EFFECTIVE STRATEGIES FOR TEACHING MIDDLE SCHOOL STUDENTS FRACTIONS: A HANDBOOK FOR TEACHERS

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EFFECTIVE STRATEGIES FOR TEACHING MIDDLE SCHOOL

STUDENTS FRACTIONS: A HANDBOOK FOR TEACHERS

IN SAUDI ARABIA

A Project

by

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Spring 2015

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TABLE OF CONTENTS

ABSTRACT

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The primary objective for all teachers is tied with the provision of proper education for all their given students in a manner that is suited specifically to their needs. One means for facilitating this is through the use of an effective handbook highlighting a number of appropriate strategies that suit the group being taught on a certain topic. With this regard, this handbook presented several effective strategies, which can be used in teaching fractions to middle school students. The underlying aim of this project is to extend the available fractions' research on middle school students struggling with fractions. This entailed the creation of a handbook that is applicable to the Saudi Arabian fractions' curriculum so as to facilitate fraction teaching for the middle school students. The author carried out extensive research from American libraries that are characterized

by rich fractional content as compared to Saudi Arabia, since the latter is considerably underdeveloped with regards to pertinent resources.

The strategies established in the handbook showcase a mixture of conventional strategies and the approved evidence based effective strategies. The handbook has made provision for pertinent information on fractions and contains four chapters, namely; an introduction chapter, a chapter on fractions' content that is part of the curriculum of middle school mathematics, a chapter on effective strategies for effectively teaching fractions, and finally a chapter on how to appropriately assess student learning of the fraction content. More importantly, there has been the identification of eight effective strategies applicable in fractions teaching, in addition to several assessment techniques appropriate in the monitoring of students' progress.

The handbook's information is expected to assist teachers in the overall teaching of fractions to the mathematics middle school students. The intention of the handbook is to offer appropriate support, in addition to complementing the overall implementation of the traditional curriculum applicable for middle school fractions,. Hence, the project does not really make provision for a fractions' textbook but rather a guide to a fraction's textbook and fractions' teacher. Via the provision of subsequent guidance with respect to the teaching of fractions, this project has attempted to efficiently illuminate a specific pedagogical framework with regards to mathematical thinking enhancement. This is with reference to the means of supporting, eliciting, and effective extending of conventional higher-order skills in mathematics such as reasoning;

connecting and integrating; expressing and communicating; in addition to problem solving and application.

CHAPTER I

INTRODUCTION TO THE PROJECT

In the curriculum for mathematics concerned with middle-school students, fraction operations are viewed as critical foundations of the overall learning of mathematics. For the appropriate teaching of these operations, it is essential for teachers to have a significant level of mastery of the same. Before getting to the middle school level the development of these fraction operations is introduced at the elementary school level. This is critical so that by the time students get to the middle school level, they already possess basic knowledge of simple operations entailing subtraction and addition. This show progress of conventional fractional operation requires all students to be guided through an effective transitioning phase from the basic simple operations such as subtraction and addition, toward more complex aspects such as proportional reasoning and multiplicative models, among others (Peppers, 2014).

A fraction can be described as a given point on the conventional or standard number line, and is represented by a part-to-whole relationship whereby the unit segment (0, 1) is a whole (NCTM, 2006). In much simpler terms, a fraction refers to a part or parts of a given whole wherein all parts are equal, often represented in standard notation as the relationship a/b where b cannot be equal to 0. It entails the number of these equal parts which a certain size has, for example, one-quarter, one-half, or three-quarters. For normal counting, natural numbers are typically used, for example, 1, 2, 3, and so on. However, for measuring purposes, such as measuring a certain and precise length, the given value might not necessarily be a whole number. This calls for numbers which are less than, or are parts of, one unit (Hill, 2004).

Fractions pose a significant challenge to middle-school students, whether they have mathematical learning challenges or not (NMAP, 2008). Numerous studies have efficiently demonstrated that middle-school students tend to have difficulties with this particular mathematical concept. Furthermore, fractions are more cognitively demanding for those students who significantly struggle in in this subject area, for example, students who may possess learning disabilities (LD) or those who regularly perform poorly in mathematics (Mazzocco & Devlin, 2008). A recent report by the National Mathematics Advisory Panel (NMAP) highlights the significance of fractions as a skill that is foundational and essential to algebra, as well as a prerequisite to overall post-high-school success (NMAP, 2008). Of immediate concern to this panel were the diverse reports of declining student performance in algebra as they proceeded to higher grades, marked by less than 40 percent as meeting expected ability levels (NMAP, 2008). Additionally, the results indicated that around 40 percent to 50 percent of middle-school students significantly struggled with the content of basic elementary-level fractional mathematics (NMPA, 2008). The performance of all students who struggle in mathematics poses a vital educational concern, considering that such students often lag two or more grade levels below their given peers (NMAP, 2008). According to Das & Zajonc (2008) similar difficulties with the learning of fractions have also been reported among school children in Saudi Arabia. A World Bank study ranked the overall performance of Saudi-Arabian children at number 43 out of the 51 countries that participated in the Trends in

International Science and Mathematics (Das & Zajonc, 2008). It was also found that around 40 percent of the Kingdom's students were unable to pass the standard mathematics knowledge assessment (Das & Zajonc, 2008). The World Bank (2009) partly attributes this failure to teach mathematics to middle-school teachers' poor content-area knowledge, in addition to the inadequate teaching strategies within the country's education system.

In many countries in the world, including Saudi arabia, middle-school students face challenges in learning concepts about mathematical fractions. Moreover, an average learners in most areas fails to obtain basic knowledge regarding the same. For instance, according to an NCTM (2006) basic national exam, half of the eighth graders in the USA properly arranged three fractions on the basis of their sizes. Furthemore, even nations such as China and Japan in which most learners have above average intelligence face challenges in the learning of fractional concepts. These are countries that are well equipped with the needed materials to effectively teach and evaluate students. The performance related to the learning of fractions with respect to middle-school students in Saudi Arabia is quite poor. This is mainly due to limited resources and effective strategies that could otherwise help in realizing improved performance. Another reason is that fractions challenge middle-school students, teaching them that many concepts which are accurate for whole numbers do not necessarily apply for all numerals. For instance, multiplication at times fails to result in a response that is greater as compared to the multiplicands while division at times fails to result in a response that is lesser as compared to the dividend. It is quite difficult to overcome the nation that characteristics, which are accurate for normal whole numerals must be correct for all numerals. Yet, adequate understanding of fractions is essential to learning the basics of geometry, algebra, and other branches of mathematics, as well as comprehending diverse aspects of higher mathematics.

The middle-school age- and content-ranges between the U.S.A. and Saudi Arabia are similar, but there are notable differences which might be important to how mathematics is taught. In the United States, for example, middle school typically includes grades six through eight, while in the KSA the grades are seventh through ninth. The middle-school mathematics content is also similar between these two systems; the only major difference is that content covered in the eighth grade in the US would be taught in the ninth grade in Saudi Arabia.

The fraction content taught in schools in both the U.S.A and Saudi Arabia is quite similar in that both educational systems focus on similar concepts, such as comparison and order fractions, fraction operations (addition, multiplication, subtraction, and division), percentages, and proportionality, among other concepts (Perkins & Flores, 2002). There is, however, one outstanding aspect which exemplifies the differences and outcomes in how these countries teach fractional concepts—aspects which the handbook will discuss: Middle-school students in Saudi Arabia demonstrated poor performance related to fractions due to limited resources, something which can suggest appropriate strategies to aid in effecting improved performance.

Hence, this project will gather vital information from diverse sources to aid in the creation of a handbook which will describe effective strategies math teachers in Saudi Arabia can use in teaching fractions to middle-school students, in addition to a providing a means of assessing content learning. The handbook will include recommendations for a variety effective teaching strategies and assessment techniques, all with a focus on significantly improving students' conceptual understanding of standard fractions.

Purpose of the Project

The essential purpose of this project is to effectively extend the current research base on fractions for the students in middle school who are struggling in mathematics. This will entail creating a handbook to be utilized in Saudi Arabia to aid in the teaching of fractions for those students in middle school. Extensive research will be conducted using American libraries, which are rich in content regarding the teaching of fractions as compared to the libraries of Saudi Arabia which are significantly underdeveloped in this regard. This project will aim to establish an appropriate and effective handbook, a teaching aid that mathematics teachers can utilize in their fractions lessons for the middle school students.

The practices identified in the handbook reflect a balanced approach to teaching fractional mathematics, mixing traditional and new, effective strategies. The handbook will describe eight practical strategies which can be utilized by middle-school mathematics teachers so as to aid their students in effectively grasping fraction content. For each of these strategies, there will be a general introduction, a description of their use, and a discussion of their importance. At the handbook's end , there will be the provision of a reference list with viable sources for the reader in case he/she might find the need to follow up on the handbook's strategies. Normally, the outcomes of pertinent studies regarding teaching strategies relay minimal gains. In education, it is critical to have sufficient knowledge on divers teaching practices prior to their application. Used together, they will increase the overall probability of aiding students' learning, as well as identifying practices that might not actually always work in classrooms.

Specifically, there will be an investigation into the following eight strategies:

a) Building on Basic Intuitive Understanding
b) Number Lines: An Essential Representational Tool
c) Concrete-Representational-Abstract Instructional Strategy (CRA)
d) Proportional and Multiplicative Reasoning
e) Ratio, Proportion, Rate Problems
f) Student Think-Aloud
g) Class-Wide Peer Tutoring
h) Direct/Explicit Instruction

There will also be a discussion about the middle school curriculum fractions content. The content relayed by the handbook was retrieved from pertinent sources including the National Council of Teachers of Mathematics and the middle school. The comparisons and contrasts made between the curriculums in Saudi Arabia and the USA were necessary for providing the author with background information on how best to write the handbook, in addition to providing the aforementioned strategies.

Finally, there will be a discussion on the assessment of student learning with respect to fraction content. It entails coming up with standards and goals for student learning before subsequently carrying out the systematic analysis and gathering of evidence for the purpose of determining how well the given performance of the student matches the set standards and expectations. Student learning assessment has two essential objectives: a) documentation of the actual learning that is occurring and, b) utilization of the assessment activities' results for student learning improvement. Several assessment techniques will be discussed in detail including misconceptions and error analysis, classifying fractions, interpreting fractions, dealing with remainders, and assessment via questioning among others.

Scope (Description) of the Project

This project is the foundation for establishing a handbook to guide Saudi-Arabian teachers in teaching fractions via effective teaching practices. It is expected that the project will provide pertinent information and create a handbook that will contain four chapters, namely; an introductory chapter; a chapter on functions that is part of the curriculum of middle school mathematics; a chapter on effective strategies for effectively teaching fractions; and finally a chapter on how to appropriately assess student learning of fraction content. Very few studies have specifically examined the overall effectiveness of the eight strategies with fractions. First, this project will significantly add to the available current research base regarding the overall effectiveness of the eight strategies, and has the goal of improving the overall performance in fractions for those middle-school students who struggle in mathematics. Secondly, all available studies with middle-school students who struggle in mathematics have been conducted in the United States, with most of the research conducted by the same given body of researchers. This project aims to effectively expand the scope the eight strategies by thoroughly examining their effectiveness in Saudi Arabia. Thirdly, previous studies have employed group designs, looking at the overall impact of the eight strategies on school children's performance in fractions. This project aims to employ single-subject design for the purpose of examining the effectiveness of the eight strategies. Middle-school students struggling in the subject of mathematics do not necessarily all share similar characteristics (Hill, 2004). Employing the single-subject design subsequently allows the researcher to effectively examine the manner in which each student individually responds to instruction, in addition to how the responses might differ with respect to students' individual characteristics. Employing the single-subject design is seemingly a first-step with regards to examining the overall effectiveness of these aforementioned approaches in Saudi Arabia. The overall flexibility which the design affords, as compared to other diverse group designs, will essentially allow the given researchers to effectively identify certain areas for the future adaptation of the program to the Saudi-Arabian context.

Significance of the Project

Over the recent past, there has been a significant increase in the number of mathematics education-based studies. These studies have been based on varied research methodologies, grade stages, and mathematics content. Thier outcome in conjunction with other relevant domains' such as cognitive psychology, are applied in the selection of appropriate teaching strategies. Mathematics poses challenges with regards to both learning and teaching. Considering the diversity of the given teaching activity and the context that is lies in, it might to quite challenging for learners to learn. Hence, teachers have to take note of the individuals students' capabilities, in addition to the context of teaching as they attempt to chance the effectiveness of the learners. In this case, it is seen that Saudi Arabia significantly lacks the resources that the US has in terms of library resources.

This project essentially targets an under-researched and significantly important area in the field of mathematics, that is, fractions. Previous research in the given area of fractions has identified fractions as a certain and even unique skill which middle-school students often repeatedly struggle with. Nevertheless, it is an essential skill for their overall academic advancement (Mazzocco & Devlin, 2008). One reason for middle-school students' difficulty with fractions is that they apparently have only a superficial understanding of the diversity of mathematical concepts, and hence are not easily able to manipulate or apply fractional quantities. This project hopes to effectively bridge this gap by thoroughly targeting the middle-school students' understanding of the concepts of fractions via a conceptual approach. Furthermore, very few studies have been conducted for the purpose of examining effective interventions to appropriately teach fractions. This project also hopes to effectively contribute to the available research base by thoroughly identifying relatively effective practices to teach fraction problems to struggling middle-school students.

Achievement in the discipline of mathematics is relatively limited for many middle-school students with learning disabilities and even lower for those middle-school students with developmental disabilities. Although it is likely that any given curriculum might account for a percentage of the achievement challenges currently found in this sample population, Crawford (2000) suggests that the deficits of some middle-school students may actually be largely due to student-intrinsic factors and not solely caused by relatively poor curricula or teaching. Fractions are a recurring area of concern for many classroom teachers of middle-school students with learning disabilities. The areas of skill deficits that are most consistently reported by middle-school teachers of students with learning disabilities are traditionally related to fractions, percentages, and decimals. These difficulties usually include both the terminology related to fractions and the diverse operations of working with fractions. Several studies of the performance of middle-school learning disabilities students on specified secondary competency tests also effectively found significant skill deficits in decimals, fractions, and percents (Rivera, 1988; Miller & Mercer, 1993; MacIntyre, 2011).

When considering the nature of partial-number mathematics, it is not surprising that the majority of middle-school students with learning disabilities experience significant difficulty in computing and reasoning with decimals, fractions, and percents. The overall development of these abilities in typical, non- learning disabilities middle-school students can be relatively slow and complicated. In fact, numerous college students in primary teacher education programs have themselves demonstrated difficulties in giving accurate explanations and in generating what can be termed as appropriate representations of diverse fractional problems. Oftentimes, students will employ correct and accurate strategies for one defined problem but essentially fall back on the less-appropriate strategies in case they are faced with more challenging problems. The experience gained by middle-school students in solving these types of problems helps them in the solving of other real life problems. Nevertheless, middle-school students demonstrate wide variability in their responses to classroom instruction in standard fractions. Given the relatively low achievement of middle-school students with learning disabilities, even when compared to their low-achieving peers, it is likely that the numerous standard instructional programs simply lack the intensity and clarity necessary for the promotion of acceptable outcomes for learning disabilities middle-school students.

