work on the cube.



Exercise #2

Cut out the cube net below and create a cube. Pick any two vertices of the cube that are opposite each other. Determine the shortest path between the points without going through the cube. Show all of your work on the cube.



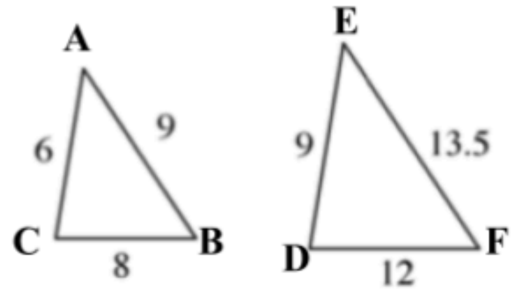
Review & Preview 7.1.3 Name ________________________

Example

Prove two triangles are similar.

Decide which similarity conjecture to use and then prove ΔABC ~ ΔEFD.

1) Look at what information we have.



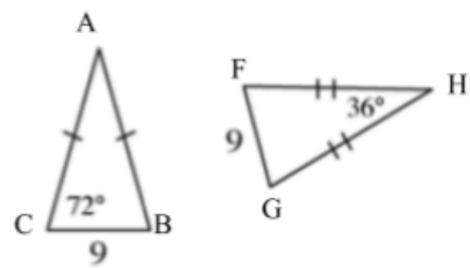
Since we know nothing about the angles, we need to use SSS~.

2) Compare the largest, smallest and medium sized sides and determine the *ratio of similarity.*

Largest Sides *Smallest Sides* *Remaining Sides*

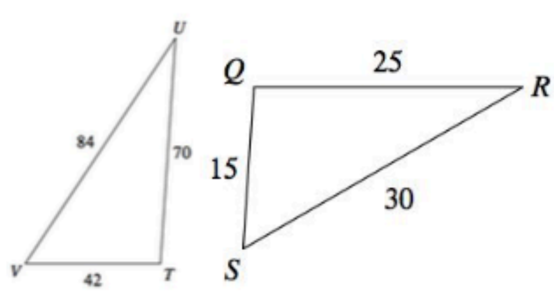
Exercise #1

Prove ΔABC ~ ΔHGF and give the ratio of similarity.

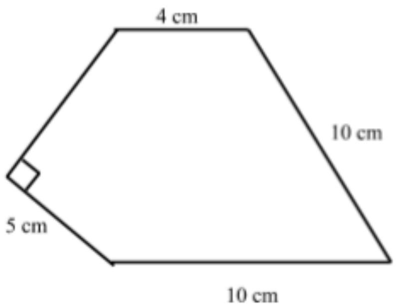


Exercise #2

Prove ΔUVT ~ ΔRSQ

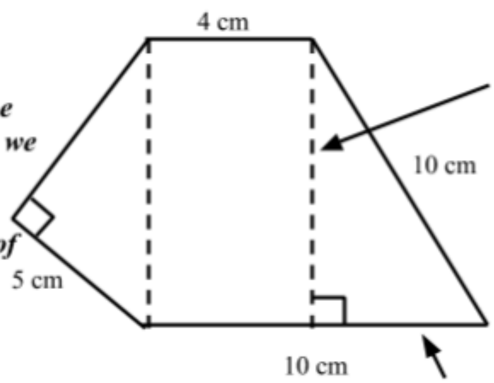


Review & Preview 7.1.3
Use auxiliary lines to find the perimeter of the composite shape.
<u>Example</u>



Since we are looking for the perimeter of the figure we need to find the missing side length. We can draw in auxiliary and perpendicular lines to use as sides of right triangles. Below is a worked example.

3) Use the Pythagorean Theorem again to find the missing leg length which we will call x:
$5^2 + x^2 = 8^2$
We get a missing length of $\sqrt{39}$ ***cm*** $\approx$ ***6.24 cm***

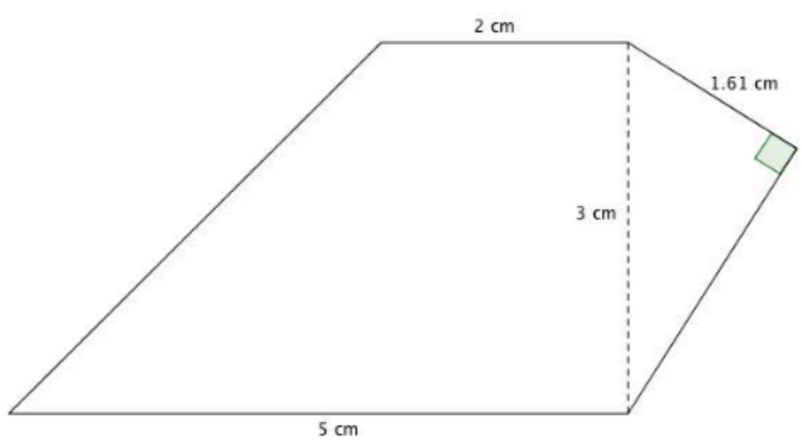


2) Use the Pythagorean Theorem to find the height:
$6^2 + h^2 = 10^2$
We get a height of 8 cm.

1) 10 - 4 = 6 cm

4) Lastly, we add up all the lengths of our composite figure.
$10 + 10 + 4 + \sqrt{39} + 5 \approx 35.24$ ***cm***

<u>Exercise #3</u>
Find the perimeter of the composite figure at right.



7.1.4 Using Symmetry to Study Polygons

Goal: Students will use their understanding of reflection and congruence to learn more about the central angle of regular polygons. Students will also learn more about the diagonals of rhombi.

Question: How can we use a hinged mirror and protractor to form equilateral polygons?

Example: Regular hexagon

1) Place the mirror on the rectangle at right as demonstrated. Looking in the mirror, adjust the mirror until you see a regular hexagon. When you see our desired polygon, measure the angle of the mirror using the protractor. This is our **central angle.**

2) Trace the mirror angle on your paper. This is the **core region** formed by the mirror.

Exercise #1: Square

a) What is the angle measure of the mirror when a square is formed?

b) What shape is the core region? Draw the core region formed by the mirror.

Exercise #2: Equilateral triangle

There are two ways to form an equilateral triangle using the mirror. Find them both.

Method 1

Method 2

Why do you think both of these methods work to form an equilateral triangle?

Exercise #3: Form any other regular polygon.

a) How many sides does your regular polygon have?

b) What is the angle measure of the mirror when your desired regular polygon is formed?

c) What shape is the core region? Draw the core region formed by the mirror.

Key Concept for *regular polygons* : Look back over the core regions of the regular polygons you formed. **How are all of these core regions related? That is, what is similar about all the core regions?**

Exercise #4: Rhombus (which is not a regular polygon)

a) What is the angle measure of the mirror when a rhombus is formed?

b) What shape is the core region? Draw the core region formed by the mirror.

c) **Key Concept for *rhombi:*** Using the space above, draw in a rhombus so that four core regions are used. Using what you know about angle measures, what are two things that are true about the diagonals of a rhombus?

Central angles of regular polygons.

Example:

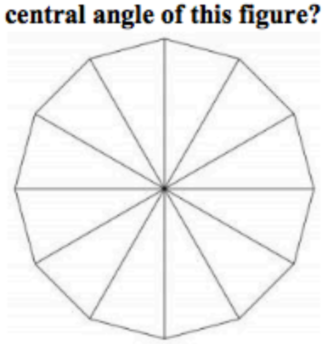
What is the central angle of a regular hexagon?

The central angle can be found without mirrors.

360º / 6 = 60º

Exercise #5

This is a 12 sided regular polygon. What is the central angle of this figure?



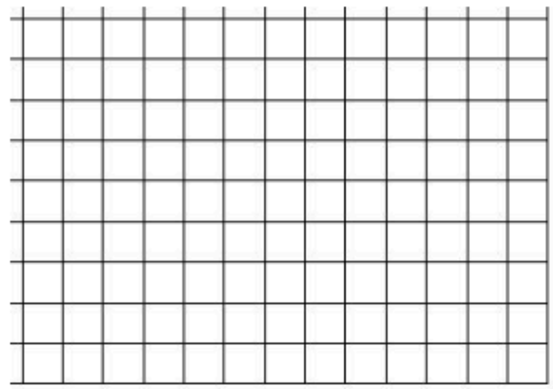
Review & Preview

Using the information you learned about the diagonals of a rhombus, to answer the following questions.

Example

Create a rhombus using four 3-4-5 special right triangle and draw it in below.

a) What is the area of the rhombus?



The area of one triangle is:

$3 \cdot 4/2 = 12/2 = 6$ units^2

Since there are four of these triangles, the total area of the rhombus is:

$6 \cdot 4 = 24$ units^2

b) What is the perimeter of the rhombus?

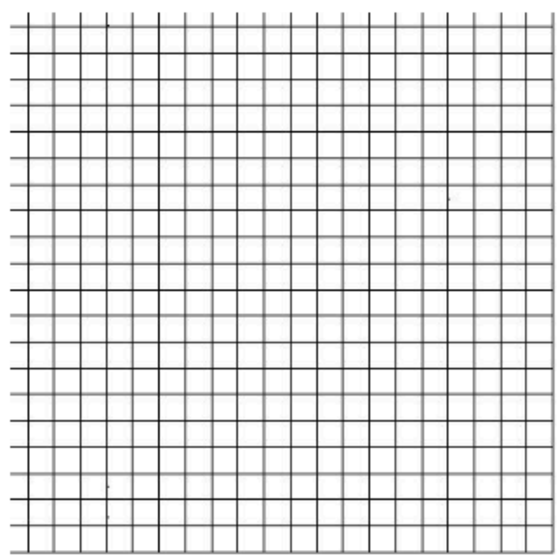
Since the 5 unit side of each 3-4-5 triangle makes up the full perimeter, the perimeter of the rhombus is: $5 \cdot 4 = 20$ units.

c) What are the lengths of the diagonals?

Since the core region of the rhombus is made up of segments of length 3 and 4 units, the diagonals are double those lengths because the diagonals bisect each other, so their lengths are 6 and 8 units.

Exercise #1

a) Create a rhombus using a 6-8-10 special right triangle and draw it in below. What is the area and perimeter of the rhombus?



b) How are the **side length ratios** of the 3-4-5 and the 6-8-10 triangles related to the **perimeter ratios** of each rhombus you created?

c) Try and find a relationship between the **side length ratios** and the **area ratios** of the rhombi.

Exercise #2

On a separate sheet of paper, draw rhombus ABCD and such that each side is 15 cm and diagonal BD = 24 cm.

a) Using the information you found from Exercise #1, find the length of the other diagonal, segment AC.

b) Find the area of rhombus ABCD.

Example How to find the midpoint and slope between two points

a) Find the midpoint P of segment TS given the coordinates T(-3, 5) and S(7, 3).

x-value average

$\frac{-3+7}{2} = \frac{4}{2} = 2$

y-value average

$\frac{5+3}{2} = 4$

P(2, 4)

b) Find the slope of line TS.

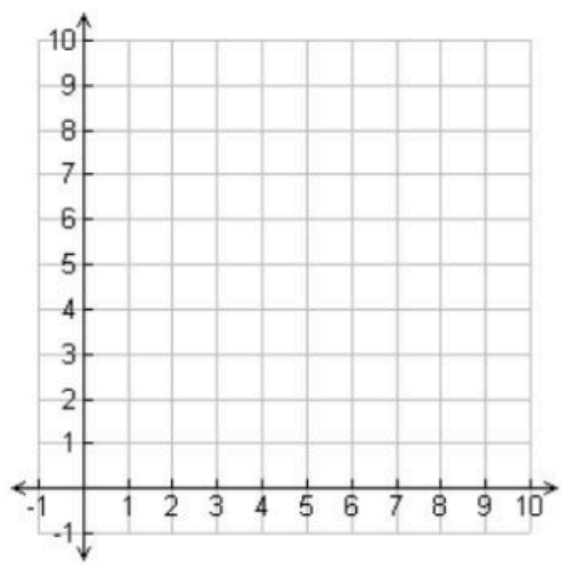
slope = $\Delta y/\Delta x = \frac{5-3}{-3-7} = \frac{2}{-10} = \frac{1}{-5}$

Exercise #3

Draw quadrilateral PQRS with the following coordinates:

P(0, 2) Q(8, 0) R(10, 3) S(2, 8)

a) Calculate each midpoint, and connect adjacent midpoints so that a new quadrilateral is formed. Label this new quadrilateral IMOK.



b) What are the slopes of each side of this new quadrilateral?

c) What kind of quadrilateral is this?

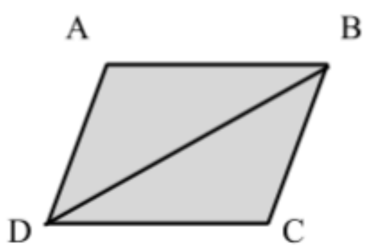
7.2.1 Special Quadrilaterals and Proof

Goal: Use properties of parallelograms and kites in proofs.

Example: Use the fact that opposite sides of a parallelogram are parallel to prove that $\Delta ABD \cong \Delta CDB$.

1) Since we know ABCD is a parallelogram, let's mark congruent angles and congruent sides.

2) We can prove triangles are congruent by using SSS $\cong$, ASA $\cong$, SAS$\cong$, or SAA$\cong$. Look at what information we have from our given information to see which of the four congruence statements we will use.



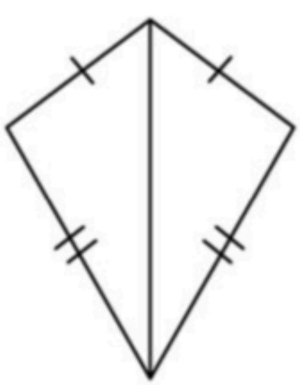
Given:

Prove:

Statement	Reason
1) BC‖AD & AB‖DC	1) Given
2) $\angle CBD = \angle ADB$	2) Alternate interior angles are congruent
3) $\angle BCD = \angle DAB$	3) Opposite angles in a parallelogram are congruent.
4) BD = BD	4) It's the same segment
5) $\Delta ABD \cong \Delta CDB$	5) ASA $\cong$

Exercise #1

Label the vertices of the kite and draw in the diagonals of the kite below. Prove that the diagonals of a kite are perpendicular.



Given:

Prove:

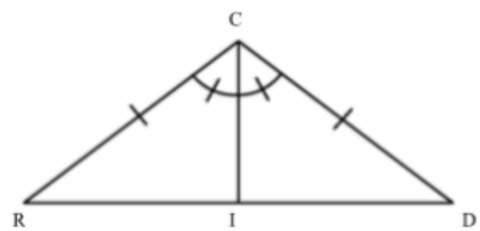
Statement	Reason

<u>Diagonals of a Kite</u>

Goal: Use congruent triangles to learn more about the diagonals of a kite.

<u>Example A</u>

Given the information in the diagram, prove segments AD and BD are equal.



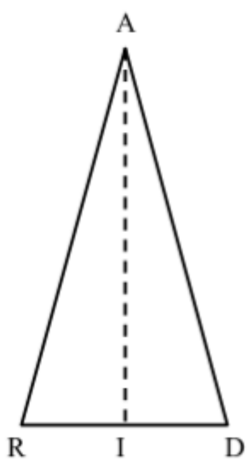
We are given that segments RC and DC are equal to each other, $\angle RCI = \angle DCI$, and IC is of course equal to itself.

This means $\Delta RCI \cong \Delta DCI$ because of SAS $\cong$.

Since corresponding parts of congruent triangles are congruent and RI and DI correspond, then segments RI = DI.

<u>Example B</u>

Given isosceles triangle RAD and segment AI bisects $\angle RAD$, prove that $\Delta RAI \cong \Delta DIA$ and that AI is perpendicular to RD.



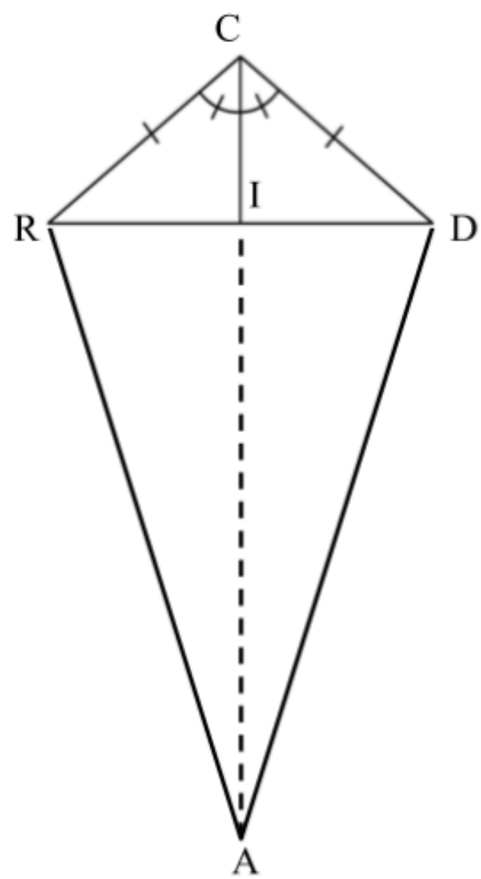
Statement	Reason
1) AR = AD.	1) Definition of isosceles triangles
2) AI = AI	2) Reflexive Property
3) $\angle RAI = \angle DAI$	3) Definition of bisect
4) $\Delta RAI = \Delta DAI$	4) SAS
5) $\angle RIA = \angle DIA$	5) CPCTC.
6) AI is perpendicular to RD	6) Since $\angle RIA = \angle DIA$ and together they form a straight line, they each measure 90º which is the definition of perpendicular.

Question:

In the above proofs there are pairs of congruent triangles. Without doing a complete proof, explain or show what we can know about the pair of angles $\angle RIA$ and $\angle DIA$.

Exercise 1
Use the above proofs to find **AT LEAST TWO properties** of the diagonals of a kite.

In pairs, make a display of your findings. Your display should include the properties followed by the proof.



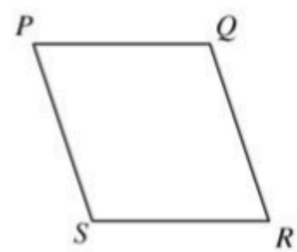
<u>7.2.2 Properties of Rhombi</u> (Taken from CPM Geometry Connections textbook)
Goal: Use properties of parallelograms and kites in proofs.

In Lesson 7.2.1, you learned that congruent triangles can be a useful tool to discover new information about parallelograms and kites. But what about other quadrilaterals? Today you will use congruent triangles to investigate and prove special properties of rhombi (the plural of rhombus). At the same time, you will continue to develop your ability to make conjectures and prove them convincing.

<u>Example</u>:

7-54 Audrey has a favorite quadrilateral - the rhombus. Even though a rhombus is defined as having 4 congruent triangles, she suspects that the sides of a rhombus have other special properties.

a. EXPLORE: Draw a rhombus like the one at right on your paper. Mark the side lengths equal.



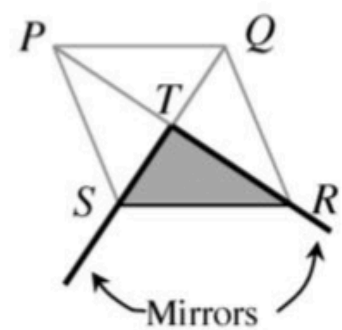
b. CONJECTURE: What else might be special about the sides of a rhombus? Write a conjecture.

c. PROVE: Audrey knows congruent triangles can help prove other properties about quadrilaterals. She starts by adding a diagonal PR to the diagram so that two triangles are formed. Add this diagonal to your diagram and prove that the triangles are congruent.

d. How can the triangles from part (c) help you prove your conjecture from part (b) above? Discuss with your classmates how to extend your proof to convince others be sure to justify any statements with reasons.

7-55 Now that you know the opposite sides of a rhombus have parallel, what else can you prove that a rhombus? Consider this as you answer the questions below.

a. EXPLORE: Remember that in Lesson 7.1.4, you explored the shapes that could be formed with a hinged mirror. During this activity you used symmetry to form a rhombus. Think about what you know about the reflective triangles in the diagram. What do you think is true about the diagonals SQ and PR? What is special about ST and QT? What about PT and RT?



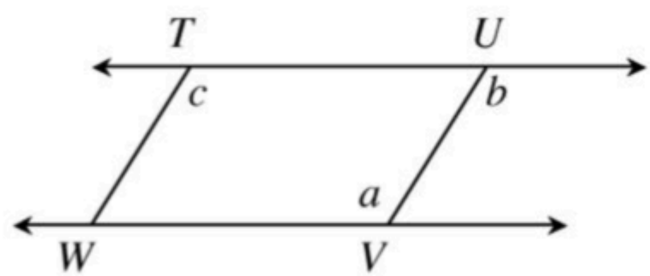
b. CONJECTURE: Use your observations from part (a) to write a conjecture on the relationship of the diagonals of a rhombus.

c. PROVE: Write a proof that proves your conjecture from part (b). Remember that to be convincing you need to justify each statement with a reason. To help guide your discussion, consider the questions below.

- Which triangles should you use? Find two triangles that involve the lengths ST, QT, PT and RT.
- How can you prove these triangles are congruent? Create a proof with reasons to prove these triangles must be congruent.
- How can you use congruent triangles to prove your conjecture from part (b)? Extend your proof to include this reasoning and prove your conjecture.

7-56 There are often many ways to prove a conjecture. You have rotated triangles to create parallelograms and use the congruent parts of congruent triangles to justify the opposite sides are parallel. But is there another way?

Ansel wants to prove the conjecture "*If a quadrilateral is a parallelogram, then opposite angles are congruent.*" He started by drawing parallelogram TUVW at right. Copy and complete his proof. Make sure that each statement has a reason.



Given: TUVW is a parallelogram
Prove: a = c

Statement	Reason
1) TUVW is a parallelogram	1) Given
2) TW ∥ UV and TU ∥ WV	2) Definition of parallelogram
3) b = c	3) ____________________
4) ____________	4) If lines cut by a transversal are ∥, then alt. Int. angles are congruent.
5) ____________	5) Substitution property

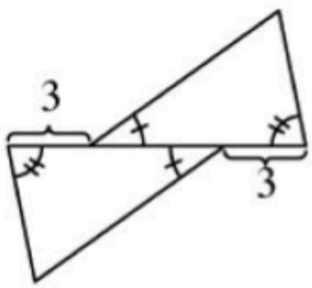
7.2.3 Proofs Using Congruent Triangles

Goal: Become familiar with identifying conjectures and conditions, as well as using congruent triangles in proofs.

Example 1

Given the information in the diagram, prove that the triangles are congruent.

Label the diagram using the given information, and fill in the missing reasons in the proof.

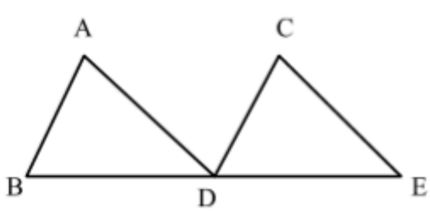


Statement	Reason
1) $\angle A = \angle D$	1) Given
2) $\angle BCA = \angle EFD$	2) Given
3) AC = DF	3) Both segments equal 3 + FC
4) $\Delta ABC = \Delta DEF$	4) ASA ≅

Example 2

Given that segments AB = CD, AD = CE, and D is the midpoint of segment BE, prove that AB || CD.

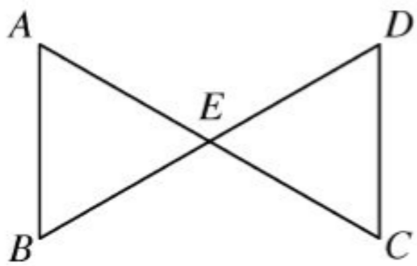
Label the diagram so that the statements correspond to the statements in the two column proof.



Statement	Reason
1) AB = CD, AD = CE, and D is the midpoint of BE.	1) Given
2) BC = CE	2) Definition of Midpoint
3) $\Delta ABD = \Delta CDE$	3)
4) $\angle B = \angle CDE$	4)
5) AB || CD	5)

Example 3

Jester started to prove that segments AB and DC are congruent. The only information he was given was that point E is the midpoint of AC and BD. Complete a proof that *segments AB and DC are congruent.*

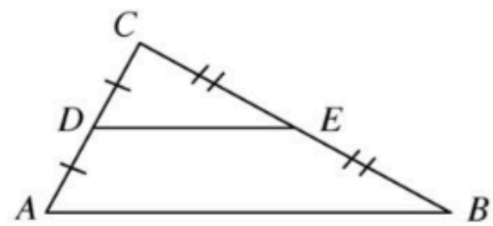


7.2.6 What can I prove? (Taken from CPM Geometry Connections textbook)

So far, congruent triangles have helped you to discover and prove many new facts about triangles and quadrilaterals. But what else can you discover and prove? Today your work will mirror the real work of professional mathematicians. You will investigate relationships, write a conjecture based on your observations, and then prove your conjecture.

7-88 TRIANGLE MIDSEGMENT THEOREM

As Sergio was drawing shapes on paper, he drew a line segment that connects the midpoints of two sides of a triangle. This is called the midsegment of a triangle. "*I wonder what we can find out about this midsegment*," he said to his team. Examine the drawing at right.



a. EXPLORE: Examine the diagram of triangle ABC, drawn to scale above. How do you think segment DE is related to segment AB? How do their lengths seem to be related?

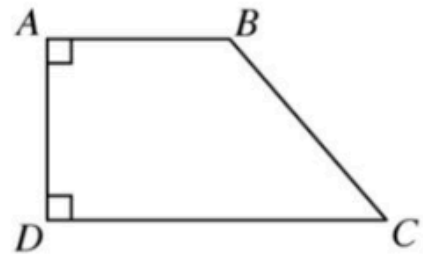
b. CONJECTURE: Write a conjecture about the relationship between segments DE and AB.

c. PROVE: Sergio wants to prove that AB =2DE. However, he does not see any congruent triangles in the diagram. How are the triangles in this diagram related? How do you know? Prove your conclusion with a proof.

d. What is the common ratio between side lengths in similar triangles? Use this to write a statement relating length DE and AB.

e. Now Sergio wants to prove that DE || AB. Use the similar triangles to find all pairs of equal angles you can in the diagram then use your knowledge of angle relationships to make a statement about parallel segments.

7-90 RIGHT TRAPEZOIDS



Consecutive angles of a polygon occur at opposite ends of a side of the polygon. What can you learn about a quadrilateral with two consecutive right angles?

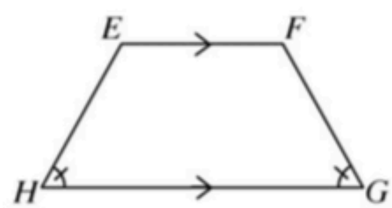
a. EXPLORE: Examine the quadrilateral at braids with two consecutive right angles. What do you think is true of AB and DC?

b. CONJECTURE: Write a conjecture about what type of quadrilateral has two consecutive right angles. Write your conjecture in conditional form. ("If..., then...")

c. PROVE: Prove that your conjecture from part (b) is true for all quadrilaterals with two consecutive right angles.

d. The quadrilateral you worked with this problem is called a right trapezoid. Are all quadrilaterals with two right angles a right

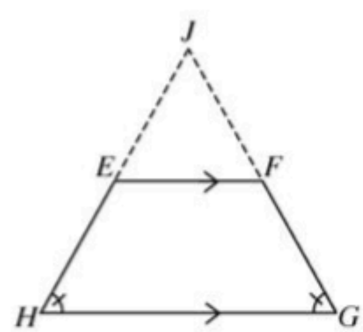
7-91 ISOSCELES TRAPEZOIDS



And isosceles trapezoid is a trapezoid with a pair of congruent base angles. What can you learn about the sides of an isosceles trapezoid?

a. EXPLORE: Examine EFGH at right. How do the side lengths appear to be related?

b. CONJECTURE: Write a conjecture about side lengths in an isosceles trapezoid. Write your conjecture in conditional form. ("If..., then...")

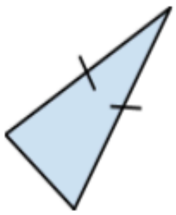
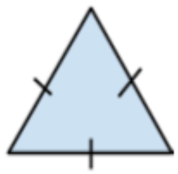


c. PROVE: Now prove that your conjecture from part (b) is true for all isosceles trapezoids. To help you get started the isosceles trapezoid is shown at right with its sides extended to form a triangle.

For each quadrilateral description say what special type the quadrilateral ***must be*** and/or ***could be***.

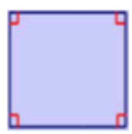
An example involving triangles
This triangle has two congruent side lengths.

It must be at least a:	It could be a:
Isosceles triangle	Equilateral triangle

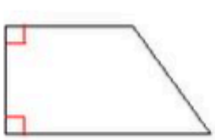
Examples involving quadrilaterals
This quadrilateral has four right angles.

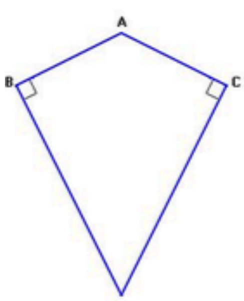
It must be at least a:	It could be a:
Rectangle	Square Rhombus (only if the angles are all 90º, at which point we have a...)

This quadrilateral has ONLY two right angles.

It must be at least a:	It could be a:
Right trapezoid or a kite	...





Answer the following questions as a group. Be prepared to share your thoughts with the class.

1) This quadrilateral has four equal sides

<u>It must be at least:</u> <u>It could be:</u>

2) This quadrilateral has two pairs of opposite parallel sides.

<u>It must be at least:</u> <u>It could be:</u>

3) This quadrilateral has two consecutive right angles.

<u>It must be at least:</u> <u>It could be:</u>

4) This quadrilateral has two pairs of equal sides.

<u>It must be at least:</u> <u>It could be:</u>

Midpoint and Slope on a Coordinate Grid **Name** ________________

Goal: Understand what midpoints and slopes are, and calculate midpoints and slopes of lines or line segments.

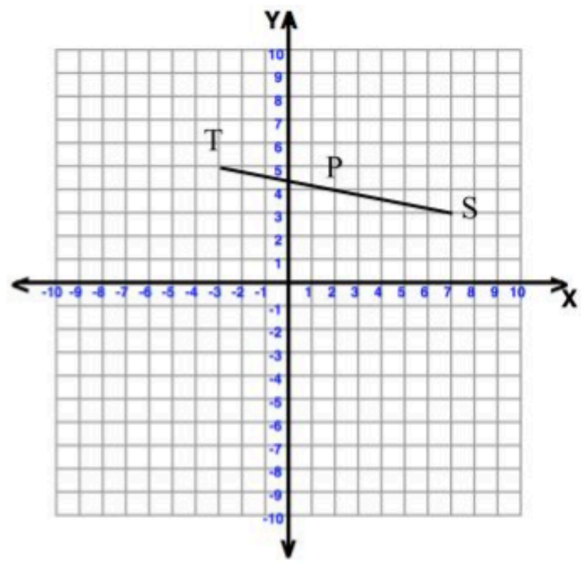
Midpoints

Midpoints are the average of the x-values and y-values.

Example 1

Find the midpoint P of T(-3, 5) and S(7, 3). Plot segment TS and point P.

x-value average	y-value average	midpoint
$\frac{-3+7}{2} = \frac{4}{2} = 2$	$\frac{5+3}{2} = 4$	P(2, 4)



Example 2

Find the midpoint C of A(2, 8) and B(10, 6). Plot segment AB and point C.

x-value	y-value	midpoint
$\frac{2+10}{2} = \frac{12}{2} = 6$	$\frac{8+6}{2} = \frac{14}{2} = 7$	C(6, 7)

Example 3

Find the midpoint F of D(-9, -3) and E(-6, 7). Plot segment DE and point F.

x-value	y-value	midpoint
$\frac{-9+(-6)}{2} = \frac{-15}{2} = -7.5$	$\frac{-3+7}{2} = \frac{4}{2} = 2$	F(____, ____)

Example 4

Find the midpoint L of J(2, -2) and K(10, -10). Plot segment JK and point L.

x-value	y-value	midpoint
$\frac{10+2}{2} = \frac{\quad}{2} =$	$\frac{-10+(-2)}{2} = \frac{\quad}{2} =$	L(____, ____)

Exercises - Complete these calculations on a separate piece of paper.

a) Find the midpoint O of segment MN given coordinates M(6, -3) and N(2, -7).
b) Find the midpoint R of segment PQ given coordinates P(-2, 23) and Q(-1, 17).
c) Find the midpoint X of segment VW given coordinates V(-2, 21) and W(-4, 8).
d) Find the midpoint X of segment VW given coordinates V(-12, 23) and W(-14, -8).

Slope of a Line

Definitions of *slope*:

a) One of the most important properties of a straight line is in how it angles away from the horizontal. This concept is reflected in something called the "slope" of the line.

<http://www.purplemath.com/modules/slope.htm>

b) The steepness of a line

We have seen *slope* before and defined it as: $\frac{\textit{change in y}}{\textit{change in x}}$ or $\frac{\Delta y}{\Delta x}$

You might remember this as $\frac{y_2 - y_1}{x_2 - x_1}$ where two points are (x_1, y_1) and (x_2, y_2).