Limitations of the Project

Firstly, most of the literature utilized is not designed for the Saudi-Arabian educational context. Although Saudi Arabia's mathematics curriculum is similar to that of the USA, it seems vital to adopt and modify the strategies found within the American context and fit them into Saudi Arabia's educational system and cultures. Secondly, there is a wide gap between the literature reviewed and the experience, and project, of the author. However, most of the articles relayed useful information, which were utilized in the creation of the handbook. Finally, there likely will be the need to translate some of the literature into Arabic so as to ease these teaching strategies into the country.

Definition of Terms

Complex Fraction

A complex fraction is defined by NCTM (2006) as a "fraction in which the denominator and/or numerator is a fraction" (p. 3).

NMAP

This acronym stands for the National Mathematics Advisory Panel which is tasked with "advising the Secretary of Education and the President on appropriate use of diverse scientifically based research so as to advance the overall teaching and pertinent learning of mathematics" (NMAP, 2008, p. 2).

Decimal

This is “a number based on the numeral 10” (Math Open Reference, 2009, p.1).

Decimal Point

This is a dot, or period, which is part of a given decimal number. According to Math Open Reference (2009), “it indicates the point where the given whole number stops and the given fraction portion begins” (p. 1).

CRA

This acronym stands for concrete-representational-abstract. It is a “systematic method utilized to effectively introduce diverse mathematics concepts, such as fractions, to students” (Hill, 2004, p. 14).

Equivalent Fractions

These are fractions, which “might appear different, but have a similar value” (NCTM, 2006, p. 3).

Improper Fraction

An improper fraction is one which the numerator has a greater value than the denominator.

Proper Fraction

A fraction in which the numerator has a lesser value than the denominator.

Proportion

An equation, which states that “two ratios are seemingly equivalent” (NCTM, 2006, p. 3).

Reciprocal

This refers to “a value in which the denominator and the numerator are switched” (Math Open Reference, 2009, p. 1).

CHAPTER II

REVIEW OF RELATED LITERATURE

Fractional concepts are retrieved from certain theories with respect to essential teacher-knowledge characteristics. There are several researchers who have proposed specific theories regarding the essential characteristics and pertinent structure related to this teacher knowledge. This knowledge has to account for traditional students and students with learning disabilities because a conventional classroom will have these two types of students and it is imperative that the teacher be prepared to meet the diverse learning requirements presented by both. With regards to mathematics, Hill (2004) has researched pedagogical content knowledge in addition to developing a model called "mathematics knowledge for teaching (MKT)" (p.1). MKT has three vital knowledge domains related to conventional mathematics teaching. The knowledge of the students, in addition to the manner in which they think about the pertinent content, is standard knowledge of standard mathematics, and specialized knowledge of the pertinent content. Hill's MKT model provides an effective theoretical base for middle school teachers to effectively teach fractions.

MKT can be divided into Subject Matter Knowledge and the Pedagogical Content Knowledge (Kappan, 2009). Subject Matter Knowledge (SMK) refers to the teacher's understandings with respect to the subject being taught. The teacher's organization and depth of knowledge impacts the manner in which the teacher implement and structure their lessons.

With regards to the role played in teaching by the subject matter knowledge, it is seen that the discipline of defined scholarly knowledge is diverse as compared to the knowledge, which is required for teaching. On the other hand, Pedagogical Content Knowledge (PCK) refers to a knowledge type, which is unique to teachers, and its basis is on the way in which teachers relate their subject matter knowledge (knowledge concerned with whatever it is that they are teaching) to pedagogical knowledge (their actual knowledge on the teaching process). Overall, PCK consists of the synthesis or integration of the teacher's subject matter knowledge and pedagogical knowledge (Cochran, 1997).

With regards to the development of MKT, appropriate MKT is seemingly tied with specific habits of mind, for example, well-explicated reasoning and careful attention to details in mathematics, in addition to the agility with a diverse variety of mathematical productions retrieved from students and textbooks (Hill, & Ball, 2009). In other given cases, there are reports by teachers on their individual development of knowledge via extensive professional development, which is focussed on mathematical aspects. Additionally, to ensure teachers possess appropriate opportunities in the learning of MKT, the individuals responsible for the preparation of teachers and the provision of professional development will personally require the possessing of adequate support. More appropriate design opportunities, more specified guidance tailored for MKT teaching, and better materials, to learn from practically are important (Hill, & Ball, 2009).

For this paper, MKT is going to be utilized in structuring the literature review through the application of Subject Matter Knowledge (SMK) of Fractions and Pedagogical Content Knowledge (PCK) for Fractions. Firstly, SMK will be discussed with respect to several topics that highlight diverse aspects of fraction content. They will include additive and multiplicative reasoning, conceptual understanding related to diverse fraction operations, and other diverse lessons that middle school students are expected to learn about fractions. Secondly, PCK will be discussed with regards to eight effective strategies of teaching fraction content and a number of assessment techniques that can be utilized to test content knowledge.

Subject Matter Knowledge of Fractions

The overall development of middle-school students' appropriate fractions knowledge commences in elementary school before progressing to middle school. This progression of diverse concepts related to fractions in the curriculum of middle school requires that all students will have properly transitioned to advanced multiplicative reasoning: moving beyond the common additive one, developing understanding that is proportional, and understanding the apparent relations that are seen between fractions, proportions, and ratios (Peppers, 2014). In relation to this literature review it is essential to understand two significant elements that are tied to teaching fractions to middle-school students, namely 1) Proportional Reasoning and 2) Fractions and Rational Numbers. These sections are followed by lessons that middle school students should learn about fractions.

Proportional Reasoning

To effectively aid students in properly understanding the given meaning of proportion and ratio, teachers have to explicitly understand the diverse ways in which students develop standard proportional understanding. They must also provide students with meaningful classroom experiences and conceptual curricula so as to become good proportional reasoners (Thompson & Saldanha, 2003). Proportional reasoning is quite significant and merits the effort and time necessary so as to assure its overall development and acquisition by learners (NCTM, 1989). Teachers in middle school should have at their disposal the appropriate content, in addition to the corresponding pedagogical skills and knowledge, so as to effectively build upon the students' informal knowledge with regards to proportionality, and promote the given aspect of proportional thinking by utilizing effective questioning. Additionally, students can be provided with multiple examples, as well as relatively accurate representations, for the purpose of conveying diverse proportionality concepts. In order for students to develop proportional reasoning, they must have 1) additive and multiplicative reasoning, 2) conceptual understanding of diverse fractional operations, and 3) an understanding of division of fractions.

Additive and multiplicative reasoning. A significant developmental step necessary for all students in middle school is usually the transition from additive reasoning to multiplicative reasoning (Dooren, 2010). For this specific transition to be scaffolded, all teachers have to properly understand certain differences between the multiplicative and the additive reasoning (Dooren, 2010). Since understanding multiplicative reasoning fosters a grasp of proportions and fractions, appropriate fraction instruction similarly and essentially requires promoting normal multiplicative reasoning (Dooren, 2010). For example, Thompson (2003) explicitly suggested that additive reasoning is often inadequate. This is because, with regards to all types of fractions, additive understanding essentially involves the viewing of the fraction a/b as "specified parts out of the numeral b specified parts." Subsequently, this might lead students to assume the given numerator is effectively included in the given denominator. This essentially makes it quite challenging to properly interpret fractions like 5/3 since the value 5 is not included in the value 3. Multiplicative understanding of all fractions is essentially based on effectively thinking about two given quantities as standardly measured in explicit units of each other.

Conceptual understanding related to diverse fraction operations. Even though the majority of all fraction operations are typically covered in the standard elementary school, often they tend to be revisited in the middle-school stage. Middle-school students seem to possess procedural knowledge with regards to diverse fraction operations instead of an essential know-how of diverse underlying concepts. Hence, middle-school mathematics teachers have to acquire an appropriate conceptual understanding related to diverse fraction operations, all for the purpose of delivering a significantly “sense-making” curriculum. For example, to meaningfully subtract or add fractions, students have to interpret fractions by making them standard numbers or values of specified quantities rather than thinking of them as two separate numbers. “Multiplication by decimals and fractions can be challenging for all middle grades students if the diverse experiences with multiplication by standard whole numbers have essentially led them to believe that standard multiplication makes bigger” (NCTM, 1989, p. 3).

Division of fractions. This is an essential skill within middle-school mathematics curricula in which students often do not have appropriate understanding. This is because the division of standard fractions is normally taught through emphasizing the standard algorithm of invert-and-multiply, and not coupled by understandable justification of its underlying principle of working (NCTM, 2006). In research on Chinese and US teachers' knowledge of dividing fractions, the US teachers tended to rely on the standard algorithm and did not have sufficient understanding for the purpose of generating appropriate representations (An, 2004). It is expected that after students grasp the true foundations of fractional division, they will be encouraged to seek ways that are more efficient, for example, the invert-and-multiply algorithm, for the purpose of performing the outlined process.

Based on this underlying knowledge, this literature review will focus on four interrelated sections, namely: description of a fraction in math; lessons that middle-school students are essentially expected to learn about fractions; the traditional strategies used for teaching fractions; and the most effective teaching strategies for helping middle-school students learn about fractions.

Fractions and Rational Numbers

This section gives a detailed description of the term “fraction” as related to math. It will also show diverse utilizations of fractions within middle-school curricula. This section will set the pace for additional discussions concerning the essential lessons that middle-school students are expected to learn about fractions. A fraction can be described as a given point on the conventional or standard number line, which is represented by a part-whole relationship whereby, the unit segment (0, 1) is a whole (NCTM, 2006). In much simpler terms, a fraction refers to a part of a given whole or in general terms, any given number of parts that are equal. It entails the number of parts which a certain quantity has, for example, one-quarter, one-half, or three-quarters. For normal counting, natural numbers are regularly used, for example, 1, 2, 3, and so on.

However, for measuring purposes, such as measuring a certain length, the given value might not necessarily be a whole number. This calls for numbers, which are less than, or are parts of, one.

A common or simple fraction is comprised of an integer numerator, which is displayed before a slash or above a line, and also a non-zero integer denominator, which is displayed after a slash or below that line. Both denominators and numerators can also be applied in fractions, which are not simple or common, for example, complex fractions, mixed numerals, and compound fractions. The numerator conventionally represents a certain number of equal parts while the denominator indicates the number of which those parts makeup for a whole or a unit. An example can be seen in the fraction 3/5. In this case, 3 (the numerator) indicates that there are three equal parts while 5 (the denominator) indicates that five parts make up the entire, or the whole, unit. One notes that fractions can be written without necessarily utilizing explicit denominators or numerators, but rather by simply using percent signs, decimals or even negative components (1%, 0.01, or 10^{-2} respectively). According to Wu (2011), “fractions are to represent ratios and to represent division.” For example, the fraction 3/5 can refer to the ratio 3:5. 3/5 can also refer to 3 ÷ 5 (three divided by five). The following section relays a brief overview of simple calculations that incorporate fractions.

Converting fractions into decimals. In order to convert fractions into decimals, one simply divides the numeral at top (numerator) by the one at the bottom (denominator) by either utilizing a calculator or manually. In the following example, to convert the given fraction into a standard decimal, one divides the top numeral (4) by the bottom numeral (5) so as to obtain a result of 0.8: 4/5 = 0.8 (Math Open Reference, 2009).

Quite often, individuals tend to read a given fraction as normal, simple division. In the aforementioned example, one can read it as the numeral 4 divided by the numeral 5 (Math Open Reference, 2009)

Improper and proper fractions. A proper fraction refers to one in which the given top (numerator) is smaller in relation to the given bottom (denominator), such as 2/5 is a proper fraction (Math Open Reference, 2009).

When converted into a given decimal, a proper fraction is always less than 1. However, an improper fraction tends to be the reverse, such that the given numerator is relatively larger as compared to the given denominator, 6/4 improper (Math Open Reference, 2009).

Additionally, one can see that an improper fraction can be essentially 'reduced', such as 6/4 =1.5 (Math Open Reference, 2009).

Fraction also refers to the term 'divide'. Sometimes, a normal fraction is simply another means of stating the term 'divide'. For example, 2/7 is a simple means of saying 'the numeral two divided by the numeral seven'. This is similar as writing the value 2 divided by the value 7 (2÷7) (Math Open Reference, 2009).

This frequently happens in the computation of large, complex fractions and in algebra, where one divides whole expressions by a certain figure or expression. For example, effectively turns out to be $2x - 3y + 8$ (Math Open Reference, 2009).

Percent. Fractions are also often written as percentages. The term "50 percent," which is written as 50%, essentially refers to many per a given hundred. Hence, 50% of a number of given students refers to 50 out of each given hundred students or alternatively, in this case, one half of all the given students (Math Open Reference, 2009).

As part of general instructional content concerning fractions, students usually acquire both procedural and conceptual knowledge. Conceptual knowledge generally refers to the way students understand a given topic and subsequently establish links and relationships with information that was previously learnt. Procedural knowledge refers to a given set of algorithms or rules, which students follow for the purpose of solving a given problem. Therefore, conceptual knowledge with respect to fractions entails understanding fractions as part of a given number line, fraction equivalence, and the magnitude comparison of the given fractions. In its turn, procedural knowledge with respect to fractions entails the computations of fractions, including subtraction, division, addition, and multiplication. According to the National Mathematics Advisory Panel, it is essential that both of these knowledge forms be applied in the classroom so that students can successfully interact with fractions (NCTM, 2000). The panel also outlines certain benchmarks to be applied for fractions with respect to diverse grade levels. The panel’s recommendations are consistent with specified focal points for all fractions as stated by the National Council for Teachers of Mathematics (NCTM, 2006).

In tandem with these essential concepts, it is also vital to explore the actual lessons that middle-school students are expected to comprehend and learn about fractions. By the fourth grade, students have to recognize, produce, and represent fractions on the standard number line or via other representations. They also will be expected to develop a relatively strong conceptual understanding of the outlined fractions, in addition to making simple comparisons between diverse representations of fractions, as seen for example in decimals and fractions. By the fifth grade, students have to be fluent in making comparisons between decimals, fractions, and percentages. By the sixth grade, students have to be generally fluent with regards to all operations involving fractions in terms of division and multiplication, and also extend this given knowledge to other operations with rational numbers, for example, with ratios. Finally, by the seventh grade, students have to be quite proficient with the diverse computations of negative and positive fractions. According to MacIntyre (2011), this form of gradual growth promoting the student's development of their individual acknowledgment of their personal cognitive and knowing how they learn.

Lessons that Middle School Students are Expected to Learn about Fractions

Middle-school students are expected to significantly deepen their given understanding of place value, in addition to their knowledge and skill in terms of the multiplication, addition, subtraction and division of whole numbers (CDE, 2011). Additionally, middle-school students are expected to develop an appropriate understanding of all fractions as numbers, the diverse concepts of area of certain given plane figures, and the diverse attributes of several shapes. The proficiency of middle-school students regarding fractions is seemingly essential to their success in other areas, such as algebra, in subsequent later grades (CDE, 2011). Students are expected to utilize visual models to effectively represent fractions as specific parts of a given whole. They also utilize these visual models, in addition to a standard number line, to effectively represent, explain, and also compare unit fractions, equivalent fractions, whole numbers, and fractions with similar numerators and similar denominators. Students are also expected to learn to recognize, accurately name, and efficiently compare fractions and utilize a standard number line to represent the given positive fractions. Operations involving decimals are usually introduced at an advanced level (CDE, 2011). The aspects, which Saudi middle school students are expected to know at these developmental/grade level stages include Fraction Definition, Compare and Order Fraction, Fraction Operations, Proportionality, Percentage, Fractions and Percentage.