Example 1

S(7, 3) and T(-3, 5) Slope of ST = $\frac{5-3}{-3-7} = \frac{2}{-10} = \frac{1}{-5}$

(x_1, y_1) *and* (x_2, y_2)

Example 2 [*Complete the calculation, too*]

B(10, 6) and A(2, 8) Slope of BA = $\frac{8-6}{2-10} = \frac{2}{-8} = $ —

(x_1, y_1) *and* (x_2, y_2)

Example 3 [*Complete the calculation, too*]

D(-9, -3) and E(-6, 7) Slope of DE = $\frac{7-(-3)}{-6-(-9)} = $ —

Example 4 [*Complete the calculation, too*]

K(10, -10) and J(2, -2) Slope of KJ = $\frac{-2-(-10)}{2-10} = $ — $=$

Questions

π) *Give at least two differences between finding midpoints and finding slopes.*

∞) *In **Example 1**, S was given as (x_1, y_1) and T was (x_2, y_2). Show what the slope would be if we made $T(x_1, y_1)$ and $S(x_2, y_2)$. Why do you think this happens?*

Exercises - Complete these calculations on a separate piece of paper.

a) Find the *slope* of MN given coordinates M(6, -3) and N(2, -7).
b) Find the *slope* of PQ given coordinates P(-2, 23) and Q(-1, 17).
c) Find the *slope* of VW given coordinates V(-2, 21) and W(-4, 8).
d) Find the *slope* of VW given coordinates V(-12, 23) and W(-14, -8).

Parallel and Perpendicular Lines Name__________________ Date ______

Goal: Learn about and use the relationship between slopes of parallel and perpendicular line.

When two lines are **perpendicular**, the slope of one is the **negative reciprocal** of the other.

If the slope of one line is ***m***, the slope of the other is ***-1/m***.

When two lines are **parallel**, the two slopes are **equal**.

If the slope of one line is ***m***, the slope of the other is ***m***.

Example

One line passes through the points (–1, –2) and (1, 2); another line passes through the points (–2, 0) and (0, 4). Are these lines parallel, perpendicular, or neither?

$$m_1 = \frac{2-(-2)}{1-(-1)} = \frac{2+2}{1+1} = \frac{4}{2} = 2$$

$$m_2 = \frac{4-0}{0-(-2)} = \frac{4}{2} = 2$$

Since these two lines have **identical slopes**, then these lines are **parallel**.

Exercise 1

One line passes through the points (–2, 3) and (1, 5); another line passes through the points (2, 3) and (-1, 1). Are these lines parallel, perpendicular, or neither?

Exercise 2

One line passes through the points (4, 3) and (6, -5); another line passes through the points (4, 0) and (-1, 1). Are these lines parallel, perpendicular, or neither?

Example

One line passes through the points (0, -4) and (-1, -7); another line passes through the points (3, 0) and (-3, 2). Are these lines parallel, perpendicular, or neither?

$$m_1 = \frac{-7-(-4)}{-1-0} = \frac{-7+4}{-1} = \frac{-3}{-1} = 3$$

$$m_2 = \frac{2-0}{-3-3} = \frac{2}{-6} = -\frac{1}{3}$$

If we were to flip the "3" and then change its sign, we would get "$^{-1}/_{3}$". In other words, these slopes are **negative reciprocals**, so the lines through the points are **perpendicular**.

Exercise 3

One line passes through the points (10, 15) and (15, 5); another line passes through the points (25, 15) and (5, 5). Are these lines parallel, perpendicular, or neither?

Exercise 4

One line passes through the points (14, 0) and (6, -5); another line passes through the points (4, 0) and (-1, 1). Are these lines parallel, perpendicular, or neither?

Exercise 5

On the graph paper provided plot the points below.

S(0, 0)   H(0, 5)   A(4, 8)   Y(7, 4)

Show whether or not this is a right trapezoid by calculating and comparing the slopes of the sides.

Exercise 6

On the graph paper provided plot the points below.

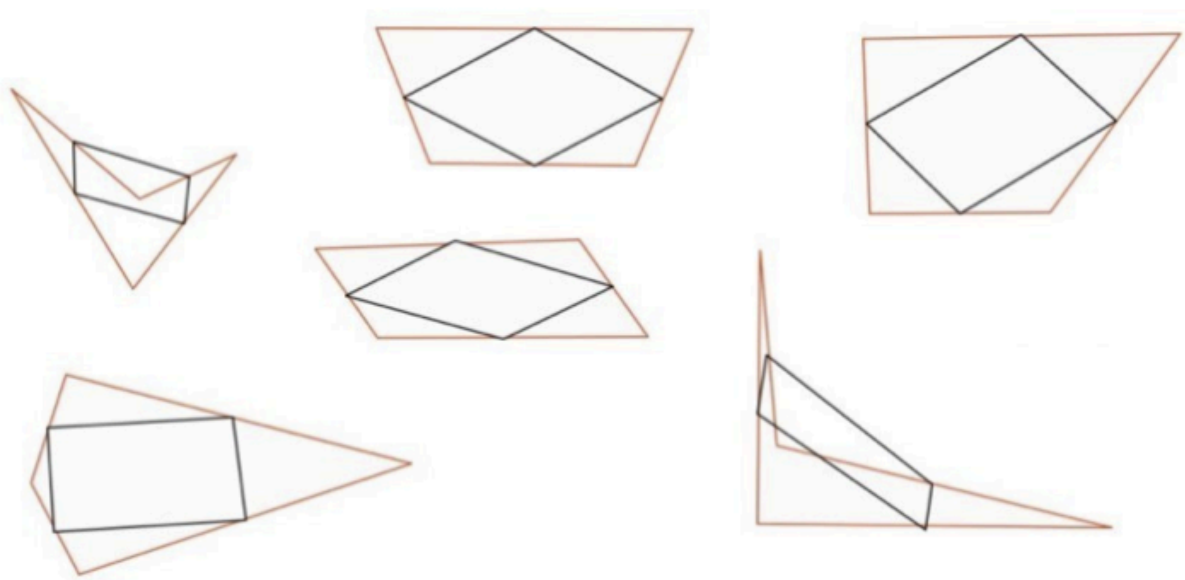
B(4, 2)   H(2, 7)   A(12, 11)   Y(14, 6)

Show that this is a rectangle by calculating and comparing the slopes of the sides.

Chapter 7 Portfolio **Name ________________ Date ____**

During our study of quadrilaterals, we've come across a very special shape: the parallelogram. But, did you know that if you were to connect the four midpoints of ANY quadrilateral, you would construct a parallelogram?

Look at the diagrams below and see if you agree.

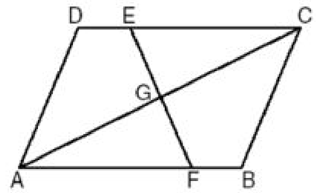


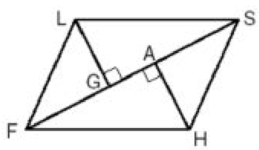
To help show that this theorem works, you will construct a quadrilateral on the graph paper provided, connect its midpoints, and prove that the new shape is a parallelogram.

a) Show the calculations for each midpoint.
b) Show that each pair of opposite sides of the new quadrilateral have the same slope.
c) Show that each pair of opposite sides of the new quadrilateral have the length.

Given: ABCD is a parallelogram
DE = BF

Prove $\Delta EGC = \Delta FGA$

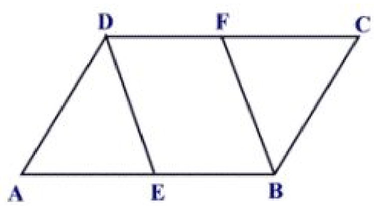




Given: $\square FLSH, \overline{FS} \perp \overline{LG}$

$\overline{FS} \perp \overline{HA}$

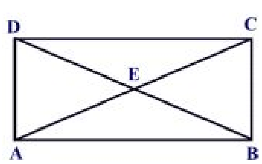
Prove: $\Delta LGS \cong \Delta HAF$



Given: $\square ABCD$,

$\overline{AE} \cong \overline{FC}$

Prove: $\square DEBF$



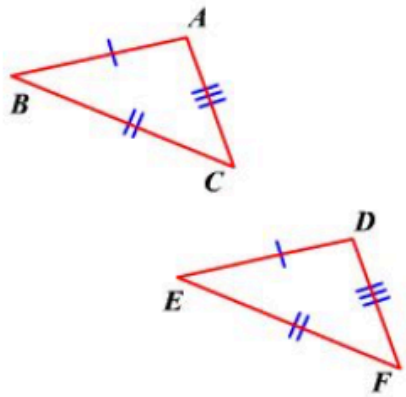
Given: $ABCD$ rectangle

Prove: ΔCEB isosceles

Quiz #1 Question 1

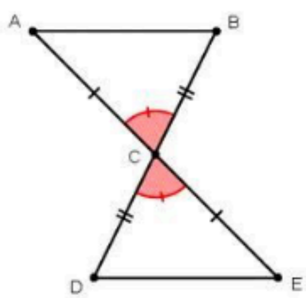
Name ___________________

A1) Given the information in the diagram, prove ∠A = ∠D.



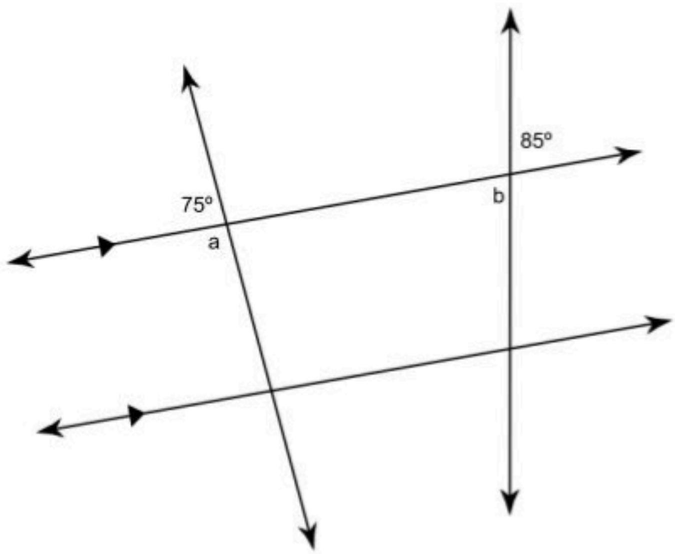
Name ___________________

B1) Given the information in the diagram, prove that segments AB = DE.

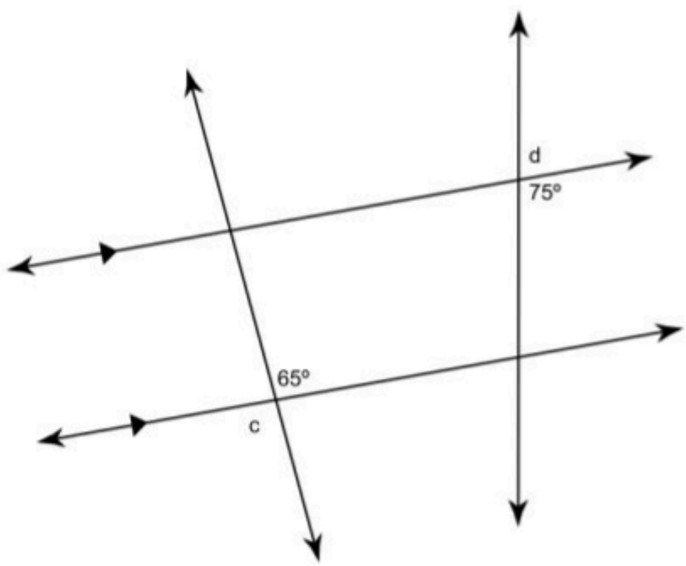


Quiz #1 Question 2

A2) Use what you know about angle relationships to determine the measures of angle a and b.



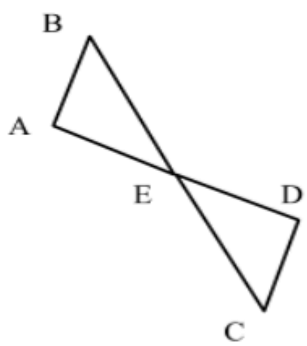
B2) Use what you know about angle relationships to determine the measures of angle c and d.



Quiz #2 Question 1

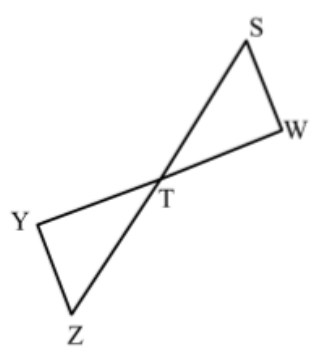
Name ________________

A1) Given that E is the midpoint of segment AD, and $\angle D$ and $\angle A$ are right angles, prove the two triangles are congruent.



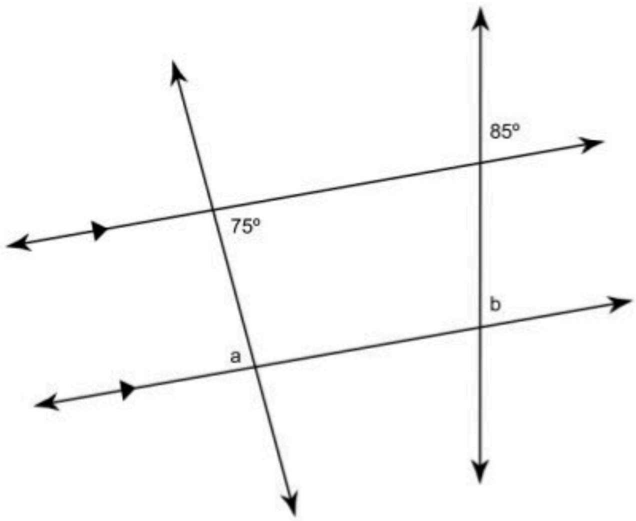
Name ________________

B1) Given that T is the midpoint of segment WY, and $\angle W$ and $\angle Y$ are right angles, prove the two triangles are congruent.

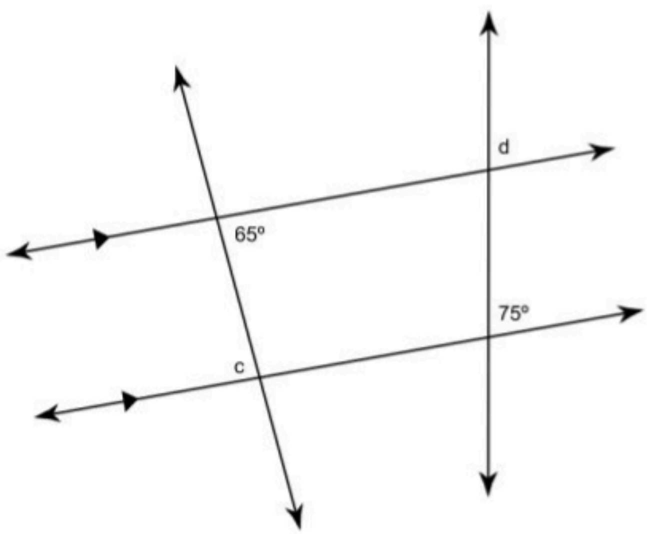


Quiz #2 Question 2

A2) Use what you know about angle relationships to determine the measures of angle a and b.



B2) Use what you know about angle relationships to determine the measures of angle c and d.



Quiz #3 (Only one version)

1) Proof of a Kite

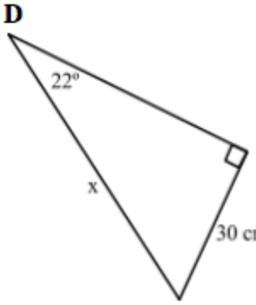
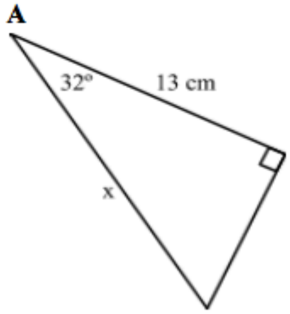
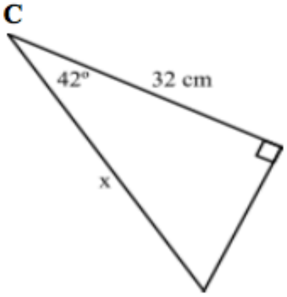
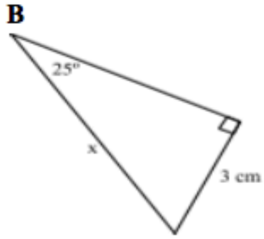
Given: CA bisects ∠XCY

XC = YC

∠X = ∠Y

Prove AXCY is a kite.