Middle-school students are expected to compute very small as well as very large numbers, whether as decimals, positive integers, and fractions, and appropriately understand the given relationships between fractions, decimals, and percentages. They must also demonstrate understanding of the apparent relative magnitudes of given numbers:

1. Estimate, round up, and manipulate both substantially large and very small numbers.
2. Accurately interpret percentages as a given part of a hundred; seek percent and decimal equivalents for standard forms of fractions and explain the reasons why they represent a similar value; accurately compute a certain percent of a given whole number.
3. Accurately identify and represent decimals, fractions, and mixed numbers on a number line, in addition to positive and negative integers (CDE, 2011).

Most middle-school students, for diverse reasons, struggle to learn fractions. For example, even early exposure to or working with standard whole numbers doesn’t guarantee that students will develop a full understanding of them. Younger students may commence to conceive the standard number line as one that consists of discrete units and essentially fail to see how they can be subdivided further when reading a given fractional number. Hence, these students usually tend to simply read the denominator and numerator as separate wholes and fail to view the relationship between the given numbers. Such errors might occur despite the teacher utilizing measures such as one-fourth, or even one-half, in daily situations. It is seemingly a failure of connecting form (numerals, symbols, and algorithms) with a conceptual understanding (connecting real-life and mathematical ideas or situations). Students may have a certain intuitive knowledge of standard fractions, but do not actually apply this to their given abstract representation. Students might also have difficulties in conceptualizing of fractions in certain ways other than the given part-whole. From the early grades, fractions are normally represented as being a section of a given whole, for example, in the division of a rectangle. Such depictions make it relatively difficult to appropriately comprehend improper fractions in a situation in which the denominator is smaller than the given numerator. The partitive model of standard fractions forms the conventional basis for the majority of fraction instruction in many schools. In this role it emphasizes the apparent part-whole relationship, which further inhibits the overall development of several other sub-constructs of standard fractions, for example, fractions as quotients (NCTM, 2006).

In addition to the diverse aspects of specific fraction content itself, which contribute to certain learning difficulties, specific underlying processes also significantly impact students their understanding of fractions. One's working memory is usually noted as a significant influence on students' abilities to compute diverse fractional problems, as well as finding solutions to diverse word problems involving fractions. Working-memory deficits might inhibit students' overall conceptualization of fractions, and the given impact is seemingly more pronounced for struggling learners who tend to have limited working-memory capacity. There is also the aspect of proportional reasoning of middle-school students. It is noted that even though proportional thinking actually evolves similarly within both of these sets of students, struggling learners tend to utilize inferior strategies while solving those problems which involve fractions when compared to their given peers. Considering the challenges that students generally face with fractions, it is essential to identify any interventions that might target and enhance student performance in this vital foundational skill (NCTM, 2006). So as to anchor such instructional methods, this project provides a guide for math teachers to effectively teach fractions to middle school students.

Pedagogical Content Knowledge for Fractions

This section will discuss two aspects of PCK. Firstly, there will be the discussion of several effective evidence-based strategies that can be utilized to teach fraction content for middle school students. Secondly, there will be a discussion of appropriate assessment techniques that can be used in testing content knowledge of whatever has been taught in the classes.

Strategies Used for Teaching Fractions

There are several strategies that have been utilized for teaching fractions to the middle-school students. They are evidence-based practices, which are usually helpful for those students who have significant difficulties in mathematics in general. The principles which have emerged from diverse research studies are appropriate for instructing students in a variety of possible settings and situations (NCTM, 2006).

<u>Graphic and visual depictions of problems</u>. The graphical representation of diverse mathematical problems and concepts, such as fractions, are normally utilized in

textbooks. These are vital components of the programs utilized in many nations and perform relatively well with regards to international comparisons, for example, with the Netherlands and South Korea. One interesting fact regarding the application of this strategy is that the particularity of the given representation of visual determines the overall impact of the given intervention (NCTM, 2006). When teachers, in teaching fractions, present proper graphic depictions of specified problem-solving sets that have multiple examples, they seem to achieve greater results as compared to when they do not use this strategy. There are two recent studies, conducted on middle-school students learning fractions, which add a different dimension to the utilization of this strategy. The researchers in these specific studies successfully developed an approach called "concrete-representational abstract" for the purpose of successfully teaching operations and concepts involving fractions (Butler, Miller, Crehan, Babbitt, & Pierce, 2003). It is essential to know that manipulatives are not applied in the skirting of defined abstraction's teaching, which is critical in gaining knowledge in fractions. They are only applied briefly to ensure students gain appropriate knowledge on the visual organizer and representations. This pro of this practice is that its concreteness and intensity ensures learners maintain a framework for the solving of mathematical fractions in their specified working memory. One essential conclusion from the aforementioned study is the significance of teachers including work involving manipulatives for their lessons.

Building on basic intuitive understanding. This strategy resides in the significance of helping middle-school students understand basic fractional concepts via building on their individual intuitive knowledge of sharing (Carpenter, 2011). It relays the manner in which learners initially solve problems through direct representation of sharing actions by utilizing counters or drawings. In extending these given sharing ideas learners next solve problems, for example, that one or more of any given objects to be shared requires that they be partitioned. Initially these learners solve fraction-sharing problems through simple halving, but eventually one sees that they learn to solve more complex sharing problems that cannot be solved via repeated halving. It is seen that this sharing concept is also utilized in the development of a more central concept of standard fractional equivalence through comparing diverse sharing arrangements. This strategy encourages teachers to allow middle-school learners to utilize their informal understanding of conventional sharing so as to explore diverse problems and subsequently explain their solutions—all by utilizing a variety of defined representations (Carpenter, 2011).

Student think aloud. Research shows that when students are faced with certain multistep problems in fractions, they tend to attempt solutions by randomly combining certain numbers rather than implementing a proper solution strategy. The process of encouraging them to verbalize their thinking by writing, drawing, or talking with respect to the steps necessary to solve the fraction problem is a consistently effective teaching strategy. "Students are encouraged to think aloud as they solve problems" (Mack, 1990). In part, this might be effective because students' given impulsive approach in solving these fraction problems is usually addressed.

Direct instruction. The Direct Instruction strategy entails the following elements: (a) devising procedural strategies; (b) specifying short- and long-term objectives; (c) sequencing skills; (d) determining necessary pre-skills; (e) designing a proper instructional format (such as establishing teacher wording, sequencing instruction, choosing learning tasks, and establishing correction procedures); (f) delineating a given teaching procedure; (g) choosing practice and review; (h) selecting examples; and (i) creating procedures for monitoring any progress (Stein, et al., 1997). There are diverse studies, which prove the effectiveness of direct instruction with respect to teaching fractions to students.

The appropriateness of this strategy in teaching fractions to students is well validated within a diverse body of literature. For example, Hastings et al. (1989) thoroughly looked at the efficacy of this approach in teaching two separate LD secondary students to accurately count money. They were effectively taught to efficiently count bills and coins by utilizing a series of specified steps. Even though the students with learning disabilities seemed to be considerably slower as compared to the control group, Direct Instruction has apparently created important math gains in the overall education of students. A research study by Hasselbring (1987) examined this strategy's overall effectiveness in the instruction of two groups of standard elementary students, with similar knowledge of fractions, in two scenarios: through the teacher and via video. A specific control group was given appropriate instruction through a standard spiral curriculum. Data was analyzed for average and high-performing students. The average-ability groups characterized by direct instruction achieved high scores, similar to the high-ability form of the spiral curriculum group. In another research study, which compared the performance of middle-school students taught through this means against their peers taught through the textbook approach, it was seen that the latter group performed better on criterion-referenced tests, in addition to tests developed by publishers (Crawford, 2000). The superior performance demonstrated by the direct-instruction group is validated by a relatively significant effect size of the value 1.15. nation they counted the money as rapidly as the given average rate posted by their peers. This strategy taught accurate and rapid skills in money counting. Furthermore, Rivera and Smith (1988) effectively taught a total of eight LD students to accurately tackle certain problems in long division—both without remainders and also with remainders—all in just one week. Vital elements of this strategy, such as modelling, established questions, imitation, and essential guide words, made essential contributions to the performance of students. Kitz and Thorpe (1995) successfully conducted a research study, which in looking at twenty-six LD students with a long record of failure in math effectively achieved a comparison of the overall effectiveness of two algebra programs. One of the groups received its instruction through a videodisc program and the other one via a standard textbook-based manner. The subsequent results concluded that the group utilizing Direct Instruction performed better when compared to the control group with respect to three post-test measures: a post-test on algebra, a videodisc post-test, and a first-year course on algebra.

Proportional and multiplicative reasoning. Proportional reasoning is conventionally characterized by the capacity to view comparison scenarios in a multiplicative sense as opposed to an additive one (Reynolds, 2015). For example, considering the numerals 10 and 2, if a question was posed to define the process of altering numeral 2 to obtain numeral 10, additive reasoning will describe the process as the numeral 2 increasing by 8 to obtain the numeral 10. However, multiplicative reasoning will describe the process as the numeral 2 being multiplied by the numeral 5 so as to obtain the numeral 10. Thus, an individual applying multiplicative reasoning can view comparison scenarios in a multiplicative sense as opposed to an additive one.

Generally, multiplicative reasoning makes provision for flexible thinking with regards to numerals and scenarios involving numerals (Reynolds, 2015).

Proportional reasoning entails the making of comparisons in multiplicative terms between entities. Hence there is the conceptualization of the relationship between two defined entities in a multiplicative relationship (Dole & Wright, n.d.). Proportional reasoning development traditionally consumes a considerable amount of time. Quality learning experiences foster it in a manner that students possess opportunities to experiment, discuss and explore with scenarios tied to proportional reasoning. With respect to middle- schooling years, an active role must be taken by instruction with regards to supporting proportional reasoning development. It is seen that approximately 50 per cent only of the overall population of adults can reason in a proportional manner. There must be queries on the reasons for this with regards case teaching (Dole, & Wright n.d.). As proportional reasoning keeps on permeating into a host of mathematics topics, there must be the prompt consideration of the teachers' roles with respect to fostering proportional reasoning skills of the students. The development of proportional reasoning does not really occur in a linear fashion, neither is it an absolute state. It continually shows development and growth with explicit teaching and experience (Dole, & Wright n.d.).

Ratio, proportional, rate problems. Appropriate knowledge on how to deal with proportions and ratios is quite essential in mathematics classes. This is especially accurate when dealing with diverse measurement units. Conventionally, a ratio is defined as the relative size given of two quantities that are expressed as a single quotient of one numeral divided by the another numeral. Proportion, on the other hand is termed as the equality found between two given ratios. Borowski & Borwein (2002) expand on these definitions by stating that proportion is the apparent 'relationship between any four numerals such that the first pair's ratio is equal to the second pair's ratio. This definition tends to account for the daily practical utilization of the term 'proportion' since it fails to make a comparison to a whole.

The strategy aids the students in seeing a single problem worked in a variety of ways. Subsequently, they gain a sense of which approach works best for a certain problem and suits their individual understanding. This strategy shows the true nature of mathematics, which entails reasoning and judgment, thus a student with more experience will eventually do a better job of effectively picking the best tool out of a defined toolbox (Lewis, 2010). The strategy might require a ratio table showcasing the two defined amounts being compared so as to aid in addressing varied questions on several topics such as the variations in the actual total quantity to be created.

Class Wide Peer Tutoring. Class Wide Peer Tutoring (CWPT) refers to a variation of peer-mediated instruction characterized by the creation of pairs of learners who alternate between taking up the roles of the learner and the tutor. The latter queries the former, notes down the response, and gives feedback with respect to the accuracy of the responses as provided for by the teacher. The response by the former can either be in writing or simply done orally (Terry, 2008). The teacher supervises the whole CWPT process and subsequently awards marks for proper tutoring. There is the active involvement by all learners in the learning process, which effectively enables them to appropriately practice the standard skills in a systematic and fun manner. However, the difference between CWPT and standard group work is that the former incorporates tutoring of each other while the latter is a simple discussion with no decisions made on who is right or who is wrong.

Daily lessons will allow each given partner to assume the tutor and tutee roles (Greenwood et al., 1997). Moreover, this strategy utilizes error correction, and immediate-response feedback, in addition to a defined technique of tutoring, which benefits both parties (Maheady, Harper & Mallette, 2003). If accurately structured, then this strategy ensures tutors engage all the given learners actively, while simultaneously ensures an effective monitoring process via assessments and tests carried out in defined periods.

Explicit instructional and Concrete - Representational – Abstract. The Concrete-Representational-Abstract (CRA) strategy is a systematic method utilized to effectively introduce diverse mathematics concepts, such as fractions, to students. This strategy essentially targets a basic conceptual understanding of the discipline of mathematics. Most studies on the CRA strategy utilize explicit teaching for the purpose of delivering instruction. Explicit instruction entails the utilization of guided practice, modelling, and multiple opportunities for practice, progress monitoring, and corrective feedback (Butler et al., 2003). The utilization of explicit-teaching procedures has been thoroughly and well documented as a significant effective teaching practice in diverse areas of learning and specifically for mathematical applications—in this case for teaching fractions. Studies investigating the overall effectiveness of the CRA, in addition to explicit teaching approaches, have effectively examined diverse content areas such as addition, subtraction, multiplication, division, fractions, and algebra. Studies have also investigated the overall effectiveness posed by combined approaches to the initial concepts in mathematics and higher-level concepts.

- Early Mathematics Concepts. There have been significant studies carried out by Miller & Mercer (1993), which have examined the overall CRA effectiveness, in addition to the effectiveness of explicit strategies, with regards to the teaching of mathematical concepts, such as fractions to middle school students. They utilized scripted lesson plans in which the explicit instruction format entailed an advanced organizer, independent practice, guided practice, model of the given skills, progressive monitoring, and corrective feedback. They also investigated the overall impact posed by the diverse approaches concerned with basic and standard mathematics skills of addition, division, and coin problems. By utilizing a multiple-baseline design, they examined when in the given CRA sequence, the given students were able to effectively transfer or crossover learning that had been gained via the utilization of representations and manipulatives so as to abstract relevant concepts.

  With a value of PND = 94%, their given division problems with a value of PND = 83% and essentially solving problems with money with a value of PND = 83%. The corresponding maintenance results, even though lower as compared to the post-test for the majority of the students, remained relatively higher as compared to the baseline; and the given students were essentially able to score more correct responses as compared to incorrect responses. With respect to the given crossover effect, the subsequent results indicated that most of the given students did not make the appropriate transfer to abstract after the described instruction in the initial concrete phase alone. Most of the given students required instruction that was utilizing both representational and concrete aids so as to create an association with the given abstract content. Another research by Harris & Miller (1995) effectively employed two given simultaneous multiple baseline designs so as to teach multiplication of fractions facts to middle school students with disabilities by utilizing the combined explicit teaching and CRA approaches. It was seen that an overall total of 13 children, whereby 12 with learning disabilities, were included in the conduction of this given research. The authors essentially conducted the given research in two defined parts. The first one entailed the research targeting fluency and mastery of multiplication facts and the students seemingly learnt an effective strategy to accurately solve pertinent multiplication problems. The given students were administered with a standard a post-test after the aforementioned first part of the overall research. The results indicated that all the given students improved their individual performances significantly with respect to multiplication. In the second part, all the students carried on working towards fluency with respect to the given multiplication facts, and also accurately solved multiplication word problems. The pertinent multiplication fluency data essentially indicated that all the students improved their individual multiplication rate, and that the students seemingly scored more accurate answers as compared to the inaccurate ones. It is also seen that the students' overall performance tended to be relatively lower when there was the introduction of complex word problems. Fuchs et al. (2005), in another randomized control trial, investigated the combined overall effectiveness of the two aforementioned approaches and compared them to the traditional or standard instruction on diverse initial mathematics concepts, in addition to subtraction and addition math fact fluency. All the participants included 127 5^{th} grade students at significant risk for learning disabilities. These authors also effectively compared the overall performance of at-risk students with the seemingly achieving peers. They employed standardized and researcher-developed tests. The subsequent results indicated that the given target group significantly outperformed the given control group of the at-risk students on standardized and researcher-developed measures of computations, story problems, and the on grade level concepts. When effectively comparing the overall performance of the given target group to the other typically achieving peers, the given authors noted that the overall improvement in the given scores from the available pre-test to the available post-test was seemingly significantly higher for the given treatment group as compared to the other typically achieving peers on the basis of the standardized measure of the pertinent grade level and computation concepts.

- Higher mathematics concepts. Several studies have thoroughly examined the combined effectiveness of the aforementioned approaches with regards to higher mathematics concepts related to fractions, geometric concepts, and algebra (NCTM, 2006). Butler et al. (2003) thoroughly examined the overall effectiveness of the given approaches to appropriately teach fraction equivalence and fraction computation. Results were seemingly in favor of the available treatment groups. Cass (2003) employed a specific multiple baseline design so as to investigate the overall combined effectiveness of the aforementioned approaches with regards to teaching area and also perimeter problems to diverse students having LD. The research utilized a geoboard for seemingly concrete materials, and also pictures of certain figures as pertinent representations. The given lessons followed the apparent model-lead-test sequence of pertinent instruction, and also specified steps for accurate error correction. All the middle school students significantly improved their overall performance in the area and the perimeter problems.

Maccini (2000) investigated the utilization of a certain strategy along with the aforementioned ones for the purpose of algebra problem solving to middle school students that were struggling in mathematics. Utilizing a certain multi-probe design, he taught the given STAR strategy, which effectively directed students to appropriately search the given word problem, accurately translate utilizing concrete materials, equations, or pictures, answer the given problem, and subsequently review. Utilizing algebra tiles, certain representations and finally abstract symbols, the given students solved the given problems with the aid of the STAR strategy at the representational, concrete, and abstract phases of the given instruction. Results were essentially in favour of the given strategy and maintenance of the given performance over time could be achieved. The author additionally examined the transfer of pertinent skills to seemingly more complex word problems. Students were effectively able to solve the near transfer problems with an accuracy level of 67% and far transfer problems with 28.7% accuracy. Additionally, Hill (2004) employed a seemingly randomized control trial so as to investigate the overall effectiveness of the aforementioned approaches versus only the explicit instruction for the purpose of teaching algebraic equations. The author expanded on the given algebraic representations by targeting the utilization of CRA and the given explicit instruction with complex algebraic equations. The given results were seemingly in favor of the combined approaches group and students in the given treatment group had the ability to maintain their overall performance over time.

Assessment of Student Learning

Generally, learning occurs in the heads of the students where it is obviously invisible to others. Therefore, the assessment of learning has to be done via performance: the application of whatever has been learnt by the students. The assessment of the performance of the students might entail assessing based on the following aspects: individual or collective, anonymous or public, high- or low-stakes, and formal or informal (Lamon, 2012). It must be noted that student assessment is an on-going process, which focuses at the improvement and understanding of student learning. It entails coming up with standards and goals for student learning before subsequently carrying out the systematic analysis and gathering of evidence for the purpose of determining how well the given performance of the student matches the set standards and expectations. Student learning assessment has two essential objectives: a) documentation of the actual learning that is occurring and, b) utilization of the assessment activities' results for student learning improvement (McLeod & Newmarch, 2009).

With respect to fraction content, the following assessment techniques have been proven to be appropriate for the middle school students;

a) Assessment via Questioning: Questioning might be utilized to seek if a certain student has knowledge on the accurate answer to a defined closed question (Lamon, 2012).

b) Sample Questions on Fractions (Utilization of Mini Whiteboards and Reasoning Understanding).

c) Misconceptions and Error Analysis: a list of possible misconceptions will be given.

d) Common Mistakes Examples: a list of possible mistakes will be given (McLeod & Newmarch, 2006).

e) Evaluation of Fractions' Statements: Students are offered some generalizations concerning fractions, before being prompted to state if they consider them to be 'sometimes', 'never' true, or 'always' (Cangelosi, 2002).

f) Links outside the Classroom Setting: aids students to utilize their knowledge of fractions in the outside context of the classroom (Egan, 1992).

g) Classifying Fractions: might be a relatively efficient means of encouraging a student to reflect on and effectively analyze their given aspects (McLeod & Newmarch, 2006).

h) Interpreting Fractions: it is essential to be able to appropriately talk about fractions with respect to the diverse appearance selections (McLeod & Newmarch, 2006).

i) Dealing with Remainders: eliminates confusion concerning what to do with the given remainders after any division operation: (Davidson, 1990).

Additionally, it is critical to make creative utilization of meta-Analysis models: for instance, students can note down their personal exam-style questions (McLeod & Newmarch, 2009). Through the insertion of their personal figures, or rather, question rewriting, they might become confident at the overall writing of their own new questions from scratch. Based on the available feedback, the instructors can appropriately adjust their teaching processes so as to aid effective learning of the students. Assessment can take a variety of forms. In a given classroom, assessment of learning is relatively summative. Although the middle school students have tackled fractions before, at times, they still identify it as a section of extreme difficulty. This means that a broader approach is required for the assessment. The formative assessment tools have to identify the aspects, which are challenging to conceptualize.

From this literature review, it is seen that most of middle school learners have a number of difficulties in mathematics and also in understanding and effectively working with fractions. These forms of difficulties might result from working with memory deficits, and also include certain difficulties in basic fact retrieval and also inappropriate strategy use. The difficulties, which struggling learners seemingly face in mathematics are also seen to impact their overall understanding of fractions. Additionally, they are further compounded by the given fraction content itself which significantly poses a difficulty for the majority of the students. Several reviews have effectively identified certain approaches that are effective in the teaching of mathematics to a host of struggling learners. Diverse studies have also effectively investigated the assessment of students learning of fraction in many ways

CHAPTER III

METHODOLOGY

Introduction

The author created a comprehensive handbook for middle school teachers to utilize in the instruction of fractions in Saudi Arabia. Due to the lack of information possessed by Saudi Arabian schools on teaching middle school students fractions, an extensive review of the literature was conducted with the aid of data from the USA libraries, which boast of superior content as opposed to the former. The author provided a brief summary of the pertinent research, which facilitates each practice and comprehensively describes the manner in which the same research can be applied in a traditional classroom setting. The content of Saudi Arabian fractions' curriculum was analyzed and notable comparisons were made with that of the US. Eight strategies were then provided from a host of reliable and viable sources, which formed the backbone of the handbook. The strategies that were identified portrayed a balanced mixture of relatively new innovative strategies and conventional strategies evidence based used in the past. Most importantly, the eight strategies were chosen because they best fit the educational environment in Saudi Arabia; they fit what might be called “the three C’s”: Saudi *classrooms*, Saudi middle-school *content*, and Saudi *culture*. Appropriate assessment techniques were also provided by the author so as to complete the learning experience for the students, in addition to measuring their overall progress. The handbook is then concluded by making provisions for all the references utilized in its creation. Furthermore, the references also serve to provide the instructor with the option of learning about the strategies in a more comprehensive manner. The sources utilized are from viable journals, peer reviewed sources, reliable textbooks, and respected websites, thus validating the reliability, authenticity and viability of the provided information.

Considering the structure of this paper, with respect to the literature review and the methodology, the MKT framework is quite valuable since it is an essential predictor of the outcomes of the student. It has the ability to impact the student outcome profoundly with respect to the given general cognitive ability. Moreover, teachers' MKT is highly related to their instruction's mathematical quality, inclusive of the utilization of mathematical representations and explanation, students' mathematical ideas' responsiveness, imprecision and the avoidance of mathematical error, and imprecision ability (Hill, & Ball, 2009).

Creation of the Handbook

Firstly, the author carried out extensive research from viable and reliable sources with respect to fraction content in middle school. The handbook was divided into four sections, namely; the introduction, fraction content in middle-school curriculum, effective strategies for teaching the fractions, and the assessment modes for the same. With respect to the fractions' curriculum content, certain comparisons were made with that of the US so as to ensure the user of the handbook has appropriate background study with respect to the subject content. For each teaching strategy, there was an introduction section, utilization section, and importance section. The provided strategies were deemed suitable for both the strong and weak learners in middle school. Since the reason for selecting these strategies entailed a mixture of innovative strategies and traditional strategies previously utilized, it is quite easy to integrate them in standard classrooms. The assessment strategies were sourced from viable resources that are suitable for the learners of middle school mathematics. The assessments are considerably interactive in nature so as to make sure that both the tutor and the learner are in sync with each other. This will ensure that the tutor can monitor the gradual development of the individual learners closely and point out vital improvement areas.

Middle School Curriculum Content

The content relayed by the handbook was retrieved from pertinent sources including the National Council of Teachers of Mathematics and the middle school math textbooks in Saudi Arabia. The comparisons and contrasts made between the curriculums in Saudi Arabia and the US were necessary for providing the author with background information on how best to write the handbook, in addition to providing the most effective strategies. Moreover, an analysis of the conventional age appropriate curriculum, in addition to the reviewing of the students' academic, physical, and mental development offered the author with the means of trusting any prior information on the subject matter that he already possessed.

Strategies for Teaching

The following strategies were adopted by the author in this handbook:

1. Building on Basic Intuitive Understanding
2. Number Lines: An Essential Representational Tool
3. Concrete-Representational-Abstract Instructional Strategy

4. Proportional and Multiplicative Reasoning
5. Ratio, Proportion, Rate Problems
6. Student Think Aloud
7. Class Wide Peer Tutoring
8. Direct Instruction

In all of these strategies, there was an introduction section, utilization section, and importance section. The strategies were appropriate for both the strong and weak learners in middle school. Since the basis of these strategies entailed a balanced mixture of relatively new innovative strategies and conventional strategies used in the past, it is quite easy to integrate them in conventional classrooms. Casting language and translation issues aside, all the strategies are relatively easy to utilize and possess practical aspects, which ensures they can be applied both in Saudi Arabia and the US.

Assessment

The author included the following assessment techniques in the handbook:

1. Assessment via Questioning
2. Sample Questions on Fractions (Utilization of Mini Whiteboards and Reasoning Understanding)
3. Misconceptions and Error Analysis
4. Common Mistakes Examples
5. Evaluation of Fractions' Statements
6. Links outside the Classroom Setting
7. Classifying Fractions

8. Interpreting Fractions
9. Dealing with Remainders

All of these assessment techniques were retrieved from viable sources suitable for the middle school mathematics learner. The assessments are quite interactive in nature to ensure both the teacher and the learner are in sync with each other. This way the teacher can closely monitor the gradual development of the individual learners and point out vital areas for improvement. Moreover, each assessment has been structured in a manner that is easy to understand and follows for the benefit of the learners.

Conclusion

Overall, the author has formulated a comprehensive handbook for middle school teachers to utilize in the instruction of fraction content in Saudi Arabia. It is the author's wishes that middle school teachers, who have both academically weak and strong learners; might find appropriate aid from this handbook. In the creation of the handbook, the author has gained numerous ideas from former experiences and mixed portions of conventional instructional strategies and new innovative instructional strategies have been utilized in the creation of the handbook. This undertaking has been carried out with the purpose of serving as an essential tool for middle school learners and their teachers. Along with the diversity in teaching strategy plans, teachers must consider the most appropriate practice for each learner at all times, in addition to adopting the best means of assessing the learners' development. The provided tutoring strategies and relevant assessment techniques are simply the author's ideas, hence; the teachers must analyze and reflect on the learners individual development and interests in effectively utilizing the handbook.

CHAPTER IV

SUMMARY

The author has long noted a distinct need for improved and specific aids to Saudi mathematics teachers, in particular those teachers engaged in the teaching of fractions and fractional concepts. These are foundational ideas for students to comprehend as they move on to more complex math. The handbook which the author has created in designed specifically for these teachers. The author has formulated a comprehensive handbook applicable to middle school tutors in the tutoring of Saudi Arabian fraction content. As a result of the minimal material on fraction content in Saudi Arabia, there was the conducting of extensive research with significant help from the American resources, which have far more superior content when compared to the former. The author has made provision of important information on the given research that facilitates all practices before describing the manner that the same research is appropriately applicable in a conventional classroom setting. The handbook made provision for the Saudi Arabian fractions' curriculum content, and certain comparisons made with that of the US so as to ensure the user of the handbook has appropriate background study with respect to the subject content. Subsequently, eight strategies were analyzed from a several viable sources, which formed the handbook’s backbone. The author also made provision for appropriate assessment techniques so as to ensure a complete learning experience for all the given students being tutored, in addition to measuring their overall progress.

The handbook’s content was retrieved from reliable sources such as the official website for the National Council of Teachers of Mathematics. It was also necessary to highlight comparisons and contrasts between the Saudi Arabian curriculums and the one from the US so as to give the author considerable background information on the subject matter and help him write the handbook appropriately, in addition to making highlighting for the most effective strategies. Furthermore, an analysis of the standard age appropriate curriculum, and a review of the students’ mental, academic, and physical development offered him with the means of trusting any prior information on the subject matter that he already possessed. Strategies adopted by the author included Building on Basic Intuitive Understanding, Number Lines: An Essential Representational Tool, Concrete-Representational-Abstract Instructional Strategy, Proportional and Multiplicative Reasoning, Ratio, Proportion, Rate Problems, Student Think Aloud, Class Wide Peer Tutoring, and Direct Instruction. For each strategy, there was an introduction section, utilization section, and importance section. The provided strategies were deemed suitable for both the strong and weak learners in middle school. Since the reason for selecting these strategies entailed a mixture of innovative strategies and traditional strategies previously utilized, it is quite easy to integrate them in standard classrooms. Apart from language and translation issues, all the strategies are simple to apply and possess practical aspects that ensure they can be applied both in Saudi Arabia and the US.

The author included the following assessment techniques in the handbook: Assessment via Questioning, Sample Questions on Fractions (Utilization of Mini

Whiteboards and Reasoning Understanding), Misconceptions and Error Analysis, Common Mistakes Examples, Evaluation of Fractions' Statements, Links outside the Classroom Setting, Classifying Fractions, Interpreting Fractions, and Dealing with Remainders.

These assessment strategies were sourced from viable resources that are suitable for the learners of middle school mathematics. The assessments are considerably interactive in nature so as to make sure that both the tutor and the learner are in sync with each other. This will ensure that the tutor can monitor the gradual development of the individual learners closely and point out vital improvement areas. Moreover, each assessment has been structured in a manner that is easy to understand and follow for the benefit of the learners.

Conclusion

In general, mathematics is quite challenging to learn and teach. The outcome on student learning considering changing a given practice of teaching might be quite challenging to discern as a result of the teaching activities' effects and the context in play, which act simultaneously. Hence, teachers have to take note of the individual students' capabilities, in addition to the context of teaching as they attempt to enhance the effectiveness of their lessons through altering their defined instructional practices. In this case, it is seen that Saudi Arabia significantly lacks the resources that the US has in terms of library resources.