2) Calculate the length of the hypotenuse. (Labeled x)
Round your answer to the thousandths place. (ex. 4.5431 --> 4.54)

D 	**A** 
C 	**B** 

Quiz #4

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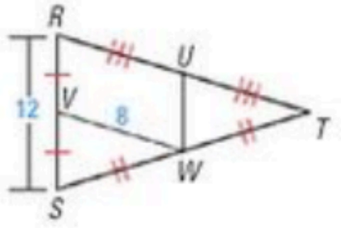
D

1) Find the length of segments RT and UW.



2) What is the slope and y-intercept of the line:
y = 9x – 3?
Write the slope as a fraction and the y-intercept as a point.

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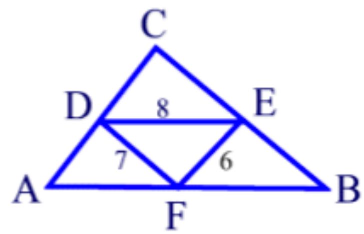
A

1) Points D, E, and F are midpoints.
Find the length of AC and FB.



2) What is the slope and y-intercept of the line:
y = 15x – 4
Write the slope as a fraction and the y-intercept as a point.

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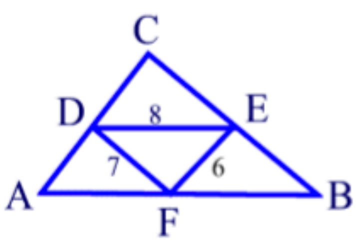
C

1) Points D, E, and F are midpoints.
Find the length of AB and CE.



2) What is the slope and y-intercept of the line:
y = 14x – 9
Write the slope as a fraction and the y-intercept as a point.

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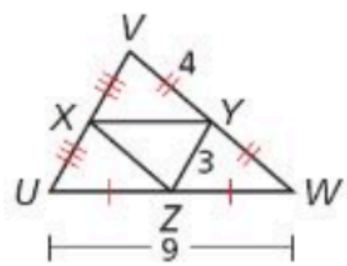
B

1) Find the length of XZ and XY.



2) What is the slope and y-intercept of the line:
y = 14x – 5
Write the slope as a fraction and the y-intercept as a point.

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APPENDIX B

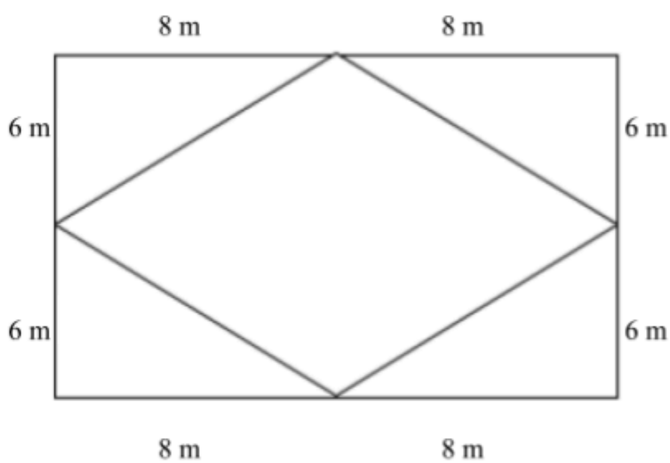
Appendix B

Worked Example Used from Hilbert, Renkl, Kessler & Reiss, (2008)

Strategies of Proving (*Translated from German*) Name: _______________ Date _____Period ____

1) **The Scene**

Elke and Katja are once again at their favorite pastime: visiting Flash and Silver on their paddock (*small field where horses are kept or exercised*). Unfortunately, they have had little time for their horses, and so the paddock had accumulated all sorts of junk in recent weeks. Because all four corners have been quite full [of stuff], and they do not want Flash and Silver to hurt themselves, they have decided to reduce the size of the paddock by removing the corners. They just connected the middle of each side of the rectangular paddock with each other, thus creating a smaller, but again four cornered paddock. Now they want to know how long the boards must be. Katja has already made a sketch:



"That's funny," says Elke , "although I do not know how long the new sides of our paddock will be, I think they are all the same length. "

"Well, one can see that ! " Answers Katya.

"Yes, exactly! So it seems. But are you sure that all four are exactly the same length? How exactly can you know that? Let us measure the sides."

Based on Katja's sketch measure the four new sides. In fact, they measure the same value for each of the sides. Now they know how long the boards need to be and could actually build it. But now Katja's interest has been aroused. She likes geometry.

"Say, Elke do you think that it's always like this? I mean, even if our paddock would have different dimensions and if we would reconnect the four middles, would we then get a rhombus? If I remember correctly, they call that namely a quadrilateral with four equal sides. "

Katja makes two new sketches and measures each. They are always the same length!

Elke : "Well, we have developed a good method to make a rhombus from an arbitrary quadrilateral."

Katja : "Are you sure? Could it be that I have drawn only those quadrilaterals where this is true? And so we cannot always expect these measures. What if the four sides are only *just about* the same size? Or completely different sizes?"

Elke : "Well, you're the geometry expert. Let us try again to prove this like a true mathematician."

So Elke and Katja try to prove the following mathematical statement:

"Combining the middle of each side of any quadrilateral with one another, the result is always a rhombus."

In your own words, write what you think Elke's and Katja's claim is.

In the following, we'll look at exactly how they have solved this math problem. You should use this not only to read, but perform the steps yourself.

2) **Study of the Problem**

First, we want to reconstruct the arguments of Elke and Katja. You need a ruler, a protractor, and three colored pencils.

a) Miss Katja's sketch. Is the inner quadrilateral really a rhombus?

b) In the following box, draw any **rectangle** ABCD and connect the four midpoints to create a new quadrilateral. Give the drawing to your partner to measure and record the side lengths of the new quadrilateral.

c) Elke claimed that works with each quadrilateral. Draw in the following field any **quadrilateral** ABCD (not a rectangle) and connect the four midpoints to create a new quadrilateral. Give the drawing to your partner to measure and record the side lengths of the quadrilateral.

Katja: "You see, Elke, here we would have been taken aback if we had presented that to our teacher. That does not work in any quadrilateral every time. It must then be a rectangle for our result to be a rhombus."

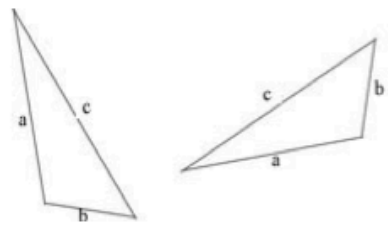
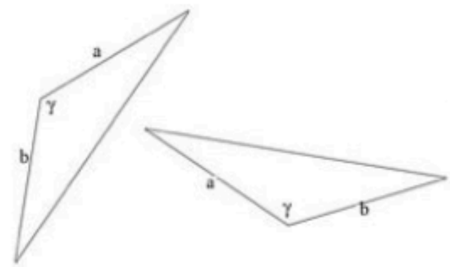
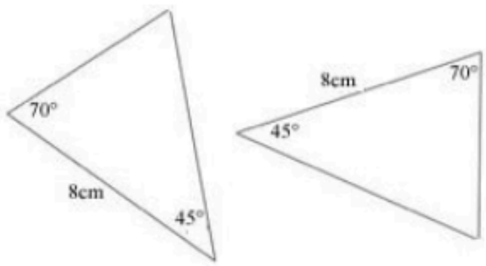
Elke: "Okay, I take back that it can be any quadrilateral. But I did not say that I would tell our math teacher that the four resulting sides are the same length. He would have certainly expressed that it was more complicated than that."

Katja: "Well, he possibly would have spoken of a diamond. Or perhaps he would have said instead congruent sides." (???)

Elke: "Just like last school year, he always illustrated it with congruence. And weren't there those strange congruence statements? I don't remember exactly, but they always had to do with triangles."

Katja: "True, with the congruence statements one can show that two triangles are congruent, so that they have equal sides and angles. Hmm, triangles appear in our sketches. Wait a second, I'll help the memory to make the jumps."

d) Try to remember as well the congruence statements (SSS, SAS, ASA, AAS). Find the congruence statements and label each congruent side and angle of the triangles with the same color.

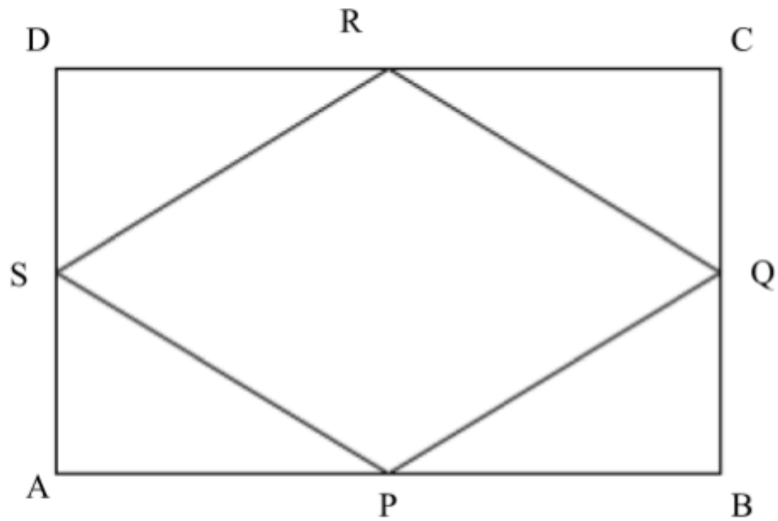
 Congruence statement: ________	 Congruence statement: ________
 Congruence statement: ________	Congruence statement: ________

e) One of the four listed congruence statements does not have an accompanying drawing. Draw a pair of congruent triangles that correspond to the missing statement in the empty box.

f) Describe what Elke and Katja were attempting to do after they stated their claim.

3) **The Claim**

Attempt to formulate Elke's and Katja's claim and add the finishing touches to the following sketch.



Let ABCD be any rectangle. For the sides, that means _______ = _______ and _______ = _______. And for the angles that means _______ = _______ = _______ = _______ = _______. Then with the midpoints labeled P, Q, R, S and connecting them we get a new quadrilateral.

We claim that this new quadrilateral is a rhombus, which means _______ = _______ = _______ = _______. Compare this with Elke and Katja's earlier claim.

Mathematical allegations must be proved. For this proof, it is important:

- *that you are thinking about what you know about quadrilaterals and congruence,*
- *that you can then collect from all the possible arguments, the evidence that could be important for the specific arguments here, and finally*
- *that you are arranging your arguments in a logical sequence (in a chain of evidence).*

4) What do you know about quadrilaterals and congruence statements?

There are many possible arguments that could be used for the proof of the claim. Particularly remember the following facts. Using a straightedge, draw an accurate sketch below each statement and give the name of each shape.

- The sum of the angles in a quadrilateral is always ________.
- A quadrilateral with only four 90º angles is called _______.
- A quadrilateral with only opposite respective sides being parallel is called _______.
- A quadrilateral with only four sides of the same length is called ______.
- A quadrilateral that is at the same time a rectangle and a rhombus is called _______.
- A quadrilateral that is at the same time a parallelogram and a kite is called _______.
- In any trapezoid there is one pair of opposite ____________ sides.
- The opposite sides in a rectangle are ___________ and ____________. (*No sketch needed*)
- In a rhombus, the opposite angles are ________.

Give the triangle congruence statement that corresponds to each statement below. Using a straightedge, sketch a pair of triangles that could be described by the description.

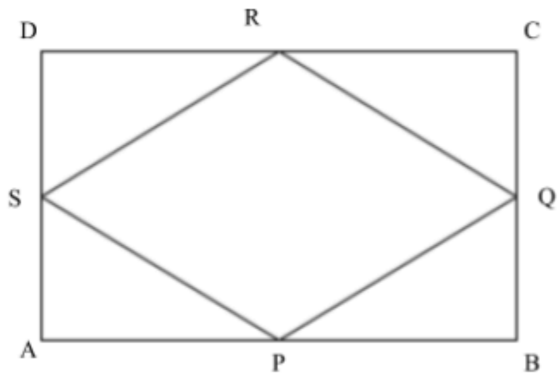
- Two triangles that have corresponding pairs of congruent sides.
- Two triangles that have two pairs of corresponding pairs and a congruent angle between them.
- Two triangles that have two pairs of congruent angles and a congruent side between each angle.

4) **The Proof**

If two triangles are congruent, they ***agree*** on all sides and angles. Maybe we can show with our congruent statements that the four sides of the inner square are all the same size.

Given any rectangle ABCD, segment AB = ___________ and segment BC = ____________.
Also the following angles are congruent, _______ = _______ = _______ = _______.

In our rectangle we will label the midpoints of each side P, Q, R, & S and connect them with another to form a new quadrilateral.



a) **We compare the triangles APS and PBQ.**
Because P is the midpoint, AP = ______.

Because S and P are also midpoints AS = ________ = 1/2 ________ = 1/2 ________.

Because we know the angles of the rectangle are congruent, we can say angle PAS = angle ________.

With that information we say the two triangles __________ and ___________ are congruent by the following congruent statement: ____________.

It follows from that statement that segment SP = ___________.

b) **In the same way as above, we compare the triangles DSR and CQR.**
Because R is the midpoint, RD = ______.

Because S and Q are also midpoints DS = ________ = 1/2 ________ = 1/2 ________.

Because we know the angles of the rectangle are congruent, we can say angle RCQ = angle ________.

With that information we say the two triangles __________ and ___________ are congruent by the following congruent statement: ____________.

It follows from that statement that segment SR = ___________.

c) **In the same way as above, we compare the triangles BQP and CRQ.**
Because Q is the midpoint, BQ = ______.

Because P and R are also midpoints PB = ________ = 1/2 ________ = 1/2 ________.

Because we know the angles of the rectangle are congruent, we can say angle RCQ = angle ________.

With that information we say the two triangles __________ and ___________ are congruent by the following congruent statement: ____________.

It follows from that statement that segment RQ = ___________.

We can now prove our claim:
Collecting our information we have the following information which will allow us to prove our claim that PQRS is a rhombus.

PQ = _______ = _______ = _______

5) **Review**

As a result of our solution we now know that the four midpoints in any rectangle connect to create a rhombus. Mathematically speaking, we have found a proof of our assertion.

We found evidence of Elke's and Katja's claim.
But they keep conversing:
Elke: "Super, Katja! That was not as hard as I thought at first. But you know what I noticed: I think if we divide up our paddock like this it is only half as large."
Katja: "Are you sure? Let's take this back to class to see if this is true…"

6) **Your task:**
Prove that PQRS = 1/2ABCD.

APPENDIX C

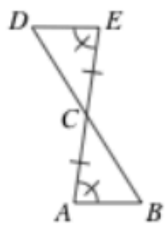
Appendix C

Worked Example Unit Test, Test Rubric and Sample Scoring

Geometer: ___________________Period: _____

Geometry Chapter Seven EXAM A

1) Use the given information in the diagram to prove that segment DE ≅ segment AB? If not possible, explain why not. *[Bonus: Prove that C is the midpoint of DB]*



2) Complete the proof at right.

Given: $\overline{TC} \cong \overline{TM}$

$\overline{AT}$ bisects $\angle$CTM

Prove: $\overline{AC} \cong \overline{AM}$

Statements	Reasons
1. $\overline{TC} \cong \overline{TM}$ and $\overline{AT}$ bisects $\angle CTM$	
2.	Definition of bisect
3. $\overline{AT} \cong \overline{AT}$	
4.	
5.	$\cong \Delta s \rightarrow \cong$ parts

3) Given the conditions, which shape(s) must it be and which shape(s) could it be?

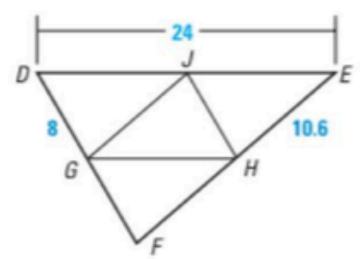
a) My quadrilateral has four equal sides.

b) The diagonals of my quadrilateral are perpendicular.

c) My quadrilateral has a pair of parallel sides.

4) Segments JH, HG, and GJ are midsegments.
What are their lengths?

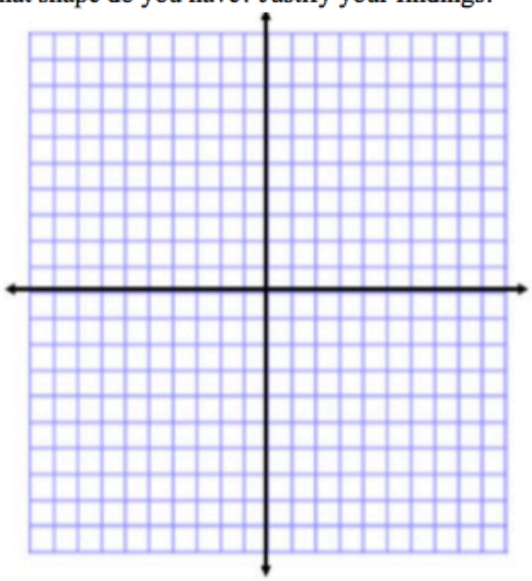
JH = _________ b) GH = _________ JG = ________



5) What do we know about points G, H, and J?