This project essentially targets an under-researched and significantly important area in the mathematics field, that is, fractions. Previous research in the given area of fractions has seemingly identified fractions as a certain skill, which middle school students usually repeatedly struggle with. Nevertheless, it is an essential skill for their overall academic advancement. One reason for middle school students' difficulty with fractions is that they apparently have a superficial understanding of the diverse concepts and hence are not really able to manipulate or apply fractional quantities. This project hopes to effectively bridge this gap by thoroughly targeting the middle school students' understanding of the concepts of fraction via a conceptual approach. Furthermore, very few studies have been conducted for the purpose of examining effective interventions to appropriately teach fractions. This project also hopes to effectively contribute to the available research base by thoroughly identifying relatively effective practices to teach the fraction problems to the middle school students struggling in mathematics.

Overall, the author has formulated a comprehensive handbook for middle school teachers to utilize in the tutoring of fraction content in Saudi Arabia. It is the author's wish that middle school tutors who have both academically weak and strong learners might find appropriate aid from this handbook. The author recognizes that simply the creation of this handbook does not, in itself, solve any problems. This in only the initial step towards improving the fractions instruction in Saudi Arabia's middle schools. Much work lies ahead, including contacting decision makers in the Saudi educational system, submitting this handbook for consideration, and making any potential modifications they might recommend or request. These are merely the beginning baby steps in this larger work to make fractions less of a challenge to schoolchildren in my home country.

In the creation of the handbook, author has gained numerous ideas from former experiences, with respect to both Saudi Arabia and the US. Mixed portions of conventional tutoring strategies and new innovative tutoring strategies have been utilized in the creation of the handbook. This undertaking has been carried out with the purpose of serving as an essential tool for middle school learners and their tutors. Along with the diversity in teaching strategy plans, tutors must consider the most appropriate practice for each learner at all times, in addition to adopting the best means of assessing the learners' development. The provided tutoring strategies and relevant assessment techniques are simply the author's ideas, hence; the tutors must analyze and reflect on the learners individual development and interests in effectively utilizing the handbook.

Recommendations

1. Further field studies must be done which might be beneficial in the development of a proper curriculum within a standard classroom.
2. Further field studies must be done to highlight feedback from middle school tutors who have students with learning disabilities in their classrooms.
3. Visual aids and presentations might be incorporated into the handbook in a bid to tutor the students in a more exciting and effective manner.
4. Any gifted middle school student has to be tracked into later years for tracking of their progress.
5. This handbook makes use of activities from certain publications. However, since there are many related books published frequently, it might be appropriate to fit the students' requirements with a number of their favourite books.

6. For each activity plan, it might be suitable to expand their ideas while simultaneously connecting other exercise areas to comply with the students' interests.
7. There might be the need to translate the given handbook into the native language in Saudi Arabia since it might be easier for the locals to understand it better.
8. The author mainly utilized American resources in the formulation of the book. However, it might be appropriate to borrow ideas from other countries for a more wholesome handbook.
9. The activity plans have given several suggestions on how to interact with the given students. Further additions must be made with regards to how to interact with students having learning disabilities.

It might be effective to review articles that do not necessarily target students in middle school so as to make provision for a more imaginative and exciting handbook.

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APPENDIX

EFFECTIVE STRATEGIES FOR TEACHING MIDDLE SCHOOL STUDENTS FRACTIONS: A HANDBOOK FOR TEACHERS IN SAUDI ARABIA

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CONTENTS

CHAPTER 1: INTRODUCTION

Approaches to the Study of Fractions

This handbook focuses on mathematics middle school teachers working with the corresponding middle school learners. It is not a conventional textbook, and neither is it a list of recipes that teaches specific aspects related to fractions. Instead, it recommends some approaches, which have been found to be effective from pertinent literature in enabling middle school students to understand fractions and subsequently link them to other mathematical aspects. It is essential that the students critically analyze the relationships between conventional fractions, as opposed to simply attempting to memorize facts and procedures (McLeod & Newmarch, 2006).

The introduction of fractions in middle school has to strongly emphasize the development of reasoning skills, the exploration of equivalence, and the comparison of fractional quantities. Most students might assume that, simply listening to their teacher and subsequently filling out their personal worksheets is the only means of learning. On the contrary, this handbook will show that, most students learn more when they honestly enjoy the given exercise at hand and are offered a proper chance to analyze and explain their work, in addition to attaining an understanding that is shared. Shared understanding in this case refers to the creation of new knowledge, which is influenced by collaboration and participation and attained via the replacement of individual knowledge with group knowledge. This ensures there is an alteration of individual perspectives in favour of a joint perspective, which is established via collective contributions. Currently, in the development of conceptual understanding, significant recognition has been given to the value of appropriate collaborative work (McLeod & Newmarch, 2006). Even though the activities, which are relayed in this handbook, might be done individually, a host of them tend to work more appropriately when taken as collaborative tasks. This strategy might be quite unfamiliar to most middle school students, especially the ones whose past experience in math was in a conventional classroom.

Generally, learning works best in the case where learners are working in a collaborative manner (McLeod & Newmarch, 2006). For example, the task might be pitched a bit higher, so that it is just outside the comfort zone of an individual learner such that, it requires a second opinion (McLeod & Newmarch, 2006). Moreover, this might entail the utilization of practical equipment, which requires additional aid. It is essential for a tutor to ensure his/her students understand the significance of collaborative learning, in addition to the conditions that are necessary for facilitating collaborative learning. This step must be taken before actual learning commences so that during the subsequent classes, all members of the defined groups have a proper opportunity to express a given opinion and subsequently challenge what other individuals say. With respect to this given context, the tutor is less of an instructor, but is an informed individual who asks appropriate questions for the purpose of moving discussions on. Such a teacher does not immediately confirm the accuracy of the answers but acts as a facilitator of learning. As a teacher, one might desire to spend significant time lending an ear to the given discussion in small defined groups, and might even chip in, but must never attempt to try to utilize such groups to replace the lectures of the whole class. This is because the groups are simply designed to be an efficient means of encouraging active learning and motivating students, in addition to facilitating the development of essential critical-thinking, decision-making, and communication skills. The group work is simply an aid to conventional lectures. After the completion of the assigned group works, it is essential for the tutor to take on a reflection session for all the groups combined to focus on the lessons learnt.

Considering the standard learning scenarios, a tutor must make some seemingly rash decisions concerned with their manner of reaction to certain scenarios, which develop, especially the ones in which a group agrees about a given aspect that is in actually incorrect (Wu, 2011). Standard learning scenarios in this case refer to the normal modes of learning with respect to the given level of learning. A comparison with other group's work where learners must justify their conclusions might be a better strategy for checking as compared to the simple validation of the teacher. It is seen that, collaborative learning scenarios actually tend to possess a common goal tied to the production of an end product, for example, a presentation to the given group, a poster, or even a questions' set retrieved for other students. The aspect of the given differences and similarities between certain posters from diverse groups might be a considerably effective means of addressing errors (McLeod & Newmarch, 2006).

Why Fractions?

Most middle school students acknowledge that the concept of 'fractions' is a math topic, which they deem to be quite challenging, regardless of the constant use of concepts related to sharing (McLeod & Newmarch, 2006). The essential question seeking to know why most middle school students perceive fractions to be this difficult is an aspect that teachers have to consider. A prime answer might be the standard means of fractions' notation. Alternatively, a prime reason might be the language utilized, which is often formal. Learners might be drowning in the traditional language of fractions at an early stage, even before they think about their properties. It is seen that learners might have the ability to label or draw fractions accurately, but fail to place them in the accurate size order, or utilize the fractions in the solving of problems. Furthermore, fractions might be quite confusing since most fractions do not necessarily act like the standard numbers. At times, they represent a given quantity, which can be visualized, while at other times a given operation. For example 8/13 might refer to a shape having 8 similar pieces and is shaded out of 13; it might refer to the given result of the division of 8 by 13, or even a section of a defined instruction to seek a value such as 3/4 of 16. Effective strategies to teaching fractions must give middle school learners a chance to appropriately explain the manner in which they view them, and the given teacher must consider this aspect in their approach (McLeod & Newmarch, 2006).

Aim of the Guide

The aim of this given handbook is to assist teachers in the overall teaching of fractions to the middle school students showcasing effective strategies that can assist them in doing so. The handbook’s intention is to support and complement the implementation of the standard curriculum of middle school fractions in Saudi Arabia, thus this is not a fractions’ textbook. Through the provision of additional guidance towards the teaching of fractions, this handbook attempts to effectively illuminate a certain pedagogical framework for the overall enhancement of mathematical thinking. This refers to the methods of supporting, eliciting, and extending standard higher-order mathematics skills like reasoning; expressing, and communicating; connecting and integrating; and finally problem solving and application (Wu, 2011).

Certain mathematical and related pedagogical aspects are worth mentioning. With regards to the mathematical front, the actual difficulty tied to the teaching of fractions seemingly lies in the given fact that the accurate definition of fractions as equivalence classes of relatively ordered integer pairs is unsuitable for tutoring at the middle school stage. Moreover, the formal definitions given to the arithmetic operations as the ones that are dictated by the given requirements of the axioms of the field are grossly improper for the overall instruction at this given level as a result of lack of motivation (Hu, 1998). With respect to the pedagogical front, inasmuch as this handbook is not a fractions' textbook, the level of the overall presentation is higher than is actually appropriate for a middle school class. This is essential in preparing the students for subsequent classes in later years, which involve learning o fractions.

Outline

a) Chapter 2: Middle school curriculum fractions content

b) Chapter 3: Strategies for teaching fractions effectively

c) Chapter 4: Assessment of student learning

CHAPTER 2: MIDDLE SCHOOL CURRICULUM FRACTIONS CONTENT

Curriculum

Focal Points

Based on the NCTM Curriculum Focal Points, there are nine conventional focal points for the middle school learners (Milgram, 2010). It must be acknowledged that the recommendation of the NCTM is that at least sixty percent of the overall instruction time in the middle school level, and preferably more, must be significantly devoted to these given topics. Six of these given topics involve the fractions as their fundamental and central focus (the last three cannot be developed appropriately without the given students having an appropriate mastery level of considerably understanding the fractions):

a) The development of fluency with and an understanding of division and multiplication of decimals and fractions

b) The development of an understanding of and the application of proportionality, inclusive of similarity

c) The development of an understanding of the given operations on all conventional rational numbers and the overall solving of linear equations

d) Connecting rate and ratio to division and multiplication

e) Writing, interpreting, and the utilization of mathematical equations and expressions

f) The representation and analysis of linear functions and the solving of linear systems and linear equations

g) The analysis of two and three-dimensional space, in addition to figures by utilizing angle and distance
h) The development of an understanding of formulas and utilizing them in the determination of volumes and surface areas of standard three-dimensional shapes
i) An analysis and summary of data sets (Milgram, 2010).

Introducing the Fractions

An initial place to commence from might be through asking the given students to critically analyze some fractions, which they have dealt with previously with respect to their daily context. This forms a means of gathering a host of accurate definitions of the term 'fraction.' It is seen that, some fraction terms and language are utilized in normal parlance, at times with seemingly diverse definitions, or even reduced accuracy; this might be explored appropriately (Wu, 2011). For instance, a standard "fraction" might at times refer to "only a tiny section" of a given thing; as in a traditional tutor only gets a sizeable fraction of the salary of a professional footballer (American Institutes for Research, 2007). (A relatively appropriate question might be to ask the given students if they perceive that 1/4 of a million might be considered as a small number or alternatively, a big number). Prior to tackling the actual written symbolic versions of any given fractions, some of them might be named and their meanings given. Some questions can be given concerning fractions and encouragement given to students to critically analyze individual questions. For example:

a) When are fractions utilized by individuals?
b) Do the fractions really matter when they are just relayed in small parts?

c) Can a given fraction be bigger as compared to a single whole unit?

d) Is a half always necessarily the same size?

e) Is it relatively possible to have thee given halves?

f) Do fractions directly correlate to division?

Teaching Points

This section gives essential teaching points that can facilitate easy learning of fractions as given by the National Council of Teachers of Mathematics (NCTM, 2006).

a) Learners have to acknowledge that one means of analyzing a given fraction is to view it as the overall answer of the equal division of the top numeral into the subsequent numeral of the given value represented by the bottom numeral.

b) An essential aspect is that the given denominator relays the number of equal sections that a given number has subsequently been divided into. It is seen that the numerator the number of those parts that are actually present.

c) Learners must be significantly familiar with the given diverse fractional representations given, and must constantly be offered more than a single representation, which might be comprised of; number lines, area diagrams that utilize numerous diverse shapes, decimal equivalents, words, symbols, and percentages, in addition to fractions as a division resultant.

d) Standard pictorial representations relaying a specific fraction might take the form of diverse sizes and diverse shapes. For instance, one must not constantly utilize shaded areas of given circles, and tiny discussions can subsequently be undertaken from the representation of half of a little square and a quarter of an enormous square and subsequently querying which among them is the larger fraction.

e) Compare diverse denominators with conventional unit fractions (for example, 1/13, 1/14, 1/15).

f) Compare a single denominator with the given fractions (for example, 1/15, 2/15, 3/15).

g) Encourage diverse approaches so as to decide whether a given fraction is relatively less as compared to a half, or equal to half, or alternatively more than half.

h) Critically analyze the apparent discrepancies between a whole unit and a unit fraction (for example, 1 – 1/14 or 1 – 1/15).

i) Properly prove that a defined fraction might be appropriately represented in a certain infinite numeral of seemingly equivalent forms.

j) Prove that the given fractions might actually be equal to, or even more enormous than a single whole.

Fraction Content in Saudi Arabia's Middle School with Sample Problems

Fraction Definition

A fraction can be described as a given point on the conventional or standard number line, which is represented by a part-whole relationship whereby, the unit segment (0, 1) is a whole (NCTM, 2006). In much simpler terms, a fraction refers to a part of a given whole or in general terms, any given number of parts that are equal. It entails the number of parts, which a certain size has. It must also be noted that fractions can be written without necessarily utilizing explicit denominators or numerators but by simply using percent sings, decimals or even negative components (1%, 0.01, or 10^{-2} respectively) (NCTM, 2006). Further terms utilized in the fraction definition are given below:

a) Proper Fractions: These fractions have a numerator that is lesser as compared to the denominator (Hatfield, et.al. 2000), for example, 5/6. Additionally, a proper fraction refers to the denominator having a higher degree as compared to the numerator.

b) Improper Fractions: These are fractions whose numerator is greater as compared to the denominator (Hatfield, et.al. 2000), for example, 6/5. Additionally, an improper fraction refers to the denominator having a lower degree as compared to the numerator.

c) Mixed Fractions: these fractions are comprised of a whole number, in addition to a fraction (Hatfield, et.al. 2000), for example, $8\,\frac{5}{6}$.

Compare and Order Fraction

Comparison and ordering enables one to know the size of a fraction among a given group. There are two essential means of achieving this; utilizing decimals or utilizing the same denominator or utilizing a number line (CDE, 2011).

1) Utilizing decimals

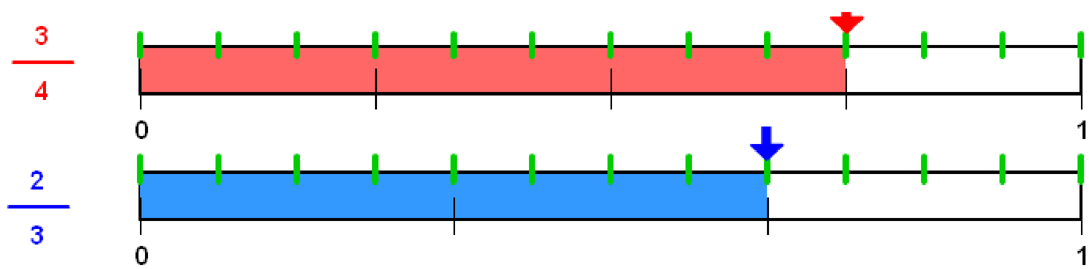
In this case, each fraction is converted into a decimal and a direct comparison made (Milgram, 2010). For example, which is bigger: $\frac{5}{6}$ or $\frac{3}{4}$. In this case, 5/6 yields 0.8333 while ¾ yields 0.750, hence $\frac{5}{6}$ is larger.

2) Utilizing the same denominator

When two fractions are characterized by the same denominator, then comparison becomes easy, for example, $\frac{3}{6}$ is less than $\frac{5}{6}$ since 3 is less than 5.

1) Utilizing a number line

In this case, the fractions to be compared are placed on a number line and a visual decision made on their relative sizes. For example, if a comparison is made for 2/3 and ¾, the numbers can be placed on a number line as shown below.



From the number lines, it is clear that 2/3 is much smaller in comparison with ¾.

Fraction Operations

Multiplication

The multiplication of fractions is conventionally done through the multiplication of the numerators followed by the multiplication of the denominators (Milgram, 2010). For example,

$\frac{2}{3} * \frac{5}{6} = \frac{2 * 5}{3 * 6} = \frac{10}{18}$.

Sometimes, there is the need for cancelling of the result so as to attain a simplified fraction. In this case, for the answer $\frac{10}{18}$, both the numerator and the denominator are divided by 2 so as to yield $\frac{5}{9}$.

Division

The division of a fraction is conventionally done via inverting it followed by multiplication (Milgram, 2010). For example,

$\frac{2}{5} / \frac{3}{9} = \frac{2}{5} * \frac{9}{3} = \frac{18}{15}$.

Sum

The addition of fractions can be carried out if the denominators have the same value by the simple addition of the numerators (Milgram, 2010). For example,

$$\frac{2}{9} + \frac{3}{9} = \frac{5}{9}$$

In case of different denominators, then the determination of the lowest common denominator (LCD) must be done to ensure the expression of each fraction in the corresponding equivalent form (Milgram, 2010). For example,

$$\frac{2}{5} + \frac{4}{9} = \frac{18}{45} + \frac{20}{45} = \frac{38}{45}$$

Subtract

Subtraction is quite similar to the addition operations with regards to similar denominators or the utilization of the LCD (Milgram, 2010). For example

$$\frac{5}{9} - \frac{2}{9} = \frac{3}{9}$$

Proportionality

Proportionality refers to the relationship between defined two quantities. It is utilized for comparison purposes in the expression of fractions (Math Open Reference, 2009). For example, when comparing the quantity of orange beads to the defined quantity of green beads, it would be orange beads / green beads. The term "**to"** is quite significant in this case here since the thing that comes before is the numerator of the given fraction while the thing that comes after it is the given denominator (Math Open Reference, 2009).

Percentage

A percentage refers to a means of expressing a fraction or a proportion as a whole number (Math Open Reference, 2009). For example, a number like "65%" (65 percent) is the shorthand for the basic fraction 65/100

Fractions and Percentage

The conversion of a fraction to a basic form of a percent is quite easy. For example, convert 6/9 to a percentage. Firstly, there is the division part; 6 ÷ 9 = 0.667 followed by its multiplication by 100 so as to yield 66.7 %.

Conceptual Nature of Fractions' Operations

Mathematical knowledge gained during schooling is a prime predictor of occupational and academic success during a child's later years. The mastering of all the aforementioned fractional operations is significant for later success regardless of the apparent controlling of other demographic and cognitive variables in the students. Apart from occupational and educational success, fractions are essential for numerical development theories. Moreover, the appropriate understanding of fractions entails the recognition that most aspects of natural numbers are not necessarily the aspect of numbers in general.

Comparison of Fraction Content between the U.S.A and Saudi Arabia

The fraction content between the U.S.A and Saudi Arabia is quite similar. They both focus on the same concepts with respect to topics such as comparison and order fractions, fraction operations (addition, multiplication, subtraction, and division), percentage, and proportionality among others. However, there are several aspects that differentiate the two countries (Perkins & Flores, 2002). For starters, the performance related to fractions with respect to middle school students in Saudi Arabia is quite poor. This is mainly because of limited resources that can offer effective strategies that can help in realizing good performance. This is mainly because of limited resources that can offer effective strategies that can help in realizing good performance as compared to the US (Alhammad, 2010). These resources include mathematical journals, textbooks, and other academic resources. The US has better resources of this type since it has older and more publications as compared to Saudi Arabia, which is still relatively young on this front. Secondly, there are variations in the notations utilized between the two countries as shown in the following table:

US	**Saudi Arabia**	
$\frac{3}{4} \div \frac{1}{8} =$ $\frac{3}{\cancel{4}_{1}} \times \frac{\cancel{8}^{2}}{1} = 6$	$\frac{3}{4} \div \frac{1}{8} =$ $\frac{24}{4} = 6$	• In the U.S. the division of fractions entails inverting of the second fraction before multiplication. • In Saudi Arabia, the division of fractions entails cross-multiplication (Perkins & Flores, 2002)
$\frac{5}{12} + \frac{3}{18} =$ $12 = 3 \times 2 \times 2$ $18 = 2 \times 3 \times 3$ $LCM = 2^2 \times 3^2$ $= 4 \times 9$ $= 36$	12 18 | 2 (Common factor of 12 and 18) 6 9 | 3 (Common factor of 12 and 18) 2 3 | 2 (Common factor of 12) 1 3 | 3 (Common factor of 18) $LCM = 2 \times 3 \times 2 \times 3 = 36$	• In the U.S. determination of the LCM entails the prime factorization method. • In Saudi Arabia, to get common denominators, there is the decomposition of the denominators primes. Provision of the LCM is via the multiplication of the available common prime factors in addition to the given prime factors, which subsequently are present in at least one denominator (Perkins & Flores, 2002)

CHAPTER 3: STRATEGIES FOR TEACHING FRACTIONS EFFECTIVELY

Strategy 1: Building on Basic Intuitive Understanding

Introduction

This strategy dwells on the significance of helping middle school students understand basic fraction concepts via building on their individual intuitive knowledge of sharing (Carpenter, 2011). It relays the manner in which learners initially solve problems such as sharing twelve cupcakes among three given plates by a direct representation of the sharing action by utilizing counters or drawings. Extending these given sharing ideas, learners next solve problems such that one or more of the given objects to be shared requires to be partitioned, for example, two children that share three cupcakes. Initially the learners solve fraction sharing problems through halving, but eventually, it is seen that they learn to solve the sharing problems, which cannot be solved via repeated halving, for example, three children that share five cakes. It is seen that, sharing also is utilized in the development of the central concept of standard equivalence of fractions via comparing diverse sharing arrangements. This strategy encourages teachers to allow the middle school learners to utilize their informal understanding of conventional sharing so as to explore diverse problems and subsequently explain their solutions by utilizing a variety of defined representations (Carpenter, 2011).

Utilization

A teacher starts with a basic division problem, which younger students can solve. For example, the following question can be posed: “I just baked twelve cupcakes, and I have to put them on three given plates. Then, how many cupcakes will each plate hold?" The students might draw some form of representation of the given plates or alternatively, they might utilize some other means of representing the given plates. Essentially, they will say that, "each plate has to hold an equal number of cupcakes, so I will deal the given counters to the plate individually." In this case, it is seen that the students do not solve the problem because they have initially been shown the means of solving the problem; this is rather an intuitive solutions. This might show an example of how powerful this strategy could be with respect to it being more effective to actually pose the problems and let the students come up with their individual means of solving them.

Older children: sharing might also be utilized with middle school children to further develop the equivalence idea. Middle school students might not necessarily draw pictures so as to represent the defined problems; but they can utilize numerical representation of the given problems (Carpenter, 2011). For example, consider 24 children that have gone to a pizza parlour and subsequently ordered 16 pizzas. The question is, "describe how these children can sit in diverse table arrangements such that they are all similar amounts of pizza?" in this case, one possibility is that all the 24 children sit at a single table and then share the given 16 pizzas. However, another possibility is the division of the table such that there are two tables having 12 children with 8 pizzas on each given table. Alternatively, the tables can be divided further to accommodate six children sharing four pizzas, in which three children are actually sharing two pizzas. Considering each of these situations, they are essentially representations of standard equivalent fractions and overly linked into the given ideas of sharing (Carpenter, 2011).

Importance

This strategy builds upon the students' intuitive knowledge of conventional sharing so as to develop the fraction concepts. The strategy's overarching theme is that effective instruction is effectively grounded in the development of understanding. The actual defining feature of appropriate learning with understanding is that conventional knowledge is connected. Specifically, it is significant that knowledge is seemingly connected to the given things that the learners already understand (Carpenter, 2011).

From these examples, it is seen that the students solve problems through certain ways that make actual sense to them since they have some level of intuitive knowledge of conventional sharing. In this strategy, the teacher poses fraction problems with respect to sharing situations that the given learners are allowed to solve through ways that they seemingly devise for themselves. A teacher can pose the problems and subsequently steer clear and let the students solve them and subsequently build on the given knowledge, which they bring to the instruction.

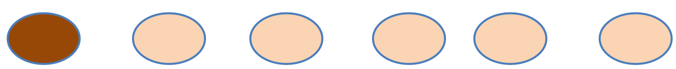
Strategy 2: Number Lines: An Essential Representational Tool

Introduction

This strategy commences with the forms of misconceptions, which children have concerning fractions, mostly the ones related to not appropriately understanding fractions as standard numbers. It must be noted that fractions are numerals, which inflate the given number representation via adding more precise measure units (Okamoto, 2011). This handbook will relay examples that showcase why part-whole area models might be confusing to some of the students, and also relay problems, which might lead students to assume all fractions consist of whole numbers. The handbook recommends the utilization of number lines as a considerably robust representational model, in addition to demonstrating the manner in which to utilize the number line for the addition of fractions. In this handbook, measurement activities are modelled with strips of paper of diverse lengths while there is also a demonstration of utilizing parallel number lines that are portioned into diverse fractional parts so as to aid students learn equivalents.

Utilization

Consider an example of a picture of six cookies in which one of them is a chocolate cookie while the others are raisin cookies.



In this case, the task is to accurately write a fraction showing the relation of the given chocolate cookie with respect to the other cookies. Fractions themselves are standard numbers having magnitudes, which extend the conventional number system. It is quite challenging to see this aspect in this example. Moreover, this example fails to offer a good sense of the given unit of measure. It is not clear if the question refers to the single cookie or if it refers to the entire cookies' set.

In this given example, the teacher has to show the students the meaning of adding two fractions, 1/3 and 1/3 by utilizing the part-whole approach. Firstly, one can shade 1/3 and shade another 1/3 before coming up with the answer, 2/3. This is a relatively proper means of utilizing a part-whole approach. However, students might incorrectly interpret this situation. They might add the given numerators together and similarly, add the given denominators together before coming up with 2/6. Therefore, this handbook strongly recommends that all given teachers utilize number lines, as conventional representational tools to aid the students understand essential fraction concepts.

Measurement activities are considerably useful for this given purpose. Consider the use of some strips of paper. By utilizing of fraction strips, it is easy to relay how many paper strips are required so as to measure diverse things. For example, one can measure a pencil by utilizing these paper strips. If the given pencil is longer than one given strip but shorter than two given strips, the problem lies on how to express the extra amount. The class can actively discuss diverse means of for doing this. Folding of the strip works in this case. Subsequently, the teacher can effectively talk about the emergent notion of 1 & 1/2 strips. During the conduction of this sort of activity, it is essential that the teacher utilizes strips of diverse length representing diverse units of measure. The teacher can query the students to measure a defined object by utilizing different strips. The overall length of the given object might be expressed differently, say, 1 & 1/2 strips in such a case.

This form of activity can effectively aid students in becoming aware of the significance of measurement units. The given number line is also quite useful in adding the students understand the actual meaning of adding fractions. The addition of standard fractions such as 1/3 + 1/3 might be introduced by utilizing the number line. In this case, the number line is effectively partitioned into three parts, such that the first 1/3 is marked here, while another 1/3 is added to come up with two-thirds as the answer.

Importance

Through the partitioning of a given number line in a repeated manner, students realize the presence of an infinite value of numerals between any given adjacent whole numbers. It is seen that this is a very significant notion that is termed as fraction density. The number lines are also quite useful in aiding students in the translation among diverse notations of fractions, for example, percents, and decimals. Number lines convey significant fractions' properties that other methods like the part-whole method do not (Okamoto, 2011).

Strategy 3: Concrete-Representational-Abstract Instructional Strategy (CRA)

Introduction

This is an effective means for appropriate mathematics instruction, which can considerably improve the overall performance of math for the middle school learners having learning disabilities. CRA is a conventional instructional strategy, in which there are three parts that aim to build on the defined instruction that was previous so as to promote proper learning of the student and subsequent grasping of the information, in addition to addressing the apparent conceptual information (American Institutes for Research, 2007). The strategy entails three essential stages.

a) Concrete: In this given stage, the teacher commences instruction via the structuring of each given fractional concept with specific concrete items (for example, purple and yellow chips, pattern blocks, base-ten blocks, cubes, and geometric figures fraction bars,).
b) Representational: In this given step, the teacher essentially changes the defined concrete aspect and turns it to a defined representational (also referred to as semi concrete) level. This might involve the drawing of pictures; utilizing dots, circles,

and tallies; or even utilizing stamps for the purpose of imprinting pictures for counting.

c) Abstract: In this given step, the teacher essentially models the defined fractional aspect at a relatively symbolic level, simply utilizing notations, fractional symbols, and numbers, so as to relay the given number of circles or defined circle groups. The given teacher utilizes operation symbols (such as +, –) so as to indicate addition, division, or even multiplication (American Institutes for Research, 2007).

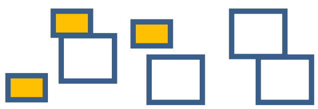
Utilization

Once the defined fraction aspects are effectively known as being "a section of a defined whole," then the middle school learners can properly practice the stages that are entailed in fractions' reading and writing. A diverse host of physical materials might be utilized so as to display the accurate representation of a given fraction as being "a section of a whole." For instance, counters, geometric shapes, fraction cubes, or fraction bars might indicate a given fraction (such as 3 purple cubes/ section out of the 5 given cubes). Numeric symbols and representations of the fraction might develop the necessary reading skills and proper fractions' writing. The abstract step is essentially formulated via relaying in accurate fraction form, a certain numeric symbol of the parts of the given whole. This given stage entails the manner in which certain values must either be written or read. Considering a standard fraction, write down the number that is usually placed at the top? Moreover, write down the number that is usually placed on the bottom? A prime example of either writing or reading the fractions in the accurate manner is as shown below.

Writing and Reading of Fractions

Aim: Kinesthetic and visual development of the given spatial organization, in addition to the accurate writing and reading of fractions

Materials: Orange squares and larger grey squares are displayed so as to aid with the placement and sequencing of numbers



= Part/whole

Tutor: “The lesson today will entail the writing and saying out loud of fractions”

Concrete: The tutor is to point out pre-arranged squares on a given table. “Define the colors in the given squares.” (The student gives the answer as orange and grey). “State the total number of whole squares.” (The student points out and subsequently counts up to eight. He/she then states 8.) “what is number of the orange squares?” (The student points out and subsequently counts to 3. He/she then states 3.)