---

6) You connect the following four points to make a quadrilateral: A (-1, 7), B (5, 9), C (3, 6), and D(-3, 4).
What shape do you have? Justify your findings!



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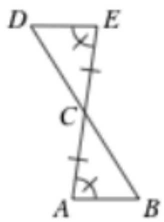
7) Given the two points J(12, -30) and K(-1, 20). What is the slope, distance, and midpoint of segment JK?

Geometer: ____________________Period: _____

Geometry Chapter Seven EXAM B

1) Use the given information in the diagram to prove that segment DC ≅ segment CB? If not possible, explain why not. *[Bonus: Prove that C is the midpoint of segment DB]*



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2) Complete the proof at right.

Given: $\overline{TC} \cong \overline{TM}$

$\overline{AT}$ bisects $\angle$CTM

Prove: $\overline{AC} \cong \overline{AM}$

Statements	Reasons
1. $\overline{TC} \cong \overline{TM}$ and $\overline{AT}$ bisects $\angle CTM$	
2.	Definition of bisect
3. $\overline{AT} \cong \overline{AT}$	
4.	
5.	$\cong \Delta s \rightarrow \cong$ parts

---

3) Given the conditions, which shape(s) must it be and which shape(s) could it be?

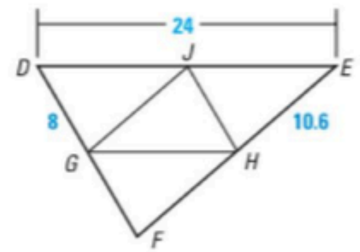
a) My quadrilateral has four equal sides

b) The diagonals of my quadrilateral are perpendicular.

c) My quadrilateral has a pair of parallel sides.

4) Segments JH, HG, and GJ are midsegments.
What are their lengths?

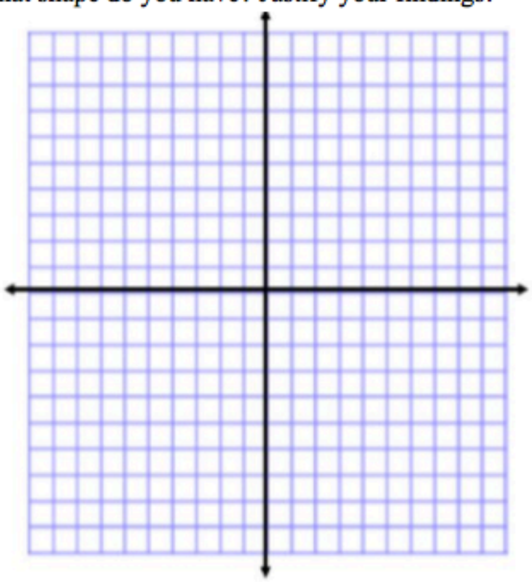
JH = _________ b) GH = _________ JG = ________



5) What do we know about points G, H, and J?

---

6) You connect the following four points to make a quadrilateral: E (-1, 7), F (5, 9), G (3, 6), and H(-3, 4).
What shape do you have? Justify your findings!



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7) Given the two points H(-30, 12) and I(20, -1). What is the slope, distance, and midpoint of segment HI?

1.

Congruent Triangles Proof	
	Points
States givens: ∠E = ∠A and segments EC = AC	**1**
Proves congruent triangles with ASA: ∠ACB = ∠ECD by vertical angles ΔACB = ΔECD by ASA triangle congruence theorem Or Proves congruent triangles with AAS: Segments DE and AB are parallel because if alternate interior angles are congruent, then the lines cut by the transversal are parallel. ∠D = ∠B because if two lines are parallel, then alternate interior angles are congruent. ΔACB = ΔECD by AAS triangle congruence theorem. Partial Credit: 1 point if students give the justifications above but leave off one reason or give an incorrect reason. No points if statements are given with out reasons, or if statements are given with incorrect reasons.	**2**
Gives final correct statement and reason: Segments DE = AB by CPCTC	**1**
Total	**4**
1 Bonus point: Gives statement and reason that C is the midpoint of DB because EC = AC.	**1**

2.

<table>
<tr><td colspan="2">Complete missing elements of proof</td></tr>
<tr><td></td><td>Points</td></tr>
<tr><td>Gives two correct missing elements of the proof.

<table>
<tr><th>Statement</th><th>Reason</th></tr>
<tr><td>1) TC = TM and AT bisects ∠CTM</td><td>1) <u>Given</u></td></tr>
<tr><td>2) <u>∠CTA = ∠MTA</u></td><td>2) Definition of bisect</td></tr>
<tr><td>3) AT = AT</td><td>3) <u>Reflexive property</u></td></tr>
<tr><td>4) <u>ΔCTA = ΔMTA</u></td><td>4) <u>SAS triangle congruence theorem</u></td></tr>
<tr><td>5) <u>AC = AM</u></td><td>5) CPCTC</td></tr>
</table></td><td>1</td></tr>
<tr><td>Gives all correct missing elements of the proof.</td><td>1</td></tr>
<tr><td align="right">Total</td><td>2</td></tr>
</table>

3.

Must Be / Could Be	
	Points
3a) Gives correct response of must be a rhombus, and it could be a square. Partial credit: 1 point for only stating it must be a rhombus, or only stating it could be a square.	**2**
3b) Gives correct response of it must be a kite, and it could be rhombus or a square. Partial Credit: 1 point for only stating it must be a kite, or only stating it could be a rhombus, or only stating it could be a square.	**2**
3c) Gives correct response of a trapezoid. Due the ambiguity of the wording, responses of a rhombus, a parallelogram, a square or a rectangle are acceptible as long as trapezoid is also listed.	**2**
Total	**6**

4.

Midsegment Theorem	
	Points
Gives all correct lengths: JH = 8GH = 12JG = 10.6	**1**
Total	**1**

5.

Midpoints	
	Points
Gives all correct response such as: G, H, and J are all midpoints OR G, H, and J all divide the length of triangle DEF in half.	**1**
Total	**1**

6.

Coordinate Geometry	
	Points
Plots the points correctly	**1**
Correctly draws slope triangles on the graph to show that opposite sides have the same slope and states that opposite sides have the same slope. Or Correctly gives slope ratios of opposites sides: 3/2 and 1/3 (or 2/6)	**1**
Correctly states that opposite sides are parallel and the shape is a parallelogram	**1**
Total	**3**

7.

Slope, distance, and midpoint	
	Points
Correctly gives the slope between the points: -50/13 or approximately -3.8.	**1**
Correctly gives distance between the points: $\sqrt{2669} \approx 51.6$ units.	**1**
Correctly gives the midpoint and shows work: (5.5, -5) x-value: $\frac{12-1}{2} = 5.5$ y-value: $\frac{20-30}{2} = -5$	**1**
Total	**3**

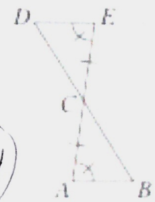
Sample Score #1

$\frac{20}{20}$ TOTAL POSSIBLE SAMPLE SCORE #1

Geometer: ____________________ Period: _____

Geometry Chapter Seven

1) Use the given information in the diagram to prove that segment DE ≅ segment AB? If not possible, explain why not. [Bonus: Prove that C is the midpoint of DB]



(4) / 4

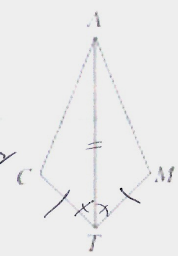
	Statement	Reason
(1)	∠E = ∠C and EC = AC	Given
(2)	∠DCE = ∠BCE	Vertical angles
	ΔDEC ≅ ΔBAC	ASA ≅
(1)	DE = AB	CPCTC

2) Complete the proof at right.

Given: $\overline{TC} \cong \overline{TM}$
$\overline{AT}$ bisects ∠CTM
Prove: $\overline{AC} \cong \overline{AM}$

(2) / 2

Statements	Reasons
1. TC ≅ TM and AT bisects ∠CTM	Given
2. ∠CTA = ∠MTA	Definition of bisect
3. AT ≅ AT	REFLEXIVE PROPERTY
4. ΔCTA ≅ ΔMTA	SAS ≅
5. AC = AM	≅ Δs → ≅ parts



3) Given the conditions, which shape(s) must it be and which shape(s) could it be?

(6) / 6

a) My quadrilateral has four equal sides.

(2) must: rhombus could: square

b) The diagonals of my quadrilateral are perpendicular.

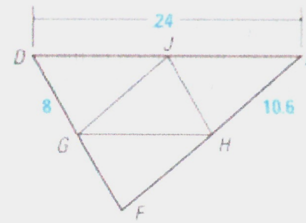
(2) must: kite could: square, rhombus

c) My quadrilateral has a pair of parallel sides.

(2) must: trapezoid

4) Segments JH, HG, and GJ are midsegments.
What are their lengths?

1/1 JH = 8   b) GH = 12   JG = 10.6

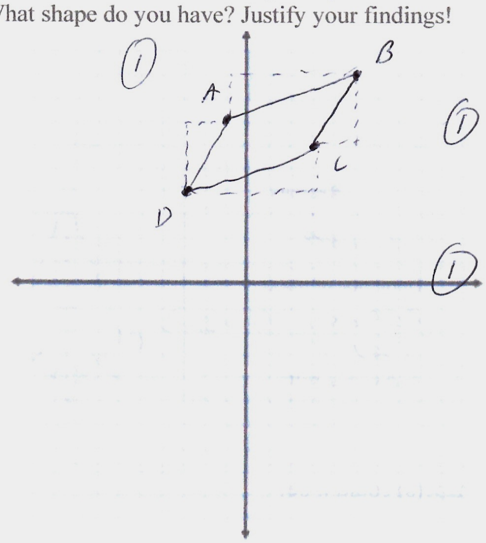


5) What do we know about points G, H, and J?

1/1 They are midpoints

6) You connect the following four points to make a quadrilateral: A (-1, 7), B (5, 9), C (3, 6), and D(-3, 4).
What shape do you have? Justify your findings!

3/3



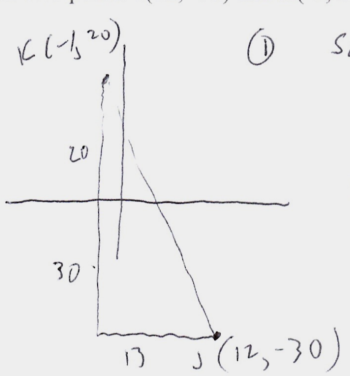
1 AD and BC have the same slope $\frac{3}{2}$

AB and DC have the same slope $\frac{2}{6}$

1 It is a parallelogram because opposite sides are parallel

7) Given the two points J(12, -30) and K(-1, 20). What is the slope, distance, and midpoint of segment JK?

3/3



1 Slope = $-\frac{50}{13}$

1 Distance = $\sqrt{50^2+13^2} \approx 51.66$

1 Midpoint (5.5, -5)

x: $\frac{12+-1}{2} = \frac{11}{2} = 5.5$

y: $\frac{-30+20}{2} = \frac{-10}{2} = -5$

Sample Score #2

$\frac{10 \text{ TOTAL}}{20 \text{ POSSIBLE}}$

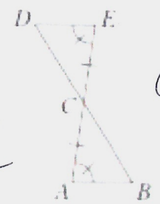
SAMPLE SCORE # 2

Geometer: ____________________Period: _____

Geometry Chapter Seven

1) Use the given information in the diagram to prove that segment DE ≅ segment AB? If not possible, explain why not. [Bonus: Prove that C is the midpoint of DB]

$\frac{2}{4}$



Statement	reason
(1) $\angle E = \angle A$ and $EC = AC$	Given
$\angle D = \angle B$	PARALLEL LINES
$\triangle DEC \cong \triangle BAC$	ASA ≅
(1) $DE = AB$	CPCTC

2) Complete the proof at right.

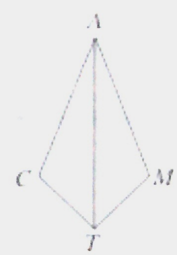
$\frac{1}{2}$

Given: $\overline{TC} \cong \overline{TM}$

$\overline{AT}$ bisects ∠CTM

Prove: $\overline{AC} \cong \overline{AM}$

Statements	Reasons
1. $\overline{TC} \cong \overline{TM}$ and $\overline{AT}$ bisects ∠CTM	Given
(x) 2. $CT = MT$	Definition of bisect
3. $\overline{AT} \cong \overline{AT}$	REFLEXIVE Prop.
4. $\triangle ATC \cong \triangle ATM$	(x)
5. $AC = AM$	≅ Δs → ≅ parts



3) Given the conditions, which shape(s) must it be and which shape(s) could it be?

a) My quadrilateral has four equal sides.

(1) It could be a Square

b) The diagonals of my quadrilateral are perpendicular.

(1) It could be a rhombus

$\frac{2}{6}$

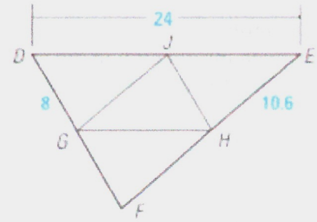
c) My quadrilateral has a pair of parallel sides.

(0) It could be a square

4) Segments JH, HG, and GJ are midsegments.
What are their lengths?

0/1

JH = 8 b) GH = 24 JG = 5.3 (X)



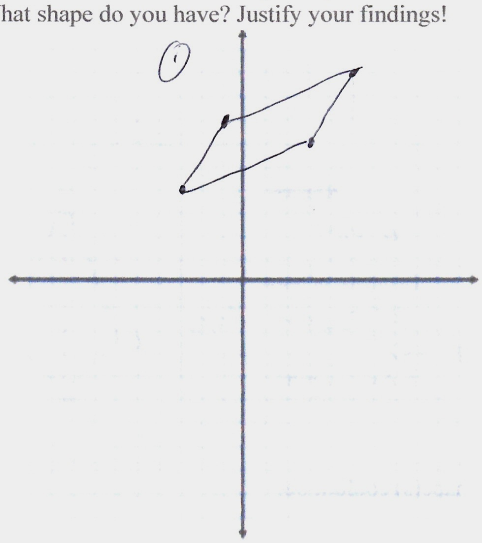
5) What do we know about points G, H, and J?

1/1

THEY ARE MIDPOINTS

6) You connect the following four points to make a quadrilateral: A (-1, 7), B (5, 9), C (3, 6), and D(-3, 4).
What shape do you have? Justify your findings!

2/3



(1)

It is a parallelogram because opposite sides are parallel.

7) Given the two points J(12, -30) and K(-1, 20). What is the slope, distance, and midpoint of segment JK?

2/3

Slope: 20 -3

(X) $\frac{20-30}{-1-12} = \frac{-10}{-13} = \frac{10}{13}$

Distance: (1)

$\sqrt{(20+30)^2+(12+1)^2} = 51.6$

Midpoint:

$\frac{20-30}{2} = -5$

$\frac{12-1}{2} = 5.5$

(5.5, -5)

(1)

APPENDIX D

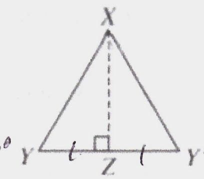
Appendix D

Homework Rubric and Sample Scoring

Homework Evaluation Scores	
1	Illegible; does not make sense
2	Some problems attempted; work does not make sense
3	Some problems attempted; with the work shown
4	Most of the problems attempted; steps are written and follow a logical progression to a correct answer
5	All problems attempted; follow a logical progression; clearly written and a correct answer is given

SCORE OF $\frac{5}{5}$

7-26. ΔXYZ is reflected across $\overline{XZ}$, as shown at right.



a. How can you **justify** that the points Y, Z, and Y' all lie on a straight line?

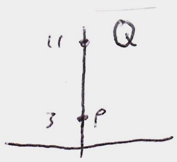
There are 2 90° angles that make 180°

b. What is the relationship between ΔXYZ and $\Delta XY'Z$? Why?

a) There are 2 90° angles that form 180° which is a line and Y, Z, and Y' are on that line

b) They are congruent by SAS $\cong$.

7-27. Remember that a midpoint of a line segment is the point that divides the segment into two segments of equal length. On graph paper, plot the points $P(0, 3)$ and $Q(0, 11)$. Where is the midpoint M if $PM = MQ$? Explain how you found your answer.



$\frac{11+3}{2} = 7$

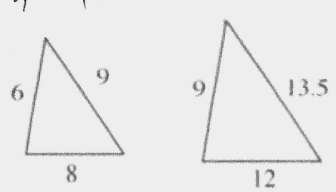
m (0, 7)

7-28. Recall the three similarity shortcuts for triangles: SSS ~, SAS ~ and AA ~. For each pair of triangles below, decide whether the triangles are congruent and/or similar. **Justify** each conclusion.

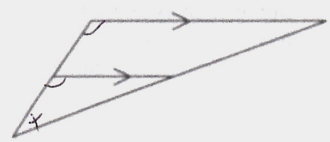
$\frac{6}{9} = \frac{8}{12} = \frac{9}{13.5}$ SSS~

AA~

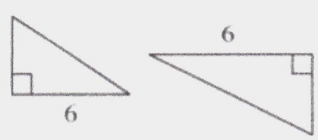
a.



b.

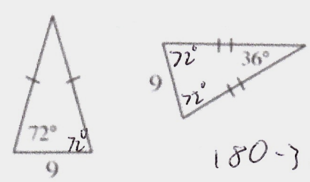


c.



NOT SIMILAR or CONGRUENT

d.



$180 - 36 = 144$

ASA $\cong$

$\frac{144}{2} = 72°$

7-29. On graph paper, plot and connect the points $A(1, 1)$, $B(2, 3)$, $C(5, 3)$, and $D(4, 1)$ to form quadrilateral $ABCD$.

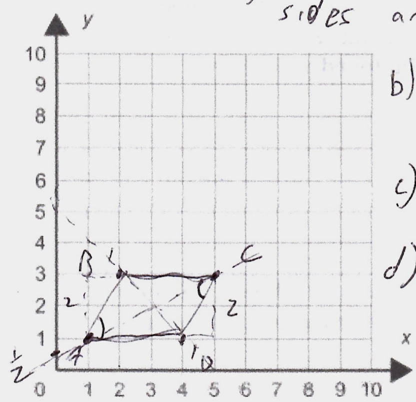
a. What is the best name for quadrilateral $ABCD$? **Justify** your answer.

b. Find and compare $m\angle DAB$ and $m\angle BCD$. What is their relationship?

c. Find the equations of diagonals $\overline{AC}$ and $\overline{BD}$. Are the diagonals perpendicular?

d. Find the point where diagonals $\overline{AC}$ and $\overline{BD}$ intersect.

a) IT is a parallelogram because opposite sides are parallel.



b) THE angles are $\cong$

c) AC: $y = \frac{1}{2}x + \frac{1}{2}$ BD: $y = -x + 5$

d) (3, 2)

7-30. Solve each system of equations below, if possible. If it is not possible, explain what having "no solution" tells you about the graphs of the equations. Write each solution in the form (x, y). Show all work.

a. $y = -\frac{1}{3}x + 7$

$y = -\frac{1}{3}x - 2$

no solution because the lines are parallel

b. $y = 2x + 3$

$y = x^2 - 2x + 3$

$2x + 3 = x^2 - 2x + 3$

$0 = x^2 - 4x$ $2(4) + 3 = 11$

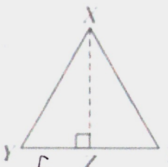
$0 = x(x - 4)$

$x = 0$ & 4

$(0, 3)$ and $(4, 11)$

SCORE OF 3/5

7-26. ΔXYZ is reflected across $\overline{XZ}$, as shown at right.



a. How can you **justify** that the points Y, Z, and Y' all lie on a straight line?

$\overline{YZ}$ and $\overline{ZY'}$ are on one SIDE of a TRIangle

b. What is the relationship between ΔXYZ and $\Delta XY'Z$? Why?

They are congruent

7-27. Remember that a midpoint of a line segment is the point that divides the segment into two segments of equal length. On graph paper, plot the points $P(0, 3)$ and $Q(0, 11)$. Where is the midpoint M if $PM = MQ$? Explain how you found your answer.

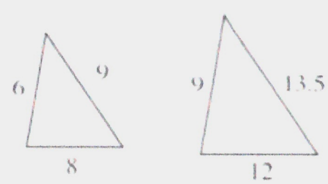
$\frac{3+11}{2} = 7$

7-28. Recall the three similarity shortcuts for triangles: SSS ~, SAS ~ and AA ~. For each pair of triangles below, decide whether the triangles are congruent and/or similar. **Justify** each conclusion.

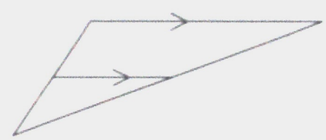
SSS~

NOT congruent because no lengths or angles are given

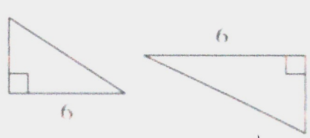
a.



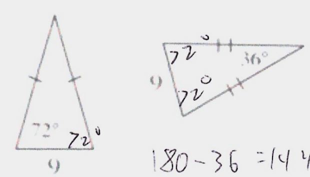
b.



c.



d.



NOT congruent only a side and an angle are given

$180-36=144$

$\frac{144}{2}=72$

ASA $\cong$

7-29. On graph paper, plot and connect the points $A(1, 1)$, $B(2, 3)$, $C(5, 3)$, and $D(4, 1)$ to form quadrilateral $ABCD$.

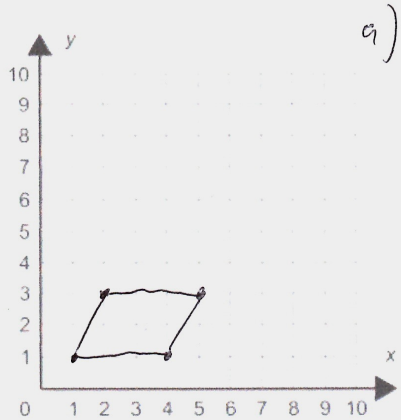
a. What is the best name for quadrilateral $ABCD$? **Justify** your answer.

b. Find and compare $m\measuredangle DAB$ and $m\measuredangle BCD$. What is their relationship?

c. Find the equations of diagonals $\overline{AC}$ and $\overline{BD}$. Are the diagonals perpendicular?

d. Find the point where diagonals $\overline{AC}$ and $\overline{BD}$ intersect.

a) It is a parallelogram because opposite sides are parallel



7-30. Solve each system of equations below, if possible. If it is not possible, explain what having "no solution" tells you about the graphs of the equations. Write each solution in the form (x, y). Show all work.

a. $y = -\frac{1}{3}x + 7$
$y = -\frac{1}{3}x - 2$

b. $y = 2x + 3$
$y = x^2 - 2x + 3$

APPENDIX E

Appendix E

Revised Worked Examples Lessons and Exercises

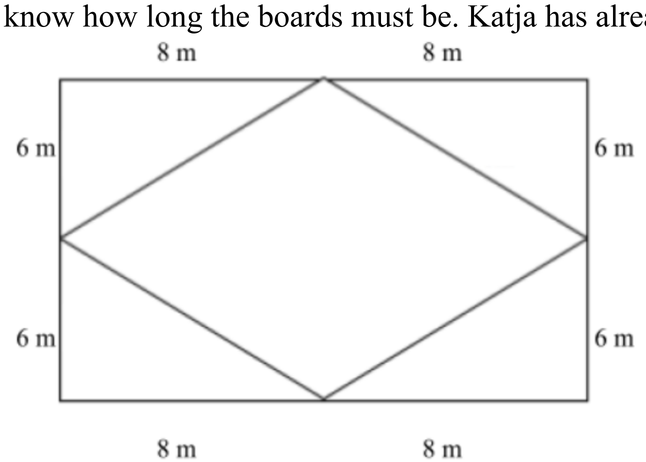
<u>Strategies of Proving</u> Name: ________________ Period ____
Read through this dialogue and complete the specified tasks.
The goal is to understand the process of proving.

We are going to consider these three steps to our proving processes:

Explore - use tools that are available to explore a share and discover any possible relationships.
Conjecture - Write a conditional statement based on these observations.
Prove - Convince yourself (and others) that your conjecture is always true with a convincing argument.

1) **The Scene**

Elke and Katja are once again at their favorite pastime: visiting Flash and Silver on their paddock. Unfortunately, they have had little time for their horses, and so the paddock had accumulated all sorts of junk in recent weeks. Because all four corners are quite full of stuff and they do not want Flash and Silver to hurt themselves, they have decided to reduce the size of the paddock by removing the corners. They just connected the middle of each side of the rectangular paddock with each other, thus creating a smaller, but again four cornered paddock. Now they want to know how long the boards must be. Katja has already made a sketch:



"That's funny," says Elke , "although I do not know how long the new sides of our paddock will be, I think they are all the same length. "

"Well, that's pretty clear! " Answers Katya.

"Yes, it is. But are you sure that all four are exactly the same length? How can you be certain of that? Let's measure the sides."

Based on Katja's sketch they measure the four new sides. In fact, they measure the same value for each of the sides. Now they know how long the boards need to be and could actually build it. But now Katja's interest has been aroused. She likes geometry.

"Say, Elke do you think that it's always like this? I mean, even if our rectangular paddock would have different dimensions and if we reconnect the four middles, would all of these inner lengths be the same? If I remember correctly, that is called a rhombus. "

Katja makes two new sketches and measures each. They are always the same length!

Elke : "Well, we have developed a good method to make a rhombus from an arbitrary rectangle."

Katja : "Are you sure? Could it be that I have drawn only those rectangles where this is true? So we cannot always expect this rhombus".

Elke : "Well, you're the geometry expert. Let's try again to prove this like a true mathematician."

So Elke and Katja try to prove the following mathematical statement:

"Combining the middle of each side of any quadrilateral with one another, the result is always a rhombus."

In your own words, write what you think Elke's and Katja's conjecture is as an If/then statement.

In the following, we'll look at exactly how they solved this math problem. You should use this not only to read, but to perform the steps yourself.

2) **Explore the scenario.**

First, we want to reconstruct the arguments of Elke and Katja. You need a ruler, a protractor, and three colored pencils.

a) Miss Katja’s sketch. Is the inner quadrilateral really a rhombus?
b) In the following box, draw any **rectangle** ABCD and connect the four midpoints to create a new quadrilateral. Give the drawing to your partner to measure and record the side lengths of the new quadrilateral.

Katja: “You see, Elke, here we have provided an example of our conjecture being true.”

Elke: “Okay, but I wouldn’t tell our math teacher that our conjecture is always true yet. He would have probably said we cannot know that we would always get a rhombus. He would want us to generalize this and not use measurements.”

Katja: “Right. He would say that we have to think of the lengths being generally congruent rather a specific measurement. You know, congruent sides.”

Elke: “Just like last school year, he always illustrated it with congruence. And weren’t there those strange congruence statements? I don’t remember exactly, but they usually had to do with triangles.”

Katja: “True, with the congruence statements one can show that two triangles are congruent, so that they have equal sides and angles. Hmm, triangles appear in our sketches. Wait a second, my memory is coming back to me.”

c) Try to remember as well the congruence statements (SSS, SAS, ASA, AAS). Give the correct congruence statements and label each congruent side and angle of the triangles with the same color.

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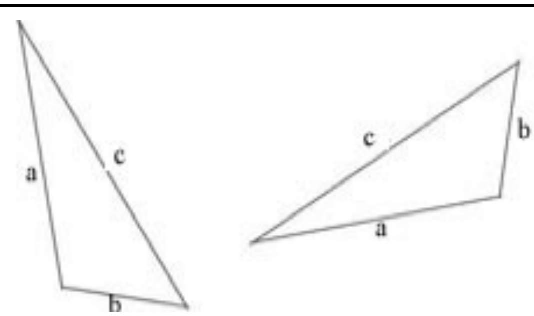

Congruence statement: ________
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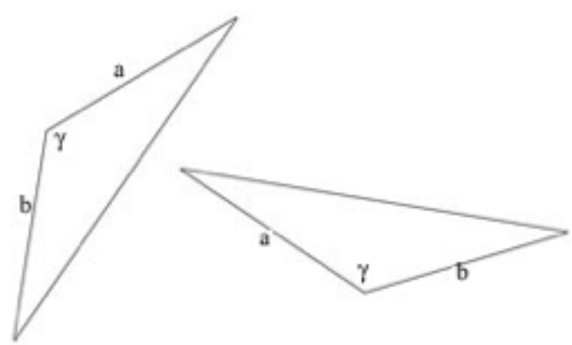

Congruence statement: ________
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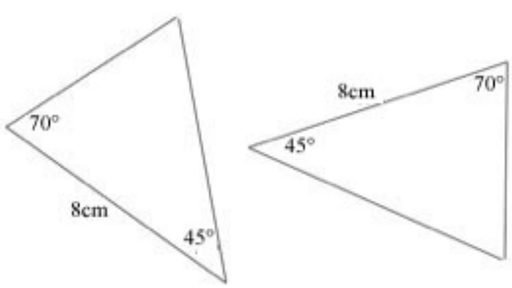


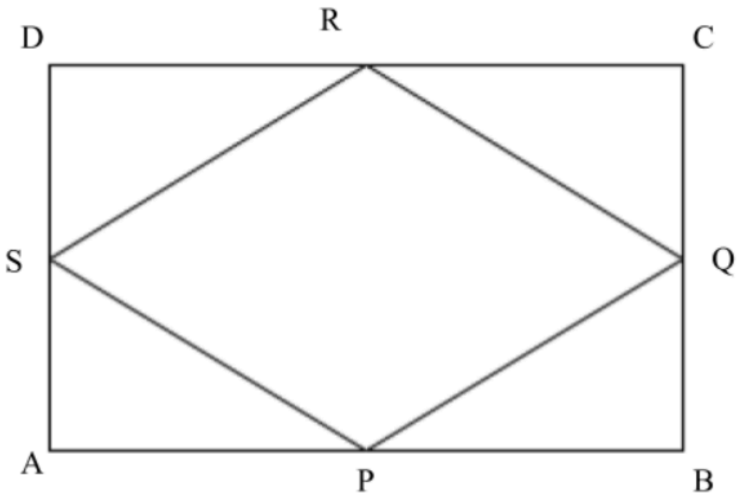
Congruence statement: ________
</td>
<td>
Congruence statement: ________
</td>
</tr>
</table>

d) One of the four listed congruence statements does not have an accompanying drawing. Draw a pair of congruent triangles that correspond to one of the missing statements in the empty box.

e) Describe what Elke and Katja were attempting to do after they stated their claim.

3) **The Conjecture**

Revisiting Elke's and Katja's conjecture.



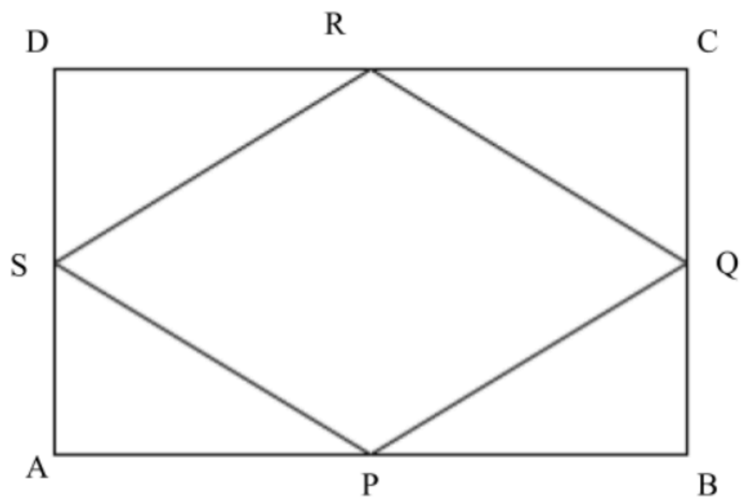
Let ABCD be any rectangle. For the sides, that means AD = BC and AB = DC. And for the angles that means ∠A = ∠B = ∠C = ∠D. Then with the midpoints labeled P, Q, R, S and connecting them we get a new quadrilateral.

Elke and Katje claim that this new quadrilateral is a rhombus, which means SP = PQ = SR = RQ.

Mathematical allegations must be proved. For this proof, it is important:
• that you are thinking about what you know about quadrilaterals and congruence,
• that you can then collect from all the possible arguments, the evidence that could be important for the specific arguments here, and finally
• that you are arranging your arguments in a logical sequence (in a chain of evidence).

4) **The Proof**
If two triangles are congruent, all corresponding sides and angles are congruent. Maybe we can show with our congruent statements that the four sides of the inner quadrilateral are all the same size.

a) **State the given information**
ABCD is a rectangle and P, Q, R, & S are midpoints.



b) **Consider what the given information can tell us.**
Because we have a rectangle, what pairs of lengths are congruent?

Because we have a rectangle, what angles are congruent?

Because we know P, Q, R, & S are midpoints, what segments are congruent?

Mark congruent segments and angles.

c) **Use the what information we now have and look for other shapes in our figure to use.**
Based on what we now know, what triangles are congruent? Give the triangle congruence statement.