Represent: “When discussing fractions, we state the ‘part of the whole.’ (Together, say ‘part of the whole’). Subsequently, we can note down a fraction that relays the part of the given whole, as highlighted above. The numeral for the part is displayed on the top while the numeral for the whole is displayed on the bottom. (State the words whole on bottom and part on top.) Now, how many squares in total were there?” (Student states 8.) “Let us refer to that as the whole.”

Abstract: “Note down the whole or the overall number of squares on the bottom where ‘whole’ is displayed.” (Student notes down 8.)

Represent: “States the number of orange squares?” (Student states 3.) “Let us refer to the orange squares as ‘part’ of the given whole.”

Abstract: “Note down the number of orange squares displayed on the top in which the word ‘part’ is displayed. (Student notes down 3.)

Summary: “As shown by the example, state what you wrote for the given fraction?” (Student states three and eight.) “We state 3 out of 8.” Practice diverse examples with the given squares, reading and writing the fractions (American Institutes for Research, 2007, p. 2)

The middle school students might also face difficulty in the interpretation of written decimals and accurately adding hundredth, tenth, or other given decimal terms. A relatively appropriate method that can aid the learner in becoming independent functionally with the appropriate decimals’ representation is to accurately display decimals with certain visual aids and subsequently link them to related familiar notation of the fractions. Consequently, the following figure effectively displays a method that can aid learners with certain learning disabilities to properly write and read decimals via linking them to standard fractions. The students must have a relatively firm acknowledgement of any fraction as a section of a defined whole and have the ability to read and properly write fractions. Furthermore, students must possess extensive previous experience in dealing with graph paper and blocks (American Institutes for Research, 2007).

Importance

CRA might be duly implemented for individual purposes, small group purposes, or even for the entire class. When utilizing it, the teacher must make provision for multiple opportunities for demonstration and practice so as to aid the students in the achievement of mastery of the defined mathematical concept. A teacher can actually prompt learners with certain tasks at each given step of the overall practice. If a learner is actively solving a defined problem, then the tutor can subsequently read it out aloud and then summarize whatever the given student completed as he/ she moves in a sequential manner through the defined stages, utilizing drawings, numerical representations, models, and verbalization, in a bid to show each stage in appropriate order. During the implementation of this approach, a teacher essentially practices appropriate instruction via referring to certain aspects or specific actions in the diverse forms. So as to reinforce these aspects, the given instruction might be cyclic, and not necessarily just a linear sequence of standard instructional tasks (American Institutes for Research, 2007).

Strategy 4: Proportional and Multiplicative Reasoning

Introduction

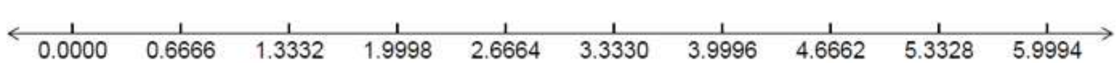
According to Sowder et al., significant step of development for any given student is the stage between additive reasoning and multiplicative reasoning type (1998). For the purpose of scaffolding this given transition, tutors must effectively know the apparent diversities between multiplicative reasoning and additive reasoning (Sowder et al., 1998). This is because; being a teacher entails a highly organized and extensive knowledge body. Tutors must possess an intimate knowledge of the given multiplicative reasoning to enable the explain difficult ideas of conceptually diverse multiplicative concepts, effectively adapt their given instruction for the numerous ability levels and learning styles, and finally, provide students with appropriate contexts to reason in a multiplicative manner without heavily relying on the given additive approaches. The comparison type that students tend to make so as to explain the quantity changes that essentially represent the discrepancies that are apparent between the reasoning of additive type and reasoning of multiplicative type.

Utilization

During additive comparison, emphasis is laid on the apparent difference between the given quantities while for the multiplicative comparisons, emphasis is laid on the given change rate. Considering additive reasoning, learners will tend to understand the apparent change in quantities, for example, how many more, that leads to retrieving the given difference, even when the multiplicative form of comparison is needed. On the other hand, students who utilize multiplicative reasoning can point out the differences between any scenarios, which entail additive transformations or transformations that are multiplicative in nature. Even though additive reasoning might effectively form in an intuitive manner, according to Sowder et al., (1998) multiplicative reasoning is quite challenging for learners to effectively develop and usually demands an instruction that is of a formal type.

Additive reasoning can be applied in the solving of the following problem: A muffin recipe entails the use of ⅔ of a cup of milk. In this case, each recipe results in the creation of 12 muffins. Therefore, find out the number of muffins that can be created through the use of 6 cups of milk. In additive reasoning, there is the repeated addition of 2/3 nine times through the use of six cups before adding 12 nine times as shown in the number line below.

2/3 is approximately equal to 0.66666, hence to get the number of recipes;



This gives six recipes. To get the number of muffins, 12 is added 6 times, hence;

This gives 108 muffins

Additionally, the tutor can challenge the students by replacing 6 cups with seven. In this case, there will be a remainder of 1/3. The students will then have to figure out what 1/3 would yield. In this case, it would yield can extra 6 muffins.

Hence, it is important for the teacher to consider the following when teaching fractions to their given students.

1. Know how the given students develop their form of additive reasoning. This entails having a clear understanding of the manner in which the student perceives additive reasoning and uses it in the solving of problems.
2. Strongly capitalize on the given learners' prior information of the related additive concepts, for example, use of the number line to solve problems for the purpose of strengthening the additive reasoning of the students.
3. Provide the given learners with appropriate learning experiences and learning contexts in a bid to prep them for other given higher-level fractions' topics.

Importance

Since multiplicative reasoning effectively enhances understanding of standard proportions and fractions, it is seen that fraction instruction also has to promote the appropriate form of multiplicative reasoning (Vanhille & Baroody, 2002). For instance, with respect to multiplicative reasoning and additive reasoning of fractions, for a teacher, the latter is frequently inadequate. The term 'additive' means of understanding fractions entails considering q / w to be q defined parts out of w defined parts. This will subsequently lead learners to perceive the numerator as being inclusive of the given denominator, and subsequently makes it challenging for them to accurately understand fractions like 42/32 since 42 is not actually included in 32. On the other hand, multiplicative understanding of standard fractions relies on considering two values as being measured in units that relative to each other. For instance, $Q = (w / e) \times V$ essentially means that "Q is actually w / e as large as the value V" or alternatively, V is actually e / w as large as the value Q (Yetkiner & Capraro, 2009). This form of explanation does not necessarily mean that there is any inclusion of the given numerator in the defined denominator and makes provision for a better form of understanding of the given fractions (Thompson & Saldanha).

Moreover, tutors can focus the given learners' attention on standard multiplicative reasoning when they dwell on basic fraction equivalency. It is seen that fractions often involve within and between-multiplicative relations. The term within-multiplicative relation essentially points out to the given relation found between the denominator and numerator of a fraction (Vanhille & Baroody, 2002). For instance, in 33/44, the defined numerator 33, is actually 33/44 of the defined denominator 44 ($33= 33/44 \times 44$). While the defined denominator 44, is actually 44/33 of the value 33 ($44= 44/33 \times 33$) (Yetkiner & Capraro, 2009). All fractions that are equivalent effectively share similar relations of within-fraction multiplication. However, the relation of between-multiplication essentially refers to the apparent relation between the defined numerators and the defined denominators of seemingly equivalent fractions. If two defined fractions are equal, it is seen that the ratio found between the given numerators is similar as compared to the ratio found between the given denominators.

Strategy 5: Ratio, Proportion, Rate Problems

Introduction

This strategy was established by Lewis and describes a defined set of problems, which involves the ratio between the water and the orange juice concentrate (2010). The strategy might require a ratio table having diverse water and concentrate amounts so as to aid in addressing varied questions on several topics such as the variations in the actual total quantity to be created. For a standard recipe that entails water and flour, Lewis relays how to establish a ratio table so as to figure out diverse proportions of each given ingredient. Based on Lewis's practical method of teaching ratios, teachers can adopt it for the middle school students. This strategy relays the significance of working out a given problem in more than a single way.

The ratio table below shows how comparison can be done for flour and water.

Flour	Water
2	6
3	9
5	15
6	18
10	X

Utilization

A teacher can adopt the following practical teaching.

Making Orange Juice

Most individuals buy orange juice via buying standard frozen concentrate. Often, the instructions require one can of the given concentrate and three similar cans of the given water. The teacher could vary the question by asking how much water is required to create enough orange juice if we used four cans of the given concentrate? Alternatively, if one commences with 15 cans of the water, then how much concentrate must be added to the water? (Lewis, 2010).

Recipe

Consider a recipe that requires three cups of flour for every one, in addition to one and one-quarter similar cups of water. One might commence in analyzing the given situation by building a standard ratio table. Therefore, one can log in three standard cups of flour, in addition to one and a quarter similar cups of water. If one makes it again, then that is six standard cups of the flour, added to two and a half similar cups of water. Then there are nine standard cups of the flour, in addition to three and three-quarter similar cups of water. If by chance one is asked about the five cups of water that matches up with the 15 standard cups of flour, then that would show up in the designed ratio table. However, if one required utilizing ten similar cups of flour, then the ratio table might not automatically relay the accurate answer (Lewis, 2010)

This requires the adaptation of the ratio table by doing what can be termed as "going by way of one" so as to put in one here. This statement will force the students to think. There has been the division of three by three so as to get one. Following up, there can be the division of one and a quarter, or alternatively five-fourths, by three so as to get five-twelfths. Subsequently, if this is a multiple of the value ten, then to get to here, it is needed to want to multiply by ten so as to decide the value that goes right there. In this case, 5/15 x 10 = 50/15 = $3\frac{1}{3}$. By utilizing the ratio table, one will be working with standard proportions. A similar action can be done with the standard number line (Lewis 2010).

Importance

The strategy aids the students in seeing a single problem worked in a variety of ways. Subsequently, they gain a sense of which approach works best for a certain problem and suits their individual understanding. This strategy shows the true nature of mathematics, which entails reasoning and judgment, thus a student with more experience will eventually do a better job of effectively picking the best tool out of a defined toolbox.

Strategy 6: Student Think Aloud

Introduction

"Thinking aloud" entails the student talking through the given decisions, details, and the reasoning behind the same. The strategy ensures the students verbalize the knowledge that they already have so as to aid them in reflecting upon a given problem in a bid to clarify it and focus on it. Struggling students can benefit from slowing down the defined process, since it affords them time to comprehend the problem fully (Power-up, 2015).

Utilization

Step 1: Provision of Clear Instruction

1. The students must be asked to reveal how they attain their answer. Students must be assured that the teacher will assist them in the "thinking aloud" process and help the students further explain their individual reasoning (Power-up, 2015).
2. While a teacher demonstrates the solving of the given problems, it is vital to properly model the thinking aloud process via stating explicitly what it actually means to explain your individual reasoning. Include any given decisions you make, regardless of its relevance. Utilization of technology resources, such as tables and I-pads so as to support note taking, in addition to creating visualizations of the essential ideas as one work towards giving the students a much more clearer understanding of the given thinking process is also vital (Cangelosi, 2002).
3. Appropriate guiding questions that can aid the students in focusing on their individual reasoning as opposed to just the solution must be asked, regardless of the accuracy of the provided answer. The students must fully explain the reason behind their choices or how their calculations were obtained. It must be emphasized that the whole decision process is an essential section of the overall mathematical reasoning. Through the highlighting of the process as a larger part of the overall thinking aloud, one aids the students in flexing and strengthening their individual reasoning skills (Cangelosi, 2002).

Sample Guiding Questions

State why chose that given operation/ number

State how you found that numeral

Who did it in a different manner?

Explain the path to getting your answer

Initially, you stated that ……………………….. What changed your mind? (Power-up, 2015, p. 1)

Step 2: Provision of Models and Strategies to the given Students

1 The students should be offered a series of prompts—sentence starters or questions —to guide them throughout the thinking aloud process. It has to be ensured that one includes questions, which require them to properly justify their given decisions.

Sample Prompts for the Process of Thinking Aloud

One aspect I can attempt is…….

I want to attempt……..because…….

I know that………

I am attempting to figure out……. (Power-up, 2015, p. 1)

2 The students can utilize diagrams and models so as to support their individual thinking. These visual supports might help them figure out if their individual thinking is faulty (Cangelosi, 2002). The supports also ensure that the teacher can aid the students to pinpoint the location and manner of their error in thinking. Students can employ supporting technology so as to improve their note taking and visualizations.

Step 3: Provide Ongoing Formative Assessment

When a given student engages in the thinking aloud visualizations, the teacher can invite a peer to carefully listen and subsequently comment on the given content while the teacher concentrates on the given student's utilization of the strategy. With struggling students, it is appropriate to establish a small group interactions that will give the students a chance to hear others prior to their sharing of their thinking (Power-up, 2015).

Importance

Thinking aloud aids the students in learning how to effectively reason through focusing on their individual thinking. Most students are unaccustomed to answering questions, which demand in excess of a single-word response or even a short-phrase response. Additionally, they might not know how to appropriately talk about the given mathematics problems or even explain their thinking. Therefore, this strategy makes provision for a strong starting point with regards to differentiating instruction.

Strategy 7: Class Wide Peer Tutoring (CWPT)

Introduction

CWPT refers to an instructional approach, which is designed to teach certain lessons to the middle school learners effectively with different skill levels (Terry, 2008). In simpler terms, CWPT refers to group work in which the given learners work collaboratively so as to understand a defined information set. This strategy utilizes several defined instructional components together, inclusive of partner pairing, immediate error correction, systematic coverage of content, team competition, point earning, and frequent testing (Greenwood et al., 1997). Each given student is actively involved in the overall learning process, which enables them to properly practice conventional skills in a fun way and systematic manner. However, the difference between CWPT and standard group work is that the former incorporates tutoring of each other while the latter is a simple discussion with no decisions made on who is right or who is wrong.

Utilization

CWPT's objective is for students to learn the presented weekly information appropriately in a bid to demonstrate their given understanding of the given information on subsequent tests (Martel, 2009). The given students will measure success through their assessments' scores. The strategy can incorporate a class game format; such that a given student can know of his/her success level via the earned points both individually and via their team. Firstly, a teacher has to utilize pre-tests so as to measure the given students' information knowledge to be taught in the preceding week. According to Greenwood et al., normally, information would be quite low (for example, 19 - 41% accurate) with respect to the pre-test and subsequently rise to approximately 90-100% correct (on average) with respect to the given post-test (1997). Considering the defined pre-test relays that items are quite easy or alternatively too challenging, there has to be a modification in the list. In this strategy there can be a division of a class into two similar groups. Then the learners in each given team can be paired with an appropriate student from the same team in any given the week. The process can be formulated in a random manner or via considering a student's skill level (Greenwood et al., 1997).

Once the pairing exercise is complete, each given partner will take a turn so as to tutor the other partner on a defined fractions' exercise. The tutors will then offer points for accurate answers, while simultaneously recording and correcting any given errors. Additionally, the tutor can make provision for 'extra points' so as to reward learners for appropriate and proper behaviour. The teams subsequently compete for marks and appropriate reinforcement in the social scenario. In this strategy, the learners spend around half an hour daily for five days, vividly focusing in teaching with the defined weekly lesson. Then the sixth day is utilized for pretesting and assessment for the oncoming weekly lesson. During the first ten minutes of each defined daily lesson, a defined learner has to partake on the teacher role while the other as the defined tutee (Martel, 2009).