Because these triangles are congruent, what do we know about the lengths PS, SR, RQ, & QP?

d) **We can now prove our claim:**
Collecting our information we have the following information which will allow us to prove our claim that PQRS is a rhombus.

5) **Review**
As a result of our solution we now know that the four midpoints in any rectangle connect to create a rhombus. Mathematically speaking, we have found a proof of our assertion.

We found evidence of Elke's and Katja's claim.
But they keep conversing:
Elke: "Super, Katja! That was not as hard as I thought at first. But you know what I noticed: I think if we divide up our paddock like this it is only half as large."
Katja: "Are you sure? Let's take this back to class to see if this is true…"

6) **Look back at our Explore/Conjecture/Prove process above.**
Prove that PQRS = 1/2ABCD.

7.2.1 Special Quadrilaterals and Proof

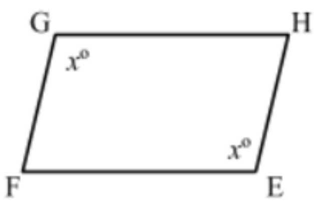
Goal: Use properties of parallelograms to communicate logical arguments supported by justifications.

Process

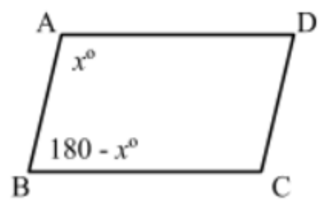
Explore the shape and look for any possible relationships.

Give a convincing argument with justification for what we are asked to prove.

Completing Proofs with Properties of Parallelogram



Opposite angles in a parallelogram are congruent.

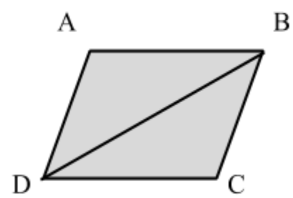


Adjacent angles in a parallelogram are supplementary.

Parallelogram Worked Example #1 - Study the example below and select the best reason for the last statement from the two properties of parallelograms listed above.

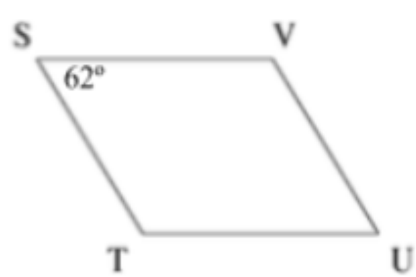
Given: ABCD is a parallelogram and ∠A = 117º
Prove: ∠C = 117º

Statement	Reason
1) ABCD is a parallelogram	1) Given
2) ∠A = 117º	2) Given
3) ∠C = 117º	3)



Parallelogram Example #2 - Study the example below and give the best reason for the last statement.

Given: SVUT is a parallelogram and ∠S = 62º
Prove: ∠T = 118º



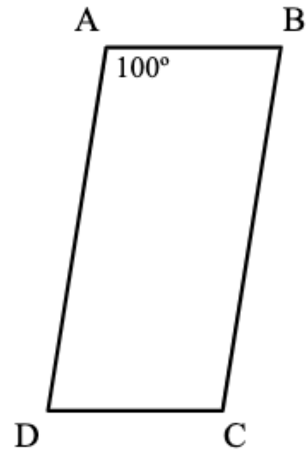
Statement	Reason
1) SVUT is a parallelogram	1) Given
2) ∠S = 62º	2) Given
3) ∠T = 118º	3)

Parallelogram Exercises - Complete the proofs with justifications based on the examples above.

1.

Given: ABCD is a parallelogram and ∠A = 100º

Prove: ∠D = 80º

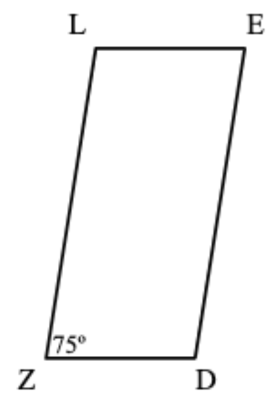


Statement	Reason
1) ABCD is a parallelogram	1) Given
2) ∠A = 100º	2) Given
3) ∠D = 80º	3)

2.

Given: LEDZ is a parallelogram and ∠Z = 75º

Prove: ∠D = 105º

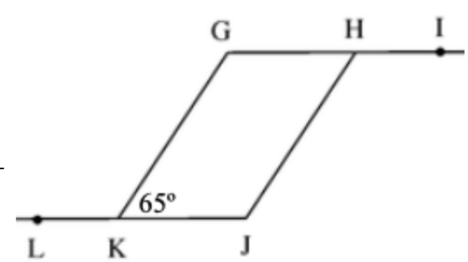


Statement	Reason
1) LEDZ is a parallelogram	1) Given
2) ∠Z = 75º	2)
3) ∠D = 105º	3)

3.

Given: GHJK is a parallelogram and ∠JKG = 65º

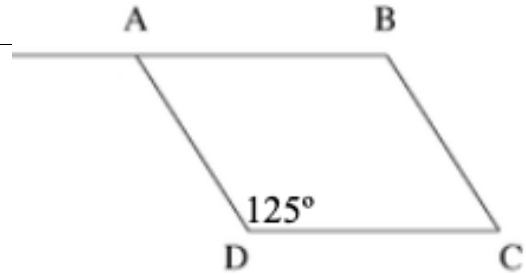
Prove: ∠GHJ = 65º



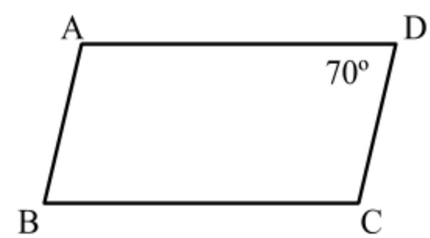
Statement	Reason
1) GHJK is a parallelogram	1) Given
2) ∠JKG = 75º	2)
3) ∠GHJ = 65º	3)

4.Given: ABCD is a parallelogram and ∠D = 125º
Prove: ∠C = 55º

Statement	Reason
1)	1)
2)	2)
3)	3)

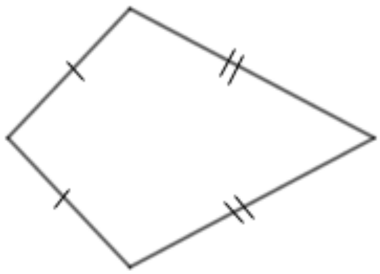


5. Given: ABCD is a parallelogram and ∠D = 70º
Prove: ∠C = 110º

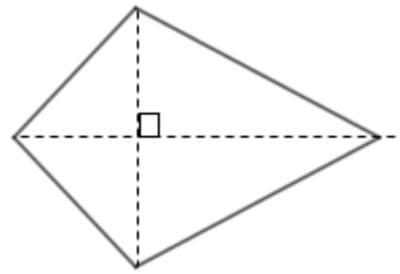


6.Look back at the work you just completed. What was the first statement in all of our proofs? What was the last statement?

Completing Proofs with Properties of Kites



Kites have two pairs of adjacent congruent sides.

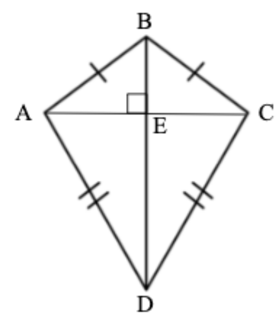


The diagonals of a kite are perpendicular.

Kite Worked Example #1 - Study the worked example below and note the subgoal.

Given: ABCD is a kite

Prove: ∠ABD = ∠CBD

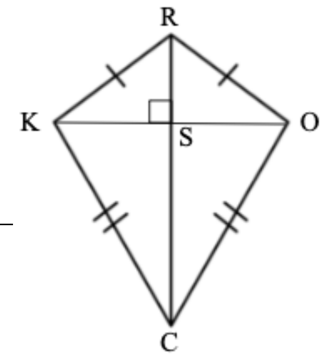


To do complete this proof we will need to prove triangles are congruent first.

Subgoal: Prove ΔABE = ΔCBE

Statement	Reason
1) ABCD is a kite	1) Given
2) AB = BC	2) Kites have two pairs of adjacent congruent sides.
3) ∠AEB and ∠CEB are 90º	3) Diagonals of a kites are perpendicular.
4) BE = BE	4) Reflexive property
5) ΔABE = ΔCBE	5) Hypotenuse Leg Triangle congruence theorem
6) ∠ABD = ∠CBD	6) CPCTC

Kite Worked Example #2 - Study and complete the example below and note the subgoal.
Given: ROCK is a kite
Prove: ∠KCR = ∠OCR



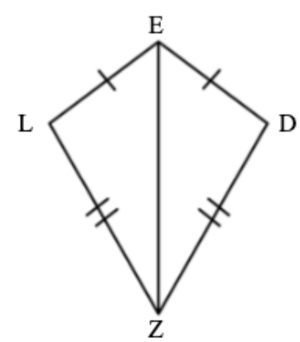
Subgoal: First prove ΔKCS = ΔOCS

Statement	Reason
1) ROCK is a kite	1) Given
2) CS = CS	2) Reflexive property
3) ∠KSC and ∠OCS are 90º	3)
4) KC = OC	4)
5) ΔKCS = ΔOCS	5)
6) ∠KCR = ∠OCR	6)

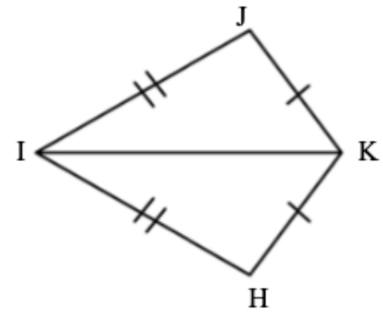
<u>Kite Exercises</u> - Complete the proofs based on the worked examples above.

1. Given: LEDZ is a kite
Prove: ∠LZE = ∠DZE
Subgoal: First prove congruent triangles

Statement	Reason
1) LEDZ is a kite	1) Given
2) EZ = EZ	2) Reflexive Property
3) LE = DE & LZ = DZ	3)
4) ΔLZE = ΔDZE	4)
5)	5)



2. Given: HIJK is a kite
Prove: ∠JKI = ∠HKI
Subgoal: First prove congruent triangles



3) After completing the kite exercices above, make a conjecture about one of the diagonals of a kite. Prove your conjecture.

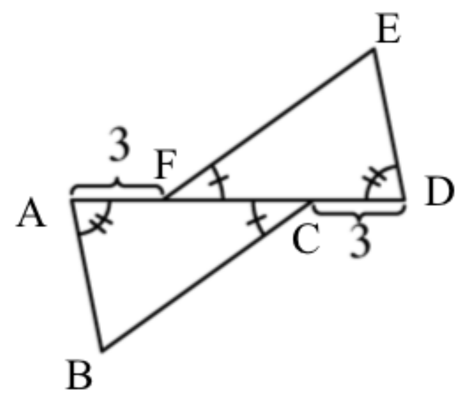
<u>7.2.3A Proofs with Congruent Triangles</u>
Goal: Communicate logical arguments supported by justifications.

<u>Process</u>
Explore the shape and discover any possible relationships.
Give a convincing argument with justification for what we are asked to prove.

Triangle Worked Example #1 – Study this completed example for 3 minutes and think about about how it addresses the process above.

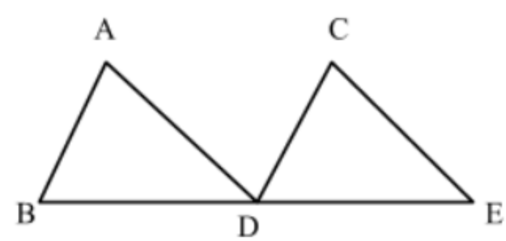
Given the information in the diagram, prove ΔABC = ΔDEF.

Statement	Reason
1) ∠A = ∠D	1) Given
2) ∠BCA = ∠EFD	2) Given
3) AC = DF	3) Both segments equal 3 + FC
4) ΔABC = ΔDEF	4) ASA ≅



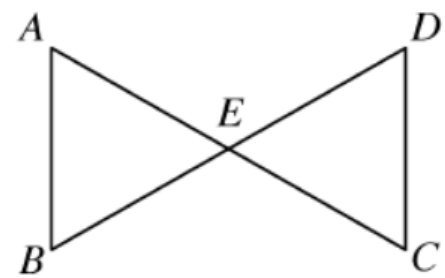
Triangle Worked Example #2 – Give justifications for the last statements.
Given that segments AB = CD, AD = CE, and D is the midpoint of segment BE, prove that AB || CD.

Statement	Reason
1) AB = CD, AD = CE, and D is the midpoint of BE.	1) Given
2) BC = CE	2) Definition of Midpoint
3) ΔABD ≅ ΔCDE	3) SSS ≅
4) ∠B = ∠CDE	4)
5) AB || CD	5)



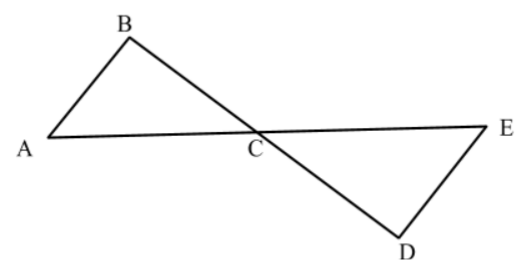
Triangle Example #3 – Give the statements and reasons to complete the proof.
Jester looked at the shape below and think that segments AB and DC are congruent. The only information he was given was that point E is the midpoint of AC and BD. Add statements and reasons to complete the proof that *segments AB and DC are congruent*.

Statement	Reason
1) E is the midpoint of AC and BD	1) Given
2) AE = EC and BE = DE	2) Definition of midpoint

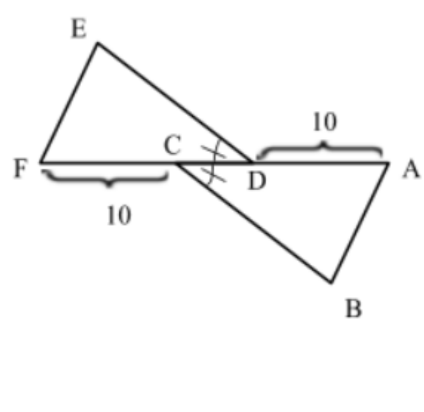


Triangle Exercises

1. Give the statements and reasons to complete the proof.
Given that C is the midpoint of segments AE and BD, prove AB || DE.

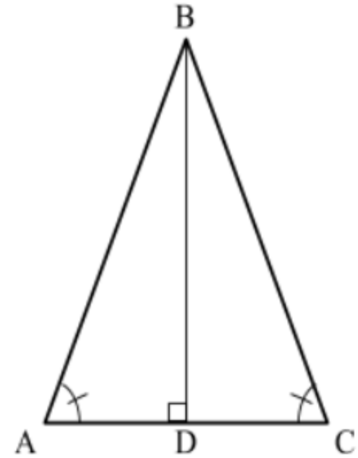


2. Give the statements and reasons to complete the proof.
Given the information in the diagram, prove FE || AB.



3. Explore, conjecture, prove.

Given the information in the diagram, what do you think is a relationship between segments AD and CD? Prove that relationship.



7.2.3B Proofs with Similar Triangles

Goal: Communicate logical arguments supported by justifications.

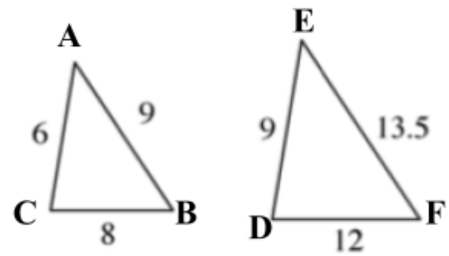
Process

Explore the shape and discover any possible relationships.
Give a convincing argument with justification for what we are asked to prove.

Similarity Worked Example #1 – Study this completed example for 3 minutes and think about about how it addresses the process above.

Given the triangles at right, prove ΔABC ~ ΔEFD.

Subgoal: Compare sides you think correspond to determine if there is the *ratio of similarity.*



Largest Sides *Smallest Sides* *Remaining Sides*

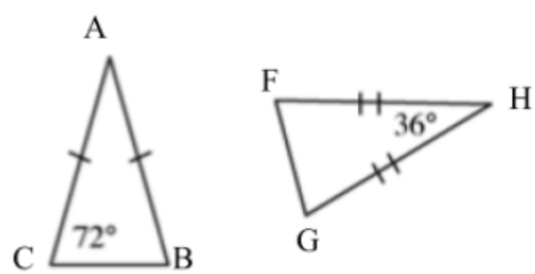
$\frac{EF}{AB} = \frac{13.5}{9} = 1.5$ $\frac{ED}{AC} = \frac{9}{6} = 1.5$ $\frac{DF}{BC} = \frac{12}{8} = 1.5$

Since the *ratio of similarity* is the same for each ratio, ΔABC ~ ΔEFD by the SSS ~.

Similarity Worked Example #2

Given the isosceles triangles at right, prove ΔABC ~ ΔHGF.

Subgoal: Try to find corresponding angles that are congruent.



∠B = 72º because the base angles of an isosceles triangle are congruent and ∠C = 72º.

∠F = ∠G = 72º because base angles of an isosceles triangle are congruent and the triangle angle sum gives us:
(180º – 36º)/2 = 72º

Now that ∠C = ∠F and ∠B = ∠G, ΔABC ~ ΔHGF by AA ~.

Proving similar triangles exercises

Study the examples above for 5 minutes and complete the exercises below.

1. Given the nested triangles, prove ΔABE ~ ΔACD.

Subgoal: Look for sides and angles that correspond.

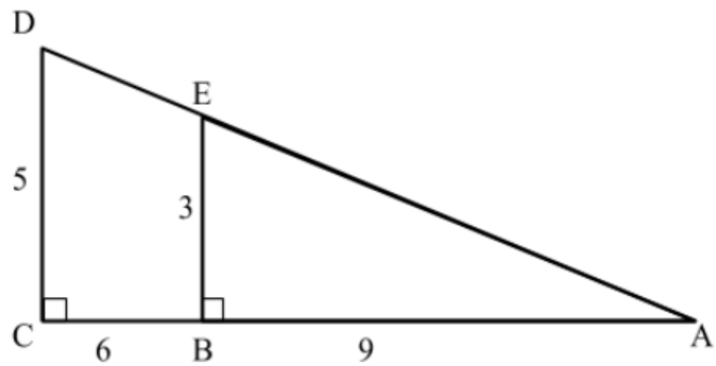
Larger Sides *Smaller Sides*

$\frac{AB}{AC} =$ _____ $\frac{EB}{DC} =$ _____

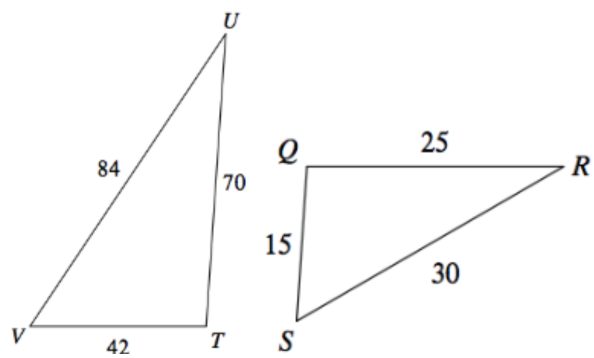
Corresponding angles

∠C = ∠EBA = _____

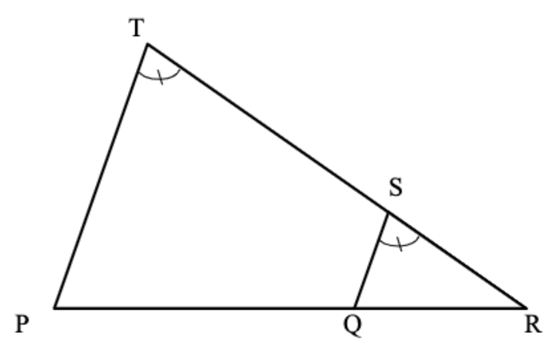
Then ΔABE ~ ΔACD by ______ theorem.



2. Given the triangles at right, prove ΔVTU ~ ΔSQR.



3.Given the nested triangles, prove ΔPRT ~ ΔQRS.



APPENDIX F

Appendix F

Student Surveys

Student Pre-Survey Name _________________ Period____

Please answer the questions below by reading the question or statement and circling which response you agree with most. There are a few short response questions. Please be as descriptive as you can on these questions so that your thoughts are clearly represented. Your responses will be kept confidential and anonymous.

1. What grade are you currently in?

   10th 11th 12th

---

2. I enjoy math.

   Always Most of the Time Once in a While Never

---

3. I am good at math.

   Yes I am okay Not at all

---

4. I keep trying in math even when a problem is hard.

   Always Most of the Time Once in a While Never

---

5. I try my best first before asking for help.

   Always Most of the Time Once in a While Never

---

6. I can solve difficult problems on my own without the help of my teacher or parent.

   Always Most of the Time Once in a While Never

---

7. I find using tools / manipulatives useful when learn about math.

   Always Most of the Time Once in a While Never

---

8. After I solve a math problem I can explain how I solved it.

   Very well Okay Not at all

---

9. I can explain and prove why something is right or wrong in math.

   Always Most of the Time Once in a While Never

---

10. I ask for help when I need it.

Always   Most of the Time   Once in a While   Never

11. I feel that I am asked to think about math, not just do math.

Always   Most of the Time   Once in a While   Never

12. I think about whether my answer makes sense or not.

Always   Most of the Time   Once in a While   Never

13. If there were no grades in math, I would still do my best in my math work.

Always   Most of the Time   Once in a While   Never

14. I work with other students on math.

Always   Most of the Time   Once in a While   Never

15. My teacher challenges me to do my best work in math.

Always   Most of the Time   Once in a While   Never

16. My parents think math is important.

Yes   No   They do not have an opinion on it.

17. My parents are able to help me with math homework.

Always   Most of the Time   Once in a While   Never

18. Someone observing the class would say that I participate in math class.

Always   Most of the Time   Once in a While   Never

19. If I work hard, I can be successful at math.

Always   Most of the Time   Once in a While   Never

20. The homework that is assigned to me is important to my math learning.

Always   Most of the Time   Once in a While   Never

21. I feel prepared to be successful in Algebra 2.

Always   Most of the Time   Once in a While   Never

22. What do you feel is most challenging about mathematics?

23. True or false: I think a person can work hard and understand challenging math concepts.

24. True or false: I think there are two kinds of people: those who understand math, and those who do not.

If you can, explain why you answered true or false.

25. How confident are you about your ability to contribute to small group discussions in math class?

Very confident   Somewhat confident   Not really confident   Not at all confident

26. How confident are you about your ability to contribute to whole class discussions in math class?

Very confident   Confident   Not really confident   Not at all confident

27. Describe your ideal math class or learning environment.

Student Post-Survey Name ________________ Period____

Please answer the questions below by reading the question or statement and circling which response you agree with most. There are a few short response questions. Please be as descriptive as you can on these questions so that your thoughts are clearly represented. Your responses will be kept confidential and anonymous. If you would like add anything not addressed in the survey please feel free to attach an additional page with your thoughts.

1. I enjoy math.

Always Most of the Time Once in a While Never

---

2. I am good at math.

Yes I am okay Not at all

---

3. I keep trying in math even when a problem is hard.

Always Most of the Time Once in a While Never

---

4. I try my best first before asking for help.

Always Most of the Time Once in a While Never

---

5. I can solve difficult problems on my own without the help of my teacher or parent.

Always Most of the Time Once in a While Never

---

6. I find using tools / manipulatives useful when learn about math.

Always Most of the Time Once in a While Never

---

7. After I solve a math problem I can explain how I solved it.

Very well Okay Not at all

---

8. I can explain and prove why something is right or wrong in math.

Always Most of the Time Once in a While Never

---

9. I ask for help when I need it.

Always Most of the Time Once in a While Never

---

10. I feel that I am asked to think about math, not just do math.

Always Most of the Time Once in a While Never

---

11. I think about whether my answer makes sense or not.

Always Most of the Time Once in a While Never

12. I work with other students on math.

Always Most of the Time Once in a While Never

13. My teacher challenges me to do my best work in math.

Always Most of the Time Once in a While Never

14. Someone observing the class would say that I participate in math class.

Always Most of the Time Once in a While Never

15. If I work hard, I can be successful at math.

Always Most of the Time Once in a While Never

16. The homework that is assigned to me is important to my math learning.

Always Most of the Time Once in a While Never

17. I feel prepared to be successful in Algebra 2.

Always Most of the Time Once in a While Never

18. How confident are you about your ability to contribute to small group discussions in math class?

Very confident Somewhat confident Not really confident Not at all confident

The next questions ask you to think specifically about the unit we just completed.

19. What was the most challenging aspect of proofs?

20. What was the most challenging aspect of coordinate geometry? (i.e. calculating midpoints, distance, slope)

21. What did you find helpful or unhelpful about the worked examples that we used in class?

APPENDIX G

Appendix G

Parent / Student Consent Letter

Dear Parent / Guardian, December 2, 2013

I am your students' Geometry teacher at Sonoma Valley High School. For the past two years I have been working on a master's degree in Mathematics Education at CSU Chico. As part of my work I will be conducting a research study this year. The study consists of facilitating learning through worked examples. In addition to learning through worked examples, I will be asking all students to complete a questionnaire about their beliefs about their abilities in math class. These questionnaires will be distributed once in the first semester and again in the second semester.

I would like to make clear that this is not a study of your specific student, but rather an investigation into mathematics education. Students participating in the study will complete the same tasks and receive the same instruction as those not participating. There are no risks to your student for participating. However, possible benefits could include your student becoming more self-aware of their behaviors in math class. I would also like to mention that your student's grade will not be affected in any way due to participation or non-participation.

The information I gain from this study will be used in the master's thesis I present to a board of CSU Chico faculty, as well as to better my teaching practice. Your consent is required so that I may use responses from your student's questionnaires and research data. Participation is voluntary and there is no penalty to your student if you chose for your student not to participate in my study. You student's name and identity will be completely anonymous. All data, including questionnaires, notes, homework, tests, quizzes, responses, etc. will be kept securely locked at school or at my residence. This data will only be reviewed by myself and perhaps my graduate advisor at CSU Chico.

Thank you for your and your student's part in my research study. If you have any questions about this process, please contact me at 707-933-4010 Ext 5282, or email me at jsoutham@sonomavly.k12.ca.us. If you would like a copy of the final write up, I am more than happy to distribute a copy to you.

Sincerely,

Mr. Southam
Mathematics Teacher, Sonoma Valley High School

Parent Permission

I, ______________________________, give my permission for my student's data and responses to be
Parent / Guardian (Please Print)
included in the research project described. I also understand that my student's name will not be used and their identity will be kept anonymous.

____
Student's Name Parent / Guardian's Signature Date

Student Permission

I, ______________________________, give my permission for my data and responses to be included
Student's Name (Please Print)
in the research project described. I also understand that my name will not be used and identity will be kept anonymous.

____
Student's Name Date

Please return this sheet to Mr. Southam and keep the first page for your records.

If you choose to opt out of the study please complete the statement below for record keeping purposes.

I do not give permission for my student's data and responses to be used in this research.

____
Student's name *Parent's name*

Carta del Consentimiento Para Padres y Estudiantes

Estimado Padres/ Guardianes,   El 2 de diciembre, 2013

Soy profesor de Geometría de su hijo/a en Sonoma Valley High School. Durante los últimos dos años he estado completando la maestría en la Educación de Matemáticas en CSU Chico. Una parte de mis estudios es una investigación formal. La investigación consiste en facilitar el aprendizaje a través de los ejemplos prácticos . Además de aprender a través de ejemplos prácticos , les pido a todos los estudiantes que completen un cuestionario sobre sus creencias acerca de sus habilidades en la clase de matemáticas . Estos cuestionarios se distribuirán una vez en el primer semestre y otra vez en el segundo semestre.

Me gustaría dejar claro que no es un estudio de ningún estudiante específico, sino una investigación sobre la educación de matemáticas. Los estudiantes que participen en el estudio completarán las mismas tareas y recibirán la misma instrucción que los que no participen. No existen ningunos riesgos para su hijo/a por participar. Sin embargo, los posibles beneficios incluirían que su hijo/a estuviera más consciente de si mismo/a y de su conducta en la clase de matemáticas. También me gustaría declarar que la nota en la clase del/ de la estudiante no se será afectada de ninguna manera por la participación.

La información que obtengo con este estudio se utilizará en la tesis de maestría que le presentaré a una junta de facultad de CSU Chico, para mejorar mi instrucción. Se requiere su consentimiento para utilizar las respuestas de los cuestionarios de su estudiante y datos de investigación. La participación es voluntaria y no hay ningún castigo para su hijo/a si usted no quiere que su hijo/a participe en el estudio. Los nombres e identidades de los estudiantes serán totalmente anónimos. Todos los datos, incluidos los cuestionarios, notas, tareas, exámenes, pruebas , respuestas , etc. se mantendrán cerrados con llave en la escuela o en mi residencia . Estos datos sólo serán revisados por mí o tal vez mi tutor en CSU Chico.

Gracias por participar en esta investigación . Con cualquiera pregunta acerca de este proceso, por favor llámeme (707)-933-4010 Ext 5282, o escríbame jsoutham@sonomavly.k12.ca.us . Además, si usted desea una copia de la obra final, será mi placer de proveerle una copia a usted.

Muy atentamente,

Mr. Southam
Profesor de Matamáticas, Sonoma Valley High School

Permiso de los Padres

Yo, ____________________________, le doy mi permiso para que se incluyan los datos y las
Padre /Madre)
respuestas de mi hijo/a en el proyecto de investigación. También entiendo que el nombre de mi hijo no va a ser utilizado y su identidad se mantendrá en anonimato.

____

El Nombre del Estudiante La Firma del Padre / Madre La Fecha

Permiso de los Estudiantes

Yo, ____________________________, le doy mi permiso para que se incluyan los datos y las
Su Nombre
respuestas en el proyecto de investigación. También entiendo que mi nombre no será utilizado y mi identidad se mantendrá en anonimato.

______________________________ __________

La Firma La Fecha

Por favor devuelva esta hoja al Mr. Southam y mantenga la primera página para sus archivos.

Si decide no participar en el estudio, por favor complete la siguiente declaración para sus archivos.

Yo no doy permiso para que los datos y las respuestas que se utilizarán en esta investigación de mi estudiante.

____

Nombre del Estudiante Nombre del Padre / Tutor

APPENDIX H

Appendix H

Human Subjects Approval Letter

California State University, Chico
Chico, California 95929-0875
Office of Graduate Studies
530-898-6880
Fax: 530-898-3342
www.csuchico.edu/graduatestudies

August 28, 2014

Jonathan Southam
2560 Wimbledon Street
Napa, CA 94558

Dear Jonathan Southam,

As the Chair of the Campus Institutional Review Board, I have determined that your proposal entitled: "CREATING, INTRODUCING AND IMPLEMENTING A WORKED EXAMPLES UNIT IN GEOMETRY" is exempt from full committee review. Your study has been given "After-the-Fact" Approval.

Now that your data collection is complete, you will need to turn in the attached Post Data Collection Report for final approval. Students should be aware that failure to comply with any HSRC requirements will delay graduation. If you should have any questions regarding this clearance, please do not hesitate to contact me.

Sincerely,

John Mahoney, Ph.D., Chair
Human Subjects in Research Committee

Cc: Yuichi Handa (525)

The California State University

HUMAN SUBJECTS IN REVIEW COMMITTEE
Post Data Collection Questionnaire

Under Federal law relating to the protection of Human Subjects, this report is to be completed by each Principal Investigator at the end of data collection.

Please return to: Marsha Osborne, HSRC Assistant
Office of Graduate Studies
Student Services Center (SSC), Room 460
CSU, Chico
Chico, CA 95929-0875

Or Fax to: Marsha Osborne, 530-898-3342

Name: **Jonathan Southam** Chico State Portal ID# **006157646**

Phone(s) **775-351-4142** Email: **jonsoutham@gmail.com**

Faculty Advisor name (if student): **Dr. LaDawn Haws** Phone **(530) 898-6111**

College/Department: **Mathematics and Statistics**

Title of Project: **CHALLENGES IN CREATING AND IMPLEMENTING A UNIT ON PROOFS AND QUADRILATERALS BASED ON WORKED EXAMPLES PRINCIPLES**

Date application was approved (mo/yr.): **8 /2014** Date collection complete (mo/yr.): **6 /2014**

How many subjects were recruited? **59** How many subjects actually completed the project? **39**

***HARM**--Did subjects have severe reactions or extreme emotional response? **No**

If yes, please attach a detailed explanation: __________

Your signature: __________ Date: **7/27/19**

***Final clearance will not be granted without a complete answer to this question.**

Approved By: Patrick Johnson Digitally signed by Patrick Johnson Date: 2019.07.30 15:05:00 -07'00' Date: 7/30/19
~~John Mahoney, Chair~~
Patrick Johnson, Chair

****

VERY IMPORTANT: If you will or have used this research in your project or thesis you are required to provide a copy of this form (with John Mahoney's signature in place) to your graduate committee.

Do you want a photo copy of this form emailed to you? **Yes**
If yes, provide email address: **jonsoutham@gmail.com**