During the next ten minutes, there will be an exchange of roles. The teacher is ultimately responsible for the presentation of each aspect on a defined teaching list on a weekly basis. Two points can be given for accurate responses. If the given tutee givens an inaccurate response, then the given teacher can make a prompt correction then later on in the lesson, he/she can ensures the tutee has a second chance to accurately respond and essentially practice the expected answer. At this juncture, in case the given learner’s answer is accurate, then the tutee earns a single mark. If still inaccurate, zero points are given for that given item. If either member has a question, he/she has to seek teacher assistance. In the course of the partner work, the teacher must tour the classroom, and subsequently award extra points for suitable and proper behaviour. The learners grade their given partner’s test and appropriate points are given for the accurate answers. After the reporting of all the given points, it is expected that the winning team will be announced and celebratory applause or a positive verbal reinforcement given. The winning team must also respectfully congratulate the other team because of their efforts. Finally, the teams and partners have to be altered on the week (Greenwood et al., 1997).

Importance

CWPT is effectively carried out in a manner, which encourages appropriate learner interaction through the utilization of peer tutoring and partner pairing. In this strategy, learners are effectively tutored by peers who are initially tutored to showcase a defined weekly information set, such that; they can make provision for immediate feedback for the accurate and inaccurate responses. Daily lessons will allow each given partner to assume the tutor and tutee roles (Greenwood et al., 1997). Moreover, this strategy utilizes error correction, and immediate-response feedback, in addition to a defined technique of tutoring, which benefits both parties (Maheady, Harper & Mallette, 2003). If accurately structured, then this strategy ensures tutors engage all the given learners actively, while simultaneously ensures an effective monitoring process via assessments and tests carried out in defined periods.

Strategy 8: Direct Instruction (DI)

Introduction

This strategy is a scientifically-based instructional approach, which is mainly effective for learners having disabilities. This strategy utilizes teaching procedures that are detailed, which are relayed in a defined order. The strategy is structured upon the given aspect that every learner can effectively learn if there are careful and that teachers can be relatively successful with appropriate instructional delivery methods (Tarver, 1999). In the DI strategy, the teacher is wholly responsible for the student learning. There are three essential aspects to the overall design and subsequent delivery of the DI strategy programs namely instruction organization, program design, and teacher/student interactions (Martel, 2009).

Utilization

The data relayed in this strategy is scripted carefully prior to the beginning of the lesson. Tutors give defined instruction by utilizing responses that are rapid-fire in combination with immediate correction of errors. Then the learners respond on an average rate of approximately 10 responses per minute. The response can be individual or alternatively, as a group. The strategy is fast paced and its ultimate success is dependent on lesson design, in addition to the action and reactions of the given teacher. Teachers make provision for frequent corrections or positive feedback. Lessons must provide diverse opportunities for learners to effectively exercise the utilization of skills, which are instructed on multiple and diverse occasions. The knowledge tutored has to be repeated over a given period so as to reinforce the overall teaching process (Tarver, 1999). For this strategy, it must be essentially noted that repetition is a significant aspect of concrete and effective learning. DI only requires a small practice period, but relays impressive progress by the given students. Teachers who tend to utilize this teaching strategy, coupled with other relevant ones are more effective (Martel, 2009).

Importance

All information that is taught through the utilization of DI is repeated multiple times for the purpose of ensuring appropriate understanding of pertinent data from given learners. DI promotes instructional formats that are crystal clear, which make provision for certain directives of student and teacher dialogue. The organization of relevant materials has to include certain parts of data on the basis of displayed skill levels. In addition, flexibility of the teacher with respect to timing ensures the given learners perform for considerably extended periods with a seemingly increased success rate.

Moreover, assessment of this given success is measured in continuous fashion (Tarver, 1999).

CHAPTER 4: ASSESSMENT OF STUDENT LEARNING

Introduction

Assessment can take a variety of forms. In a given classroom, assessment of learning is relatively summative (Stein, 1997). Although the middle school students have tackled fractions before, at times, they still identify it as a section of extreme difficulty. This means that a broader approach is required for the assessment. The formative assessment tools have to identify the aspects, which are challenging to conceptualize. Class questions can provide a significantly valuable starting point for any given discussion. Silence is also an appropriate assessment tool. Simply standing back and keenly lending an ear to learners as they explain their thinking might be a very effective assessment tool as compared to the numerous written diagnostic tests. Additionally, it is critical to make creative utilization of summative tests: for instance, students can note down their personal exam-style questions. Through the insertion of their personal figures, or rather, question rewriting, they might become confident at the overall writing of their own new questions from scratch (McLeod & Newmarch, 2006).

Assessment via Questioning

Questioning might be utilized to seek if a certain student has knowledge on the accurate answer to a defined closed question, for example creation of a shape before querying on what fraction is completely colored. Nevertheless, it might be utilized more thoroughly for an appropriate formative assessment, in addition to encouraging enhanced thinking in mathematical terms (Lamon, 2012).

Prior to the introduction of a topic, a few predetermined questions might aid in the identification of what learners already know. Having asked questions, a given teacher has to think of what to make of the student responses. It might be suitable to attempt to analyze what has essentially resulted in an inaccurate answer. It is necessary to be flexible in choosing how just far to follow an inquiry option that was not considered for the given lesson. Querying the students on the reason of performing a certain activity is an appropriate means of uncovering individual thinking techniques. Devil's advocate queries can aid one to view if the students have acquired an appropriate knowledge of a given aspect. Devil's advocate in this case refers to taking the opposing side for argument's sake so as to put the student on the spot and gauge his/her knowledge content. The students might already feel acquainted with querying the teachers, although it is possible to formulate certain scenarios in which they have to query each other. In any given scenario, it is vital to ensure everyone has considerable time to think prior to giving of a response (McLeod & Newmarch, 2006).

Sample Questions on Fractions

Utilization of Mini Whiteboards

"Point out a fraction (always promote the stating in words, utilizing diagrams and numerals). Point out a relatively challenging fraction."

"Point out any query in which the response is six. Then utilize 1/2 in the same query. Then utilize 1/4. Then make a relatively hard query. Point out any query in which the response is 21/2"

“This is half a shape (to be presented on a mini white board), what might the overall shape appear like? Ask the same for 1/5 or 1/4 as deemed necessary” (during the collection of responses on the board, promptly query the students to state the appearance of the shapes as a means of reiterating the notion of equivalent pieces). Now point out a fraction, which is larger than 1/4. Now point out a fraction, which falls within 1 and 1/2, or alternatively, within 1 and 3/4 (Ensure the students to draw pictorial representations and subsequently relay then on number lines).

“Among these fractions, point the odd one out: 1/2, 2/4, 1/4?”

Reasoning Understanding

“Why are 2/6 and 1/3 similar?”

“Why did you alter 9/12 from 3/4? Suppose the comparison was between 3/4 and 5/8: What might you do then?”

Misconceptions and Error Analysis

Some fractions’ misconceptions might be clearly apparent. Nevertheless, it is quite essential to seek approaches to uncover the thinking procedure of students and viewing of discrepancies, which may be hidden. Sometimes students appear to offer an accurate answer, but their overall reasoning techniques fails (Lamon, 2012). It is essential to ensure students to attempt to realize their ideas, regardless of whether their basis is on misconceptions or not because it is only when they are revealed that they can be addressed effectively.

Some students might identify the figure below as one, which relays 1/3 since they view it one shaded rectangles over three un-shaded rectangles (Lamon, 2012, p.4)

Some students view some fractions such as 1/4 and 1/3 as being interchangeable probably since the terms ‘third’ and ‘quarter’ fail to imply the numerals 3 and 4. It might be viable to check out to find out if the given students have such misunderstandings. Normally, diagrams representing common fractions might be utilized limitedly. A number of traditionally utilized fractions often tend to be relayed in specific shapes. Thus the students might have seen 1/4 showcased in a square, but the fraction 2/3 only showcased in a circle. Images and diagrams must be utilized in a careful manner so as to avoid confusing students and to subsequently prompting them to make relatively suitable comparisons. Vivid fractions’ representations on standard number lines, in addition to properly reinforcing their decimal equivalents and size are also significant (McLeod & Newmarch, 2006).

Even when similar shapes have been utilized to aid comparison, students might find it challenging to reconcile anything that they view with the manner in which they viewed the different numerals in a single fraction. Some students assume that even though a visual representation might portray that 1/8 is much larger as compared to 1/16, in their minds, the value 16 tends to take over and attempts to confuse them that the fraction is a relatively larger number. Considering two whole numerals that are positive, it is quite obvious which among them is bigger. With 1/8 and 1/16 this might also be obvious. Nevertheless, disregarding unit fractions, this ‘rule’ might not necessarily be applicable. For example, 3/8 is quite larger as compared to 5/16; however, 3/16 is quite larger as compared to 1/8. Students attempting to utilize the top numerals and bottom numerals size as the overall sign of comparison might have a diverse view of the aspect now.

Common Mistakes Examples

2/5: assuming the numerals in the fraction are two unrelated numbers

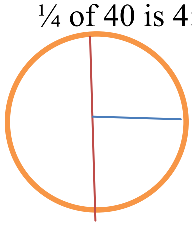
1/3 + ¼ = 2/7: treating fractions in a similar manner as whole numerals

1/5 = 1.5: being greatly influenced by appearances

2/5 > ½ since 5 > 2: assuming that the only the denominator determines the fraction size

½ + ½ = 2/4: having the inability to judge when an answer does not actually make sense

¼ of 40 is 4: having inadequate fractions' experience as parts of sets of objects



One portion in this figure represents 1/3: Inaccurate partitioning of fractional area (McLeod & Newmarch, 2006, p. 3)

Evaluation of Fractions' Statements

Students are offered some generalizations concerning fractions, probably showcased on different cards, before being prompted to state if they consider them to be 'sometimes', 'never' true, or 'always', before justifying the selections, with proper examples and relatively strong reasoning. The relayed responses might knowingly entail traditional misconceptions. Such an exercise has to be discussion that is significantly in-depth, and is initially in small groups or pairs, and finally with the entire group. Then all students in turn must select a single statement. Others must be prompted to argue, challenge, and redefine their given reasoning. Once a verdict has been agreed, they might create a banner in the class setting for display. They might even consider suitable statements for other students to think of in a similar way (Cangelosi, 2002).

An example is relayed below:

a) A conventional fraction is a relatively small piece of a defined whole
b) When one multiplies a given numeral by another numeral, the answer has to be bigger at all times
c) It is impossible for a fraction to be greater than one
d) A fraction can be relayed in diverse ways
e) Fractions and decimals are diverse forms of numbers
f) A fraction can be displayed as a decimal and vice versa (McLeod & Newmarch, 2006, p.4)

Links outside the Classroom Setting

Past experience proves that, most students fail to utilize their knowledge of fractions in the outside context of the classroom. They are also unable or unwilling to apply it to certain ‘real life’ challenges (Egan, 1992). For instance, a student might accurately perform the problem of 1/4 of 40 million, but in reality had no clue on how to find the answer for 2/4 of40 million. It might also be appropriate to take an advert or headline and create a banner that reveals its meaning. For instance, if a certain headline states that one in three 12-year-olds smokes, a suitable question might be, what fraction does this relay? Moreover, what quantity of students does the value relay in any learning institution? To link with this given problem, the students have to be encouraged to consider the 12-year-olds who have not smoked.

Another exercise might entail the fact that a healthy diet has to include less than 60g of fat daily. Food labels and other relevant nutritional information can be collected before finding out what accurate fraction of the actual daily fat allowance is found in a standard bag of crisps. By utilizing a 60 square grid, and by rounding, students might accurately calculate the given fraction of their expected limit in diverse foods, and subsequently relay this by utilizing equal fractions. For instance, a packet of crisps has around 1/6 of the recommended daily amount.

Classifying Fractions

The classification of diverse of fractions' representations, or even basic statements concerning them might be a relatively efficient means of encouraging a student to reflect on and effectively analyze their given aspects. Students might select diverse representations of the same fraction: in words, in a pictorial representation, on a standard numeral line, or as a basic decimal. They might be classified in accordance to their size, or alternatively statements concerning fractions can be classified as either false or true. This is a clear approach, which results in proper differentiation in the given group, with the students utilizing diverse classifications with respect to previous experience of the lessons learnt on fractions. One probable activity might entail all students in a given group to be offered cards representing fraction and, through working in pairs, accurately sort them into diverse groups. This might essentially ignite a discussion regarding whether all the fractions having denominators 3 and 2 might be put on this given grid. Alternatively, they might select their own fractions, before writing their own cards, so as to go in each defined category (McLeod & Newmarch, 2006).

As a follow-up, the student might select their personal groups for the purposes of classification. Moreover, to the defined fractional categories, it is quite interesting to prompt the given students to include selections such as ‘simple and ‘challenging before vividly discussing the reason why some standard fractions seem to be more challenging as compared to others. Much more difficult problems might be devised for others via including the given fractions in spaces on the defined classification grid such that other given students must accurately guess the utilized headers.

Interpreting Fractions

Considering their diverse representations, in addition to the manner in which they at times refer to a given numeral or an operation, it is essential to be able to appropriately talk about fractions with respect to the diverse appearance selections. A basic multiple representation exercise, inclusive of diverse visual and numerical representations, is one effective way of doing this.

Sample Notes

a) Sharing of food stuff is an appropriate mean of introduce diverse aspects concerning fractions. For instance, utilizing a candy bar and dividing it into several pieces.

b) A tape measure might be a valuable means of illustration for diverse forms of fractions. For instance, students can write on 50 cm, 0.5 m, and ½ for their individual portable chart

c) A clock face accurately displays what quarters and halves appear like, and this might be extended to other given fractions with crucial discussions as to why some of them are much easier to display as compared to others (McLeod & Newmarch, 2006, p. 5)

It is recommended that teachers vividly utilize the fractions' language in other sections of the defined curriculum for the purpose of reinforcement. For instance, when examining diverse shapes, one might consider a 'third of a circle' or 'half a square'. One might also consider a scenario when a certain fraction talk is not actually appropriate, for instance, "it is half as hot as compared to yesterday" does not really make sense.

Dealing with Remainders

Students can sometimes be quite confused concerning what to do with the given remainders after any division operation. They might be unsure when to ignore the given remainders, when to round them up, or alternatively to show whatever is left over as a standard fraction or alternatively as a decimal. Diverse decisions have to be made with regards with regards to the interpretation of the given remainder, and what actually makes sense, with respect to the context given (Davidson, 1990). Students must to explore questions in a diverse situation range, whereby, all involve a similar division calculation, for example, a query that entails the division of 10 by 4, with regards to the defined context, might have as a relatively simple solution 3, 2, 2.50, 21/2 or alternatively as "2 remainder 2".

Sample

a) One needs to purchase pens with each costing $4. If he/she has $10, how many can be bought?

b) Twelve individuals plan to travel by a given car from the city centre to the train station. The can can only accommodate five passengers. How many cars will be required?

c) You are tasked with sharing $20 between three individuals. How much will each receive?

d) If one has 12 chocolate bars and shares them equally among 5 individuals, what is the quantity that each will receive if the bars are not split? What if they are split?

(McLeod & Newmarch, 2006, p. 6)

Students can practice accurately posting their personal contexts' sets for diverse division queries, which will entail diverse decisions on how to deal with the given remainder.

